Growth Spline Modeling

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Introduction

- Interest in the influence of individual differences when investigating the progression of a quantity over time is ubiquitous across many research disciplines.
  - Consumer differences on product lifecycle
  - Patient factors on dose response curves
- The goal is to go beyond examining influences on the overall, average response pattern as differences among increasingly minute groups of patterns can provide additional insights.
- It may be desirable to model change at specific time points (e.g., time-varying treatments).
- Growth models allow us to shift focus to increasingly individual level analyses.
- Spline models allow a more appropriate fit and allow change to be modeled at points specified by the researcher.
- Therefore, we describe an extensible, hybrid statistical approach comprised of spline modeling and growth modeling which allows an examination of dynamic antecedent-outcome relationships while properly controlling for past effects.

Data Requirements

- This family of analyses focuses on examining influences on patterns.
- The simplest pattern is a line between two points.
- Therefore, the elemental child of this family consists of two linear trends and requires three longitudinal observations per subject.
- Of course, additional longitudinal observations are always desirable
- Therefore, this family of analyses is better suited for data sets containing an increased number of longitudinal observations, one or more covariates, and a response pattern at least nominally composed of segments (e.g., time-varying treatments, phases, states, or steps).

Example Data Set Background

- Extant skill acquisition theory posits that both the relative and absolute contributions of abilities to skill acquisition change through time.
- Previous tests of theory inadequately controlled for past acquisition.
- Participants were trained on a complex and dynamic computer-based task.
- Four performance observations are used to model skill acquisition using three additive, linear acquisition trends.
- The influence on each trend of a time-invariant, individual-level ability covariate (i.e., general mental ability), which was standardized to aide in model interpretation, is modeled.

Figure 1. A Basic Hypothetical Example of Additive Splines

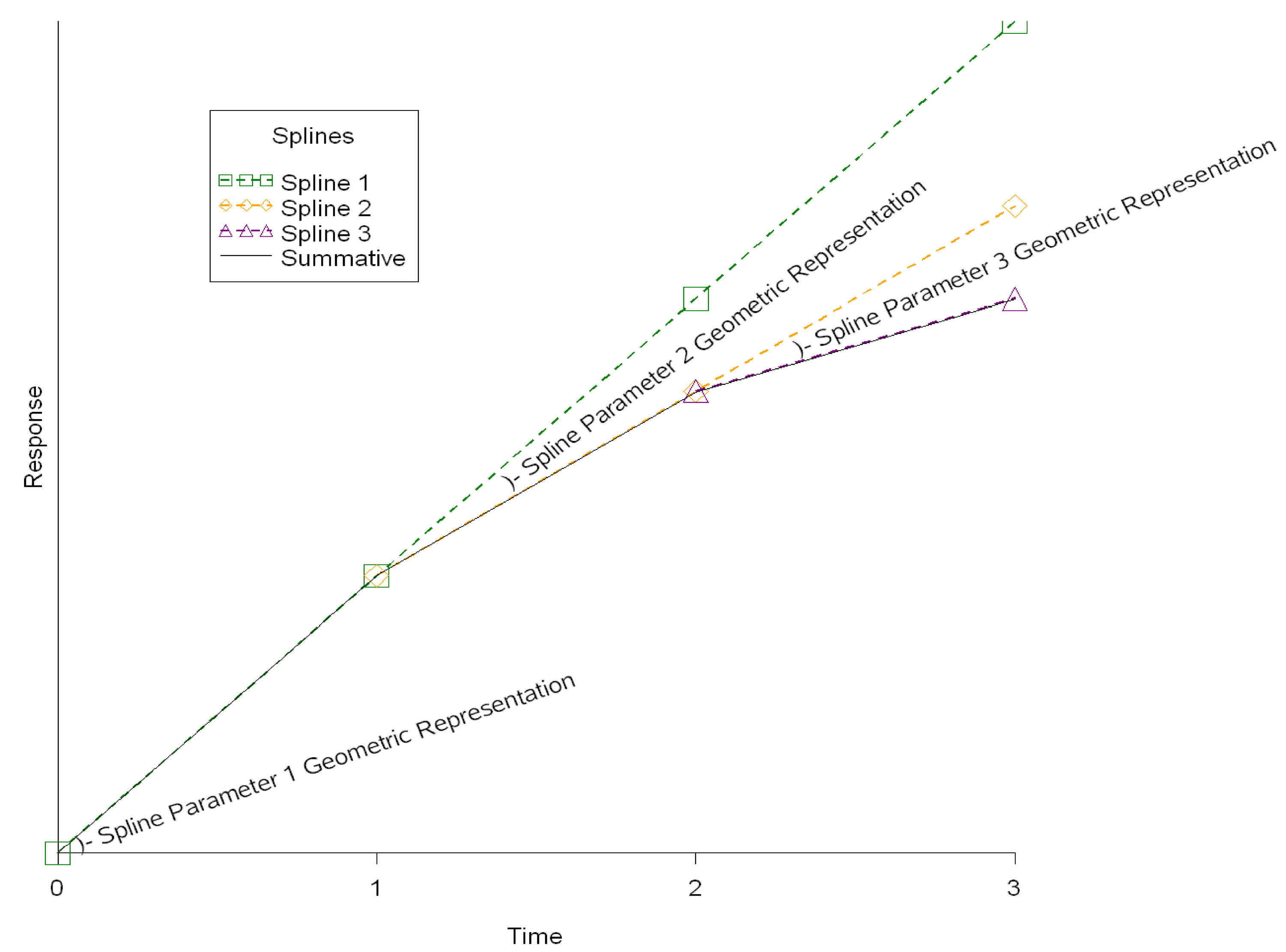


Table 1. Coding and Interpretation of Hypothetical Spline Variables

	Measurement Occasion				
Variable	1	2	3	4	Interpretation
Spline 1	0	1	2	3	Underlying linear change (e.g., base acquisition rate)
Spline 2	0	0	1	2	Linear deviation to the underlying linear change (e.g., change from base acquisition starting in second time period)
Spline 3	0	0	0	1	Deviation to the linear deviation to the underlying linear change (e.g., change in acquisition rate starting in the third time period)

Figure 2.Predicted Performance

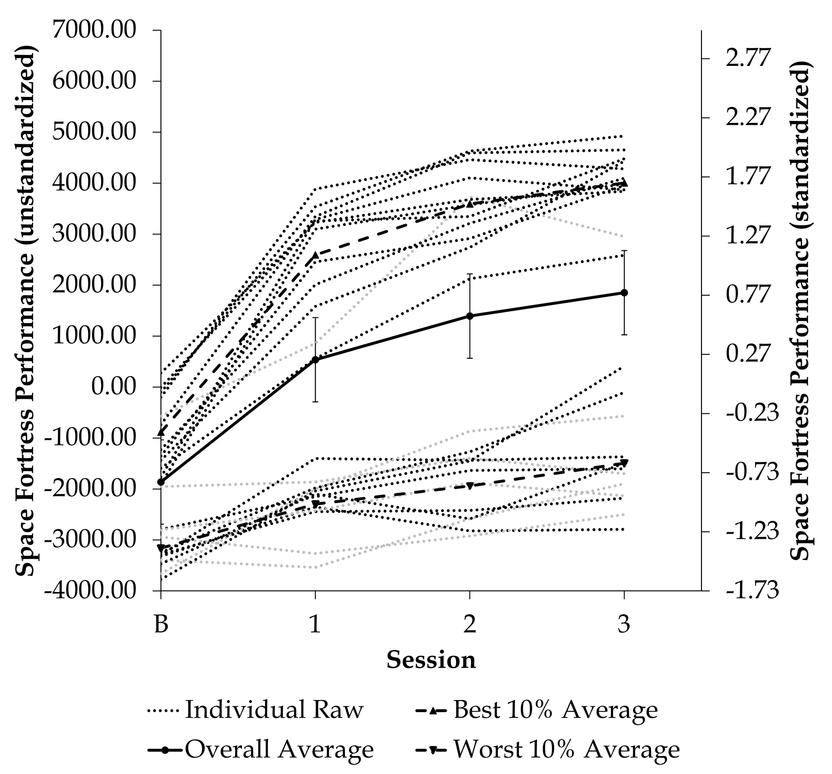
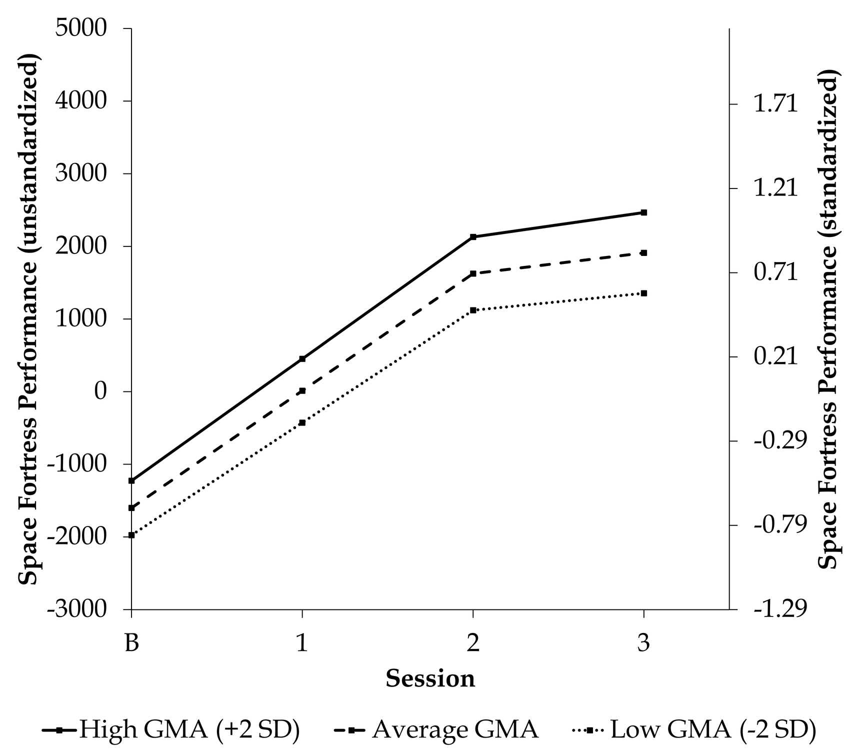


Figure 3. Performance As a Function of Ability



Model

$$Y_{ij} = \pi_{0j} + \pi_{1j}(TIME)_{ij} + \pi_{2j}D_{1i}(TIME - TIME_1)_{ij} + \pi_{3j}D_{2i}(TIME - TIME_2)_{ij} + r_{ij}$$

where $r_{ij} \sim N(0, \sigma^2)$

and

$$\pi_{0j} = \beta_{00} + \beta_{01}(z_{COVAR}) + u_{0j}$$
$$\pi_{1j} = \beta_{10} + \beta_{11}(z_{COVAR}) + u_{1j}$$
$$\pi_{2j} = \beta_{20} + \beta_{21}(z_{COVAR}) + u_{2j}$$
$$\pi_{3j} = \beta_{30} + \beta_{31}(z_{COVAR}) + u_{3j}$$

where

$$\begin{pmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \\ u_{3j} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} & \tau_{02} & \tau_{03} \\ \tau_{10} & \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{20} & \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{30} & \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}\right]$$

Model Explanation

- Although there are many ways to code time in longitudinal models, time is coded here as 0, 1, 2, 3.
  - The intercept estimates the value of the outcome variable, in this case performance (i.e., skill attainment), at occasion 0 (i.e., initial status).
  - The slopes estimate rate of change in the outcome across occasions, in this case skill acquisition.
- Y_{ij} is an individual's response at a given time point.
- π_{0j} is an intercept coded as response at origin.
- π_{1j} represents the initial spline and the underlying linear trend, conditional on the other splines, throughout the response period.
- $TIME_1$ and $TIME_2$ variables are set times since the origin denoting spline starting points (e.g., knots).
- The *D* variables (i.e., D_{1i} and D_{2i}) are dummy variables equal to 0 when the amount of time from the origin, is less than $TIME_1$ and $TIME_2$ respectively, and equal to 1 when *TIME* is greater than i_1 and i_2 respectively.
- The remaining parameters (π_{2j} and π_{3j}) become summative deviations to the underlying trend when enough time passes according to the *D* variables.
- Individually the splines represent response in a particular segment of time while controlling for past effects, but collectively they can capture the nature of the overall response trend. When applied to data with a decelerating logarithmic response pattern as in the current example, the resulting model is akin to that represented in Figure 1.

Conclusion

The possibilities are endless.

Relevant SAS Code

```
proc mixed noclprint covtest;
class id;
model y = scovar spline1 spline2 spline3
          scovar*spline1 scovar*spline2 scovar*spline3/solution ddfm=bw notest;
random intercept spline1 spline2 spline3/subject=id type=un;
```

Output

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	22988.96993683	
1	1	21016.85219819	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	id	588779	105807	5.56	<.0001
UN(2,1)	id	228106	54421	4.19	<.0001
UN(2,2)	id	297471	54852	5.42	<.0001
UN(3,1)	id	–264216	69585	–3.80	0.0001
UN(3,2)	id	–289927	67092	–4.32	<.0001
UN(3,3)	id	331785	91639	3.62	0.0001
UN(4,1)	id	34109	37747	0.90	0.3662
UN(4,2)	id	–44341	27565	–1.61	0.1077
UN(4,3)	id	2410.34	40355	0.06	0.9524
UN(4,4)	id	51734	28354	1.82	0.0340
Residual		307539	15513	19.82	<.0001

Fit Statistics

-2 Res Log Likelihood	21016.9
AIC (smaller is better)	21038.9
AICC (smaller is better)	21039.1
BIC (smaller is better)	21070.5

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
10	1972.12	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	–1598.83	80.2414	129	–19.93	<.0001
ccovar	402.17	80.5494	129	4.99	<.0001
spline1	1606.33	57.7497	1173	27.82	<.0001
spline2	–1284.13	74.0876	1173	–17.33	<.0001
spline3	–76.3486	40.7541	1173	–1.87	0.0613
scovar*spline1	146.06	57.9714	1173	2.52	0.0119
scovar*spline2	–118.73	74.3720	1173	–1.60	0.1107
scovar*spline3	–14.8630	40.9105	1173	–0.36	0.7164

Interpretation

- Random Effects
  - There appears to be significant variance between people in their initial skill attainment, τ_{00} = 588,779, z = 5.56, p < .001, base skill acquisition, τ_{11} = 297,471, z = 5.42, p < .001, linear deviation from base acquisition, τ_{22} = 331,785, z = 3.62, p < .001, and final deviation from previously established acquisition, τ_{33} = 51,734, z = 19.82, p < .001, which might all be predicted with additional covariates.

- Fixed Effects
  - The average individual has an initial skill attainment score of $ß_{00}$ = –1598.83, t(129) = –19.93, p < .001, and starts acquiring skill at a rate of $ß_{10}$ = 1606.33, t(1173) = 27.82, p < .001, per testing occasion. The average individual then experiences an acquisition deceleration of $ß_{20}$ = –1284.13, t(1173) = –17.33, p < .001, per testing occasion and then another deceleration of $ß_{30}$ = –76.35, t(1173) = –1.87, p < .10. With respect to the covariate, individuals who differ by a standard deviation in general mental ability have performance scores which differ by $ß_{01}$ = 402.17, t(129) = 4.99, p < .001, points on average and have initial skill acquisition rates which differ by $ß_{11}$ = 146.06, t(1173) = 2.52, p < .05, points on average. However, general mental ability appears to have little effect on later acquisition decelerations, $ß_{21}$ = –118.73, t(1173) = –1.60, p = .11, and $ß_{31}$ = –14.86, t(1173) = –0.36, p < .72.

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