

2ⁿ CONFOUNDED FACTORIAL

Confounded factorial is a design technique for arranging a complete factorial experiment in block, Where the block size is smaller than the number of treatment combinations in a full factorial design.

•To construct 2ⁿ factorial design confounded in 2ⁿp blocks

•Select p independent effects to be confound with block.

•There will be 2ⁿp – p – 1 other effects that called generalized interactions also confounded with blocks.

2ⁿ PARTIALLY CONFOUNDED FACTORIAL DESIGN IN CONJOINT CHOICE EXPERIMENT

In this study, 2ⁿ partial confounded factorial designs for CCE will be developed to

1. avoid confounding main effects and lower order interactions with blocks if possible

2. to identify choice sets for which no two alternatives in a choice set have the same levels of a factor.

The solutions are:

1. p independent effects must be higher order interaction (higher order > 3)

2. p independent effects must be even order interaction.

Since some of the effects will be confounded with the blocks in confounded factorial design, the conjoint choice designs were extended to the use of partially confounded factorial designs, which are called as Partially Confounded Factorial Conjoint Choice Experiment (PCFCCE) designs. In this method, there will be at least two replicates and each replicate does not share the same confounded effects.

ANALYSIS FOR PARTIALLY CONFOUNDED FACTORIAL CONJOINT CHOICE EXPERIMENTS

Conjoint choice experiment start with the assumption that an individual has utility u_i for a choice alternative i which is divided into two component, a deterministic component (v_i) and random component (ε_i)

$$u_i = v_i + \varepsilon_i \quad i = 1, 2, \dots, J$$

The random component are assumed to be independent with a Gumbel distribution, the multinomial logit model follows. It can be shown that for an alternative i from a set of J alternative, the probability that alternative i is chosen is:

$$P_i = \frac{e^{v_i}}{\sum_{j=1}^J e^{v_j}}$$

The form of the PCFCCE models is

$$y_{iscj} \sim \text{indep. multinomial}(\pi_1, \pi_2, \pi_3)$$

$$f_{y_{iscj}}(y_{iscj}) = \prod_{i=1}^{16} \prod_{s=1}^S \prod_{c=1}^{16} \prod_{j=1}^3 \pi_{iscj}^{y_{iscj}}$$

The density function can be modified

$$f_{y_{iscj}}(y_{iscj}) = \prod_{i=1}^{16} \prod_{s=1}^S \prod_{c=1}^{16} \prod_{j=1}^3 \pi_{iscj}^{y_{iscj}} \left\{ \exp \left[\sum_{j=1}^2 y_{iscj} \log \left(\frac{\pi_{iscj}}{\pi_{isc3}} \right) + \log(\pi_{isc3}) \right] \right\}$$

$$\text{where } \theta_{iscj} = \log \left(\frac{\pi_{iscj}}{\pi_{isc3}} \right), b(\theta_{iscj}) = \log(1 + \sum_{j=1}^2 \exp(\theta_j)), a(\phi) = 1.$$

$$\text{Let } \pi_{iscj} = E[Y_{iscj}], \text{ then } g(\pi_{iscj}) = \log \left(\frac{\pi_{iscj}}{\pi_{isc3}} \right) = \mathbf{x}_{iscj}'\beta.$$

$$f_{y_{iscj}}(y_{iscj}) = \prod_{i=1}^{16} \prod_{s=1}^S \prod_{c=1}^{16} \exp \left[\sum_{j=1}^2 y_{iscj} \mathbf{x}_{iscj}'\beta - \log(1 + \sum_{j=1}^2 \exp(\mathbf{x}_{iscj}'\beta)) \right]$$

- Example**
- To construct 2⁸ factorial design confounded in 2³ blocks.
 - Need to select 3 independent effects to confound with block.
 - There will be 2³ – 3 – 1 = 4 generalized interactions also confounded with block.

Work

- 2⁸ design (256 treatment combinations) with 8 factors (A, B, C, D, E, F, G & H).
- Select effects ABCDEF, DEFG & CDEH to be confound with blocks.
- 3 effects that selected to be confound with the block, can be represented by a linear combination. The linear combinations for the ABCDEF(L1), DEFG(L2) & CDEH(L3).
L1 = x1 + x2 + x3 + x4 + x5 + x6
L2 = x4 + x5 + x6 + x7
L3 = x3 + x4 + x5 + x8
ri = Li (mod 2)
- There are 8 combinations of possible residues (r1, r2, r3): (0,0,0), (1,0,0), (0,1,0), (1,1,0), (0,0,1), (0,1,1), (1,1,1).

Application of using 2ⁿ partial confounded factorial design used in the conjoint choice experiment was constructed to study consumer's preferences about Tablet for University Students.

In this study, eight attributes of tablet with two level each were used to evaluate consumer's purchasing decisions toward tablet by using PCFCCE. The associated attributes were price (factor A), 3G (factor B), warranty (factor C), internal memory (factor D), flexibility (factor E), life of battery (factor F), quality of camera (factor G) and Ram (factor H).

Attributes	Low level	High level
1 Price	RM 2500	RM 1750
2 3G	No	Yes
3 Warranty	1 year	2 year
4 Memory	32GB	64GB
5 Flexibility	No	Yes
6 Battery	8 hours	10 hours
7 Camera	3-Megapixel	5-Megapixel
8 Ram	512MB	1GB

Independent effect vs generalized interaction

Selected effects

Generalized interaction

Selected effects

Generalized interaction

The log likelihood is nonlinear and hence the Newton-Raphson method is needed to estimate the parameters.

$$\beta^{(k)} = \beta^{(0)} + [-(\ell''(\beta^{(0)}))]^{-1} \cdot \ell'(\beta^{(0)}) \\ = \beta^{(0)} + (X^T W X)^{-1} \cdot X^T (Y - \mu)$$

To illustrate the iterative procedure of Newton-Raphson as it applies to the multinomial logistic regression model, we need an expression for the first and second derivative of β .

Example

Survey form	Option A	Option B	Option C	Survey form	Option A	Option B	Option C
3	Option A	Option B	Option C	35	Option A	Option B	Option C
Price	1800-2500	1500-2200	1500-2200	Price	1800-2500	1500-2200	1500-2200
3G	Y	N	Neither	3G	Y	N	Neither
Warranty	2year	1year	or	Warranty	2year	1year	or
Memory	32GB	64GB	or	Memory	32GB	64GB	or
Flexible	Y	N	Option B	Flexible	Y	N	Option B
Battery	8hour	10hour	or	Battery	8hour	10hour	or
Camera	3-Megapixel	5-Megapixel	or	Camera	3-Megapixel	5-Megapixel	or
RAM	500MB	1GB	or	RAM	500MB	1GB	or
Choice	Y			Choice	Y		

$$y_{ij} \sim \text{indep. Multinomial}(\pi_1, \pi_2, \pi_3)$$

$$f_{y_{ij}}(y_{ij}) = \prod_{i=1}^n \exp[y_{i1}x'_{i1}\beta + y_{i2}x'_{i2}\beta - \log(1 + \exp(x'_{i1}\beta) + \exp(x'_{i2}\beta))]$$

$$L(\beta) = \sum_{i=1}^n [y_{i1}x'_{i1}\beta + y_{i2}x'_{i2}\beta - \log(1 + \exp(x'_{i1}\beta) + \exp(x'_{i2}\beta))] \\ = [x'_{i1}\beta - \log(1 + \exp(x'_{i1}\beta) + \exp(x'_{i2}\beta))] + [x'_{i2}\beta - \log(1 + \exp(x'_{i1}\beta) + \exp(x'_{i2}\beta))] + [-\log(1 + \exp(x'_{i1}\beta) + \exp(x'_{i2}\beta))]$$

$$\ell'(\beta) = \frac{\partial L(\beta)}{\partial \beta}$$

$$= X^T (y - \mu)$$

$$= \begin{bmatrix} x_{11} & x_{12} & x_{21} & x_{22} & x_{31} & x_{32} \end{bmatrix} * \begin{bmatrix} y_{11} - \mu_{11} \\ y_{12} - \mu_{12} \\ y_{32} - \mu_{32} \end{bmatrix}$$

$$\ell''(\beta) = \frac{\partial^2 L(\beta)}{\partial \beta^2}$$

$$= (X^T W X)^{-1}$$

$$W =$$

$$\begin{bmatrix} \pi_{11}(1 - \pi_{11}) & -\pi_{11}\pi_{12} & 0 & 0 & 0 & 0 \\ -\pi_{11}\pi_{12} & \pi_{12}(1 - \pi_{12}) & 0 & 0 & 0 & 0 \\ 0 & 0 & \pi_{31}(1 - \pi_{31}) & -\pi_{31}\pi_{32} & 0 & 0 \\ 0 & 0 & -\pi_{31}\pi_{32} & \pi_{32}(1 - \pi_{32}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{31}(1 - \pi_{31}) & -\pi_{31}\pi_{32} \\ 0 & 0 & 0 & 0 & -\pi_{31}\pi_{32} & \pi_{32}(1 - \pi_{32}) \end{bmatrix}$$

$$\beta^{(0)} = \beta^{(0)} + (X^T W X)^{-1} X^T (Y - \mu)$$

The new value of $\beta^{(0)}$ is the next approximation for the root. We let $\beta^{(0)} = \beta^{(1)}$ and continue in the same manner to generate $\beta^{(1)}, \beta^{(2)}, \dots$, until successive approximations converge.

	[0, 0, 0]	[1, 0, 0]	[0, 1, 0]	[1, 1, 0]	[0, 0, 1]	[1, 0, 1]	[0, 1, 1]	[1, 1, 1]
→	11111111	11111100	11111101	11111101	11111100	11111101	11111101	11111100
	11111100	11111101	11111100	11111100	11111101	11111101	11111100	11111101
	11111101	11111100	11111101	11111101	11111100	11111100	11111101	11111100
	11011011	11011010	11011001	11011011	11011000	11011010	11011010	11011000
	11011010	11011001	11011010	11011000	11011011	11011000	11011011	11011001
	11001101	11001100	11001101	11001100	11001101	11001101	11001100	11001101
	11001100	11001101	11001100	11001100	11001101	11001100	11001101	11001100
	11000011	11000010	11000011	11000010	11000000	11000010	11000010	11000000
	10111001	10111000	10111001	10111000	10111001	10111001	10111000	10111001
	10111000	10111001	10111000	10111000	10111001	10111000	10111001	10111000
	10101101	10101100	10101101	10101100	10101101	10101101	10101100	10101101
	10101100	10101101	10101100	10101100	10101101	10101100	10101101	10101100
	10100101	10100100	10100101	10100100	10100101	10100101	10100100	10100101
	10100100	10100101	10100100	10100100	10100101	10100100	10100101	10100100
	10011101	10011100	10011101	10011100	10011101	10011101	10011100	10011101
	10011100	10011101	10011100	10011100	10011101	10011100	10011101	10011100
	10001101	10001100	10001101	10001100	10001101	10001101	10001100	10001101
	10001100	10001101	10001100	10001100	10001101	10001100	10001101	10001100
	10000011	10000010	10000011	10000010	10000000	10000010	10000010	10000000
	11111011	11111010	11111011	11111010	11111011	11111011	11111010	11111011
	11111010	11111011	11111010	11111010	11111011	11111010	11111011	11111010
	11101101	11101100	11101101	11101100	11101101	11101101	11101100	11101101
	11101100	11101101	11101100	11101100	11101101	11101100	11101101	11101100
	11100101	11100100	11100101	11100100	11100101	11100101	11100100	11100101
	11100100	11100101	11100100	11100100	11100101	11100100	11100101	11100100
	11100011	11100010	11100011	11100010	11100000	11100010	11100010	11100000
	11111101	11111100	11111101	11111100	11111101	11111101	11111100	11111101
	11111100	11111101	11111100	11111100	11111101	11111100	11111101	11111100
	11101101	11101100	11101101	11101100	11101101	11101101	11101100	11101101
	11101100	11101101	11101100	11101100	11101101	11101100	11101101	11101100
	11100101	11100100	11100101	11100100	11100101	11100101	11100100	11100101
	11100100	11100101	11100100	11100100	11100101	11100100	11100101	11100100
	11100011	11100010	11100011	11100010	11100000	11100010	11100010	11100000
	11111111	11111110	11111111	11111110	11111111	11111111	11111110	11111111
	11111110	11111111	11111110	11111110	11111111	11111110	11111111	11111110
	11101111	11101110	11101111	11101110	11101111	11101111	11101110	11101111
	11101110	11101111	11101110	11101110	11101111	11101110	11101111	11101110
	11100111	11100110	11100111	11100110	11100111	11100111	11100110	11100111
	11100110	11100111	11100110	11100110	11100111	11100110	11100111	11100110
	11100011	11100010	11100011	11100010	11100000	11100010	11100010	11100000
	11111111	11111110	11111111	11111110	11111111	11111111	11111110	11111111
	11111110	11111111	11111110	11111110	11111111	11111110	11111111	11111110
	11101111	11101110	11101111	11101110	11101111	11101111	11101110	11101111
	11101110	11101111	11101110	11101110	11101111	11101110	11101111	11101110
	11100111	11100110	11100111	11100110	11100111	11100111	11100110	11100111
	11100110	11100111	11100110	11100110	11100111	11100110	11100111	11100110
	11100011	11100010	11100011	11100010	11100000	11100010	11100010	11100000
	11111111	11111110	11111111	11111110	11111111	11111111	11111110	11111111
	11111110	11111111	11111110	11111110	11111111	11111110	11111111	11111110
	11101111	11101110	11101111	11101110	11101111	11101111	11101110	11101111
	11101110	11101111	11101110	11101110	11101111	11101110	11101111	11101110
	11100111	11100110	11100111	11100110	11100111	11100111	11100110	11100111
	11100110	11100111	11100110	11100110	11100111	11100110	11100111	11100110
	11100011	11100010	11100011	11100010	11100000	11100010	11100010	11100000

effect	betahat	secovbha	t.ratio	p.value
Intercept	0.92689	0.03507	26.42762	0.00000
Price	0.11397	0.01416	8.05111	0.00000
3G	0.08731	0.01415	6.17202	0.00000
Warranty	0.12285	0.01415	8.68283	0.00000
Memory	0.12059	0.01416	8.51771	0.00000
Flexibility	-0.07838	0.01415	-5.53956	0.00000
Battery	0.08832	0.01416	6.23712	0.00000
Camera	0.09579	0.01415	6.76905	0.00000
Ram	-0.11192	0.01416	-7.90222	0.00000
Price 3G	0.19353	0.03441	5.62402	0.00000
Price Warranty	0.08812	0.03443	2.55915	0.01052
Price Memory	0.01583	0.03437	0.46072	0.64502
Price Flexibility	-0.07190	0.03460	-2.07821	0.03773
Price Battery	-0.08420	0.03434	-2.45198	0.01424
Price Camera	0.05000	0.03435	1.45529	0.14564
Price Ram	-0.02352	0.03426	-0.68668	0.49231
3G Warranty	0.06080	0.03440	1.76720	0.07724
3G Memory	0.02052	0.03430	0.59815	0.54976
3G Flexibility	-0.10804	0.03446	-3.13516	0.00173
3G Battery	-0.01656	0.03429	-0.48360	0.62869
3G Camera	-0.00136	0.03430	-0.03954	0.96847
3G Ram	-0.02493	0.03419	-0.72908	0.46598
Warranty Memory	0.09663	0.03404	2.83833	0.00455
Warranty Flexibility	-0.07001	0.03432	-2.04022	0.04137
Warranty Battery	0.00540	0.03413	0.15816	0.87433
Warranty Camera	0.01055	0.03411	0.30939	0.75704
Warranty Ram	-0.01479	0.03400	-0.43485	0.66368
Memory Flexibility	0.01308	0.03424	0.38208	0.70242
Memory Battery	0.00248	0.03403	0.07288	0.94190
Memory Camera	0.04090	0.03399	1.20329	0.22891
Memory Ram	-0.08077	0.03387	-2.38458	0.01713
Flexibility Battery	0.13209	0.03416	3.86670	0.00011
Flexibility Camera	-0.10305	0.03417	-3.01543	0.00258
Flexibility Ram	-0.02435	0.03412	-0.71363	0.47548
Battery Camera	-0.06257	0.03403	-1.83881	0.06599
Battery Ram	0.00895	0.03393	0.26383	0.79192
Camera Ram	-0.02396	0.03392	-0.70637	0.47998

Effects	Estimates	Least Square Mean	Odds	Odds Ratio
Intercept	0.926886679			
Price	0.113965147	1.040851828	2.811628042	1.255997772
High level	-0.113965147	0.812925152	2.254484824	
3G	0.087314743	1.014205422	2.75716071	1.190804925
High level	-0.087314743	0.839572336	2.315375636	
Warranty	0.122854432	1.049741111	3.85691399	1.78527294
High level	-0.122854432	0.804032247	2.234532975	
Memory	0.120592269	1.047478948	2.850455905	1.27275885
High level	-0.120592269	0.80626441	2.295935376	
Flexibility	-0.078379526	0.848507153	2.316154723	0.854910023
High level	0.078379526	1.095266205	2.712634617	
Battery	0.088317865	1.015206404	2.759926446	1.191935175
High level	-0.088317865	0.838569146	2.31305352	
Camera	0.095793106	1.022679785	2.780636296	1.211169246
High level	-0.095793106	0.831093573	2.255628024	
Ram	0.111922628	1.038893007	2.424562029	1.250877439
High level	-0.111922628	0.814968051	2.259094459	

Interaction effects	Estimates	Least Square Mean	Odds	Odds Ratio
Price	1 Warranty	1	1.251828107	3.498729534
Price	0 Warranty	0	0.829875545	2.29330334
Price	0 Warranty	1	0.847054124	2.334164742
Price	0 Warranty	0	0.778189495	2.177225083
Price	1 Flexibility	1	0.89057188	2.43453279
Price	1 Flexibility	0	1.191129782	3.290796991
Price	0 Flexibility	1	0.71288461	0.856440434
Price	0 Flexibility	0	0.71288461	0.856440434
Price	1 Battery	1	0.84195327	1.044973864
Price	1 Battery	0	0.84195327	1.044973864
Price	0 Battery	1	0.84195327	1.044973864
Price	0 Battery	0	0.84195327	1.044973864
3G	1 Flexibility	1	0.10807335	0.827780461
3G	1 Flexibility	0	0.10807335	0.827780461
3G	0 Flexibility	1	0.10807335	0.827780461
3G	0 Flexibility	0	0.10807335	0.827780461
Warranty	1 Memory	1	0.096626246	0.82522596
Warranty	1 Memory	0	0.096626246	0.82522596
Warranty	0 Memory	1	0.096626246	0.82522596
Warranty	0 Memory	0	0.096626246	0.82522596
Memory	1 Flexibility	1	0.070013099	0.901344848
Memory	1 Flexibility	0	0.070013099	0.901344848
Memory	0 Flexibility	1	0.070013099	0.901344848
Memory	0 Flexibility	0	0.070013099	0.901344848
Ram	1 Flexibility	1	0.08077351	0.988902548
Ram	1 Flexibility	0	0.08077351	0.988902548
Ram	0 Flexibility	1	0.08077351	0.988902548
Ram	0 Flexibility	0	0.08077351	0.988902548
Battery	1 Flexibility	1	0.110935008	1.068918034
Battery	1 Flexibility	0	0.110935008	1.068918034
Battery	0 Flexibility	1	0.110935008	1.068918034
Battery	0 Flexibility	0	0.110935008	1.068918034
Camera	1 Flexibility	1	0.103048133	0.841252129
Camera	1 Flexibility	0	0.103048133	0.841252129
Camera	0 Flexibility	1	0.103048133	0.841252129
Camera	0 Flexibility	0	0.103048133	0.841252129

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OVERVIEW

•To remain competitive, companies must create new products that satisfy consumer needs in order to maximize acceptance. In addition, desirable new products will usually lead to sales growth and stability. Thus, companies need information about consumer preferences to develop new products that consumers want to buy.

•Conjoint analysis is an efficient, cost-effective, and most widely used quantitative method in marketing research to understand consumer preferences and value trade-off. Value can be interpreted by consumer as the received of multiple benefits from a price that is paid. In reality, a consumer wants the most preferable attributes or features at the lowest possible price while an organization wants maximize profits by minimizing cost of providing those features and to ahead of its competitors.

•In conjoint choice experiments, respondents choose from a set of product alternatives in choice set. A base alternative which is normally a “none” option is added to the set of product alternatives to make the choice more realistic.

Table 1- Choice Set			
Survey form	Option A	Option B	Option C
Price	RM 2250	RM 1750	
3G	No	Yes	
Warranty	1 year	2 year	
Memory	32GB	64GB	
Flexibility	Yes	No	
Battery	8 hours	10 hours	
Camera	3-Megapixel	5-Megapixel	
Ram	512MB	1GB	

•Completely Confounded Factorial Conjoint Choice Experiment (CCFCE) design allows estimation of main effects and interaction effects. Since some of the effects will be confounded with the blocks in completely confounded factorial design, the conjoint choice design was extended to use of partially confounded factorial design, which is call Partially Confounded Factorial Conjoint Choice Experiment(PCFCE).

•The use of PCFCE is consistent with random utility theory. For each choice set a consumer must choose between two products each with a different set of product attributes or neither. In this study, all the responses were assume to be independent and hence the multinomial logit model follows. The log likelihood is nonlinear and hence the newton-raphson method is needed to estimate the parameters.

$$\begin{aligned}\beta^{(1)} &= \beta^{(0)} + [-I''(\beta^{(0)})]^{-1} * l'(\beta^{(0)}) \\ &= \beta^{(0)} + (X^T W X)^{-1} * X^T (Y - \mu)\end{aligned}$$

OBJECTIVE

•To introduce 2^n PARTIALLY CONFOUNDED FACTORIAL DESIGN IN CONJOINT CHOICE EXPERIMENT

•Eight attributes of tablet with two level each were used to evaluate consumer’s purchasing decisions toward tablet by using PCFCE.

•Provide PCFCE Newton-Raphson to estimate the parameters by SAS/IML®

