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Integrated Framework for Stress Testing in SAS®

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ABSTRACT

Stress testing is an integrated part of enterprise risk management and is a regulatory requirement. Stress testing is especially useful for integrating forward-looking views into risk analysis. Indeed, stress tests can provide useful information about a firm's risk exposure that statistical risk methods, calibrated on the basis of history, can miss. However, traditional stress testing is done on a stand-alone basis. This makes the interpretation of risk obtained from stress events vs. from risk analysis with statistical models difficult to interpret. We consider a Markov model and innovative implementation in SAS® that integrates rare stress events into regular statistical risk models. The model allows a consistent integration of the information in backward-looking historical data.

INTRODUCTION

Capturing tail events, especially those that incur severe loss, is an important objective in modern risk analysis. Historically, a substantial part of finance research has been devoted to developing models that extend beyond the normal distribution and that capture the stylized facts of financial time series. In the asset pricing literature, the first notable attempt of this was the introduction of stochastic volatility, fractional Brownian motion, and jump diffusion. The GARCH model captures the well-known volatility clustering of financial time series. However, even under a GARCH model, conditional return distributions might not be normal. Consequently, risk managers often consider non-normal models for conditional financial returns in Value-at-Risk (VaR) models. Stylized facts of multivariate financial returns have also shown that the normal correlation might not be a good descriptor of dependence. Skoglund et al. (2010) consider back testing performance of different VaR models during the recent financial crisis. VaR models that perform the best capture the stylized facts of volatility clustering, non-normality, and, stronger dependence than the normal correlation using copulas. However, past performance is no guarantee for future results.

Still, current VaR-type risk models rely almost exclusively on historical data. At the same time, stress testing is viewed as a forward-looking risk analysis tool that should complement risk measures based on historical calibration such as VaR. Financial institutions are encouraged to think forward - to assess the potential impact of hypothetical economic conditions - on all the risk aspects of the business operations. Recent regulations including Basel III (2011), Comprehensive Capital Adequacy Review (CCAR) by the Board of Governors of the Federal Reserve System (2012), the stress testing implementation for the Dodd-Frank Act by the Office of the Comptroller of the Currency (2012), and, Solvency II, which was adopted by the Council of the European Union and the European Parliament (2009) as the new regulatory directive for insurance and reinsurance business, have either included stress testing as a complementary risk analysis or directly used a stress scenario based approach to measure tail risks.

In practice, there are two types of stress testing. The first type is stress scenarios. The scenarios are based either on economic conjecture or on historically severe loss events. Many regulatory macroeconomic scenarios such as the recent Board of Governors of Federal Reserve System (2012) scenarios for Dodd-Frank Act are examples. Historically significant loss events are frequent choices to use as scenarios for stress testing (for example, the 1987 stock market crash, 1998 Russian financial crisis and the September 11 2001 terrorist attack).

The second type of stress testing, although not as well-known as the first, is stress testing financial model specifications. There are two subcategories. The first is to superimpose a stress period of historical data into model calibration. The purpose of the superimposition is to avoid optimistic projection when the historical data used in the analysis happens to be more economically positive than usual. Basel III recently added a stressed VaR component into the market risk capital requirement. The second subcategory is to guard again model specification risk. Model specifications, especially model parameters, are usually shocked.

In the approaches to stress testing, in order to create reasonable scenario or stress model, adequate economic and financial knowledge about the economic system and the business environment of the financial institution is required. Therefore, expert judgment is critical. The practice of reverse stress testing aids the risk analyst in understanding the core events or core risk factors that influence the risk of the financial institution most. (See Skoglund and Chen, 2009). Reverse stress testing is also a core practice by regulators.

Despite the recent development in stress testing practices, stress testing is still considered a separate task from model-based VaR type risk analysis. This disconnection can prevent a comprehensive view of the risk profile of a financial institution. The idea of stress testing is to form a forward-looking loss distribution that captures potential scenarios that have not necessarily occurred in the past. From that perspective, stress scenarios should be regarded as scenarios that can occur in the risk model itself and should not be regarded as a separate task.

This paper proposes applying an innovative multi-period switching simulation model in the SAS risk management products to develop an integrated stress testing framework that incorporate plausible events that is not necessarily captured in history or historical stressed calibration in risk models. Berkowitz (2000) proposed a similar integrated stress testing framework by adding rare stress events into an existing one-period simulation with certain probabilities. The introduction of the rare events is exogenous and unconditional. Our multi-period switching simulation model extends the Berkowitz model in several respects. The switching decision can be both endogenous and exogenous. It also supports a wide range of structural break econometric models. The model can be used to introduce a stress scenario at any point on the multi-period path-dependent risk horizon. The integration of stress testing and model analysis is not only important for comprehensive tail risk analysis but also takes stress testing into advanced risk management decision making analysis such as scenario-based portfolio optimization. For example, Rockafeller and Uraysev (2000) develop a general scenario-based portfolio optimization framework. More recently, Chen and Skoglund (2012a and 2012b) consider cash flow gap management and optimal funding liquidity risk hedging in a multi-horizon stochastic cash flow context that is also based on scenarios. The integrated risk analysis with stress testing can be part of the projected scenarios that will affect the optimal outcome.

Section 2 introduces the Markov switching simulation method. Our model for integrated stress testing uses structural break models to integrate forward looking stress views into risk models. The risk analyst can integrate any set of stress events to the risk model by using either exogenous triggers for the events or endogenous triggers driven by the underlying market variables. Although we make use of a structural break or switching simulation method to integrate stress views to risk models, we do not anticipate that the parameters and switching rules are necessarily estimated from historical data. Rather, they are supplied as expert forward looking views on events that can happen in the future but are not part of the historical performance used when calibrating the base risk model. The method of switching regimes is appropriate for the integration of regular risk models and stress. This is because the stress events represent events in the future that typically are not counted for in the current historical data. Hence, they are in that sense regime shifts or structural breaks in comparison with the base risk model.

Section 3 considers two important applications of the Markov switching simulation. The first is inclusion of exogenous stress events as rare events and the second is the case of model parameter stress. Stress testing does not necessarily only take the form of event-based stress scenarios. Stress testing should also accommodate stress of model parameterization such as correlations. The switching simulation proposed in this paper provides a general framework that incorporates event and model stress as well as mixtures of the two.

Section 4 is concerned with applications of the switching simulation in SAS Risk Dimensions. We consider an equity portfolio with a covariance matrix as the base risk model. Event-based multi-period stress is added to the base model and the complete risk distribution - integrating the base model and the stress events - is obtained. We also consider model-based stress as a second example of the switching simulation. In this case we let a switching function drive the covariance matrix so that model correlation parameters can change according to exogenous or endogenous events.

Finally, section 5 presents our conclusions.

This SAS Global Forum paper is a summarized and less technical version of Chen and Skoglund (2013).

MARKOV SWITCHING SIMULATION METHOD

In this section we introduce the Markov switching simulation method that underlies the integrated stress testing model. The simulation algorithm considers a multi-period, path-dependent, model over a discrete time horizon, $t = 1, \dots, T$.

Consider a stochastic vector, $x(t) = (x_1(t), \dots, x_1(t))$. The realization $x(t)$ at time t follows a true distribution, f. Let the base model of the random vector be g_0. In addition, there are a few alternative distributions conditional on the economic states at time t. These alternative distributions are denoted by g_i where $i = 1, \dots, m$. Therefore,

$$x(t) = g_i\big(x(t)\big) \; if \; S = S_i$$

where there are $m + 1$ possible economic states and S_i , $i = 1, \dots, m$ is a particular state. The probability of the occurrence of a particular state is

$$p_i = p\,(S_i)$$

and,

$$\sum_{i=0}^{m} p_i = 1$$

The functions $g_i, i = 1, \dots, m$ are probability mass or density functions for state S_i . In the context of integrating stress testing into classical risk models, we can think of g_0 as the base risk model. The $i = 1, \dots, m$ alternative distributions represents stressed events that can happen but are not captured in the recent performance on which the base risk model, g_0, is calibrated.

INTEGRATED STRESS TESTING USING MARKOV SWITCHING SIMULATION METHOD

A significant advantage of the switching simulation model is its possible integration of forward-looking hypothetical models into classical risk models that are calibrated based on data. Indeed, the Markov switching simulation can support the typical structure break time series as well as many other deviations from a regular model setting. This is an important model feature, as a stress test is essentially a deviation from the base model, that is, a structural break from the base risk model and its parameters implied from the historical period of model calibration.

In our model, the base model deviations might or might not be based on historical information. For example, in a forward looking view, stressed events that have not happened before can be projected as possible to happen. Or, a historical crisis that is not covered in the current base model could happen again. In this section we will discuss how to integrate scenario and model-based stress testing using the switching simulation model. Berkowitz (2000) proposed a single period algorithm that superimposes a probability weighted exogenous rare event scenario to a classical risk model. In Berkowitz's model, stress-testing is embedded within the VaR model such that g_0 is the base risk model and $g_1, \dots, g_m$ are point mass (stress) events. This model integration is motivated by the fact that stress events should represent potential future economic states and hence be part of the risk model forecast. In current risk practice, VaR risk model analysis and stress testing are often two separate risk analysis tools. The VaR risk model is based on financial economic models calibrated from data. Stress tests are forward-looking risk analysis based on hypothetical assumptions and expert knowledge based economic projections or past experience. Clearly, the comprehensive analysis of the tail behavior of a risk portfolio requires combining the empirical and expert views. We consider two important applications using the multi-period path dependent switching simulation, developed in Chen and Skoglund (2013), to integrate multi-period event stress and multi-period model stress.

EVENT-BASED STRESS

In case of event-based stress such as $\boldsymbol{g_1}, \dots, \boldsymbol{g_m}$ being point mass (stress) events at time $\boldsymbol{t = 1, \dots, T}$ the switching simulation method incorporates path-dependency. The rare events are always conditional on the previous horizon realization. When a rare event state occurs at time t, the corresponding scenario is a singleton mass. The realization at time t can either be from a normal state, $\boldsymbol{S_0}$, or an event $\boldsymbol{S_i, i = 1, \dots, m}$. A series of rare events can be chained together on a simulation path, $\boldsymbol{t = 1, \dots, T}$. In this case, the path might see bigger than usual losses and impose more hedging or capital coverage. Suppose there are $\boldsymbol{i = 1, \dots, m}$ events that have a causal relation. Consider a Markov chain with transition probability matrix from state $\boldsymbol{S_i}$ to $\boldsymbol{S_j}$ being $\boldsymbol{P = [p_{ij}]}$, where $\boldsymbol{i = 0, \dots, m}$ and $\boldsymbol{j = 0, \dots, m}$. For example, consider the event that a too-big-to-fail institution experiences a significant loss due to a fraud. Such an event at time t can lead to various subsequent time $\boldsymbol{t = t + 1, t + 2}$ ... market disruptions that can take different paths. The occurrence of a rare stress event cannot only be specified by an exogenous hidden process but also be triggered by the underlying risk factor realization from the base risk model. For example, as was experienced in the sub-prime mortgage crisis, when interest rates returns from a low-level regime they not only affect consumer financial situation directly but subsequently also buyers' incentive for real estate properties, eventually leading to lower property prices. As a further event lower house prices and increased interest rates might trigger a cycle of substantially increased defaults. Obviously, the occurrence of a severe loss, distributed at $\boldsymbol{t} = \mathbf{1}, \ldots, \boldsymbol{T}$ is the outcome of several chained events. The rare event considered here bears a similarity to extended jump processes. However a jump process is usually calibrated from historical data where an unprecedented large loss rarely can happen. The inclusion of stress events in the model admits consideration of "Black Swan" events into the risk model.

MODEL-BASED STRESS

Stress testing does not necessarily only take the form of rare events. A stress testing model could accommodate a parameter change versus the base risk model. The switching simulation method in Chen and Skoglund (2013) can handle this case and the switching can be either exogenous or endogenous. In a base risk model it is natural to consider stochastic volatility as well as time-varying correlations. The multivariate GARCH model and its variants are popular models in practice. Many multivariate GARCH models are feasible only for a few assets. However, the dynamic conditional correlation method of Engle (2002) is feasible for a larger set of assets. Still, multivariate GARCH models for the base risk model are calibrated on historical performance and do not capture events that have not yet happened, or are not included in the period of calibration. It is therefore prudent to consider potential switching stress events where base model parameters can change suddenly to an extreme level. For example, a realized market downturns might induce sudden large increases in volatilities and correlations. While the GARCH models are designed to respond with higher volatility and correlation in case of large shocks, it cannot readily accommodate sudden regime shifts if those are not part of historical performance.

APPLICATIONS

In this section we illustrate the effect of integrating stress tests with regular or base risk models using the switching simulation method. The method is implemented in SAS Risk Dimensions. For the best illustration, our examples are using a linear portfolio with multivariate normal distribution as the base model.

The first example is focused on event-based stress. However, the bank's economic experts believe that a set of possible stress events can cause extreme losses for some positions in the portfolio, and as a result the aggregate portfolio profit and loss will be affected significantly. The risk manager is concerned that the base risk model cannot incorporate these events[1].

Our second example is focused on significantly stressed model parameters in stress events, specifically, stressed volatilities and correlations. In this case, the risk manager is concerned that the base model volatility and correlation do not seem to capture the banks view that for a stressed event for an economic indicator, the correlation will not only increase in the portfolio but increase significantly, that is, jump to a new stressed regime. Hence, with high probability, it causes much larger portfolio profit and loss than implied by the base model specification.

In our applications we consider risk as measured over $t = 1, \ldots, 10$ days for the portfolio. As mentioned above, we will consider a simple linear portfolio. It is not necessary to consider a more complex portfolio because our focus is on demonstrating applications of the integrated stress testing model using the Markov switching simulation method. However, as the integrated stress testing framework is simulation based, it can be applied to any portfolio. The portfolio has six positions with a current mark to market of zero. The distribution in the base risk model is multivariate normal with correlation matrix Ω for the 6 positions with equal correlation parameter of 0.5, that is, an equi-correlation matrix. The standard deviation, σ, is common for each position and is set such that σ=1%. The resulting portfolio distribution is analytic and the base model portfolio risk VaR and ES at 99% and 99.9% confidence level respectively are given in table 1. Figure 2 displays the base risk model portfolio distribution at the last days risk horizon. Because of the multivariate normal distribution for portfolio positions the resulting portfolio profit and loss distribution is normal for any $t = 1, \ldots, 10$ days risk horizon and the risk at $t = t + n$ can be obtained by multiplying the risk at t by $\sqrt{n}$ (See for example, Diebold et al,1997). In this normal setting we also have that VaR and ES are equivalent risk measures and only differ by a constant. However, as we will see when we introduce stresses into the simple normal setting, the scaling approach to the tail risk measures will become invalid.

[1] A similar example for credit risk that we will not consider here is that the banks standard portfolio credit risk model uses a multi-factor (normal) model for firm's returns that is estimated based on historical returns data for credit (indices) factors. However, the bank does not believe the backward looking statistical model captures the potential future market stress in several Euro countries defaults. Hence, the models are complemented with stressed events with significant negative factor (index) returns for some Euro countries.

Time/Measure	VaR(99)	ES(99)	VaR(99.9)	ES(99.9)
Day 1	0.21	0.24	0.28	0.31
Day 2	0.30	0.34	0.40	0.44
Day 3	0.37	0.42	0.48	0.53
Day 4	0.43	0.49	0.56	0.62
Day 5	0.48	0.54	0.63	0.68
Day 6	0.52	0.60	0.70	0.75
Day 7	0.56	0.65	0.75	0.81
Day 8	0.60	0.69	0.80	0.88
Day 9	0.64	0.73	0.85	0.93
Day 10	0.68	0.77	0.90	0.98

Table 1. Base Model Risk Profile. VaR and ES Risk over t=1,...,10 Days

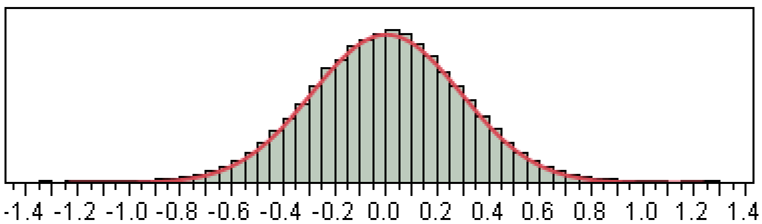

Figure 1. Terminal (t=10 days) Portfolio P/L Distribution for the Base Risk Model

RARE EVENT SCENARIOS

In the case of rare events we consider six stress scenarios $S = (S_1, \dots, S_6)$ for the portfolio with positions, $P = (P_1, \dots, P_6)$. We will denote a rare event scenario shift, S_i, of position j as $P_j = -X$ where X is the mark to market value of position j in the scenario. We consider the following rare events:

$S_1 \Rightarrow \{P_1=-0.5, P_2=-0.5\}$

$S_2 \Rightarrow \{P_1=-0.25, P_2=-0.25\}$

$S_3 \Rightarrow \{P_1=-0.4, P_2=-0.4\}$

$S_4 \Rightarrow \{P_1=-0.1, P_2=-0.1\}$

$S_5 \Rightarrow \{P_1=-0.15, P_2=-0.15\}$

$S_6 \Rightarrow \{P_1=-0.18, P_2=-0.18\}$.

Hence, only positions P_1 and P_2 are exposed to rare events. The unconditional probability of event i is common for all rare events $i = 1, \dots, 6$ and is 0.1%. The conditional probability of rare event i′ happening after rare event i has happened is set to 0.1% if i′=i and to zero otherwise. Clearly, the assignment of conditional migration probabilities from one rare event to another depends on the exact relationships between the events. If the rare events are such that they represent i=1,…, unrelated events, then it is natural to assign the conditional probability of migrating from event i to i′ zero when i′≠i. However, if event i′ is regarded as an event that can follow as a consequence of event i, but cannot happen by itself, then the unconditional probability of event i′ is zero and the conditional probability of migrating from event i to i′ is nonzero.

For example, a consumer credit stress might immediately, at t, give rise to loss in positions with exposure to credit market. It might also be followed by a subsequent, t+1, systematic downturn and hence affect more positions if the crisis spreads. However, note that even if a rare event is not followed by a new rare event the effect of the rare event at scenario n and time t is to move the stochastic realization of the vector x at scenario n and time t. Hence, at scenario n and time t+1 the starting point is the rare event realization. With a GARCH model, the impact of the event is even more significant as the volatility impact is exponentially decaying. Therefore, in this rare event model, two effects are generally seen as a result of a rare event at t. First, the rare event might change the probability of that event being persistent (conditional probability of event different from unconditional). It might also be the case that once a rare event has happened; other rare events might likely follow. Second, even if the rare event is not followed by a rare event the impact on risk over time is still substantial. Of course, the assignment of unconditional and conditional probabilities to events can be complex in practice. However, this is a core requirement to ensure proper integration of the stress events in the risk model and hence a single consistent risk view that integrates all the information. Rebonato (2012) proposes analytical tools to assist in the conditional probability assignment.

Table 2 displays the integrated rare events risk model portfolio VaR and ES at 99% and 99.9% confidence level respectively. The risk measures are calculated using 100,000 simulation replications. Risk, as measured by VaR and ES, has significantly increased when adapting the rare events to the base risk model. At t=1, the 99.9% risk level VaR and ES have the same value of 1 unit of loss. This is the same loss as in scenario S_1.

This is an unsurprising outcome because the 99.9% VaR coincides with the probability of the scenario S_1, that is, 0.1%. The VaR 99% in the rare events model for t=1 is more than 3.5 times as high as for the base risk model. However, as we move forward in time the relative VaR 99.9% difference between the rare event and base risk model decreases. At t=10 the ratio is approximately 1.48. This is because, in our model specification, once the large loss has happened there is no even larger loss that can happen. This is a consequence of the conditional probability of rare event i' happening after rare event i being zero if $i' \neq i$.

It is interesting to consider a rare event realization in this model and the corresponding profit and losses observed over time. Consider for example a scenario where event S_3 happens at t=1. Clearly, at t=1 the impact of the rare event is to generate a portfolio loss of -0.8. After the rare event a new rare event might happen or not. If a new rare event does not happen the impact is to conserve a higher risk profile than normal for t=2. This is because the starting point, at t=2, is the rare event in t=1. This path-dependent model behavior is consistent with how stress events behave in reality.

The impact of a stress should not be assessed at a single time horizon. Indeed, the evaluation of portfolio loss for a given stress event might require multiple horizons, and specifications of potential sequential evolution of stress events for t=1,...,T using conditional migration probabilities. The relevant risk horizon considered should weigh in the ability of the bank to properly liquidate or hedge positions adequately during that time. Indeed, the time horizon for liquidation might be significantly longer under stress than under normal situations.

Figure 2 displays the event risk model portfolio distribution at t=10 days risk horizon and normal quantile plot. The red line indicates a fitted normal distribution. In contrast to the corresponding figure 1 for the base risk model the normal distribution does not fit the loss tail, as can be seen from both the normal quantile plot and the profit and loss distribution, which shows significantly larger losses than implied by the normal base risk model.

Time/Measure	VaR(99)	ES(99)	VaR(99.9)	ES(99.9)
Day 1	0.23	0.43	1.00	1.00
Day 2	0.36	0.63	1.02	1.10
Day 3	0.47	0.77	1.09	1.17
Day 4	0.58	0.86	1.12	1.22
Day 5	0.68	0.94	1.17	1.26
Day 6	0.76	0.99	1.20	1.30
Day 7	0.81	1.02	1.24	1.34
Day 8	0.86	1.06	1.27	1.39
Day 9	0.91	1.09	1.31	1.42
Day 10	0.94	1.13	1.34	1.46

Table 2. Rare Event Model Risk Profile. VaR and ES Risk over t=1,...,10 Days

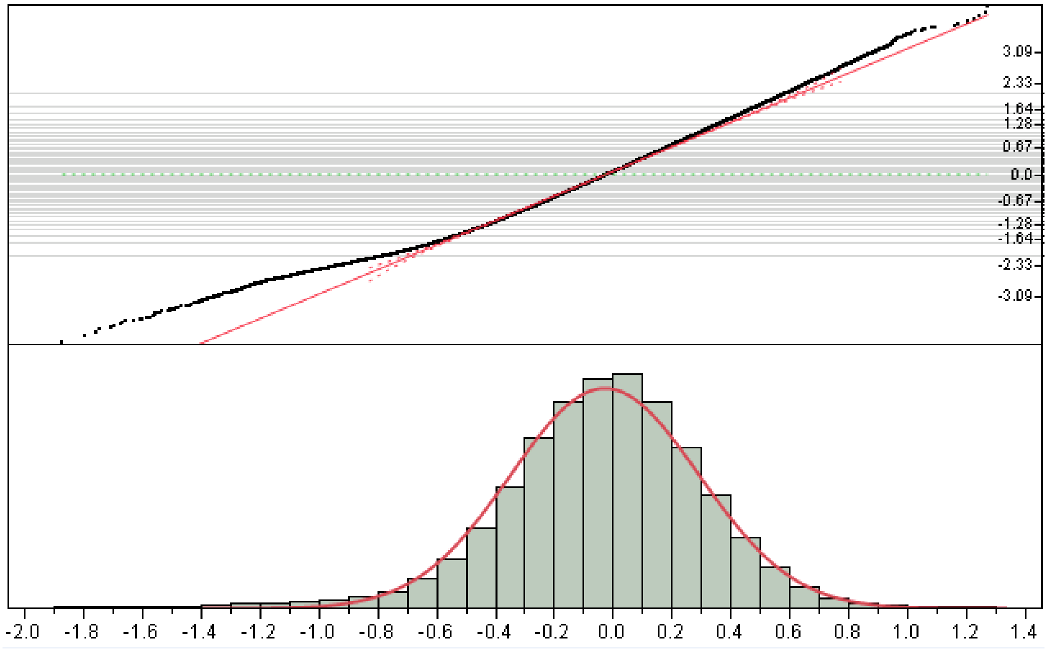

Figure 2. Terminal (t=10 days) Portfolio P/L Distribution for the Event Risk Model

MODEL REGIME SWITCHING

In our second example we consider a risk model with regime switching for the model parameters in case of stress. Specifically, we consider regime switching of volatilities and correlations given a switching function that depends on an economic indicator, u, distributed as standard normal, N(0,1), for all time horizons, t=1,...,10. The economic indicator variable is correlated with the portfolio positions, $P=(P_1,...,P_6)$, using the same correlation as between the positions[2]. It is natural to assume in this setting that the portfolio is correlated to the economic indicator (for example, if the portfolio is an equity portfolio and the economic indicator is a broad equity index). Two different switching functions will be used to switch between the correlation matrices, Ω, that is, the base risk model correlation matrix, $\Omega(S_1)$, the stressed regime 1, and, $\Omega(S_2)$, the stressed regime 2. In the first case the switching function is simple such that the actual correlation matrix, Ω,used at t+1is determined by,

$\Omega(t+1) = \Omega$ if $u(t) \geq 0.05$

$\Omega(t+1) = \Omega(S_1)$ if $0.01 \leq u(t) < 0.05$

$\Omega(t+1) = \Omega(S_2)$ if $u(t) < 0.01$.

where u=Φ(u) is the probability transformation of u to a uniform(0,1) random variable. The second switching function uses a Markov conditional transition probability, p_{ij}, between the states i,j. We let state 1 represent the base risk model correlation matrix, state 2 represent stressed regime 1 correlation matrix, and state 3 represents stressed regime 2 correlation matrix. The Markov transition probability matrix

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

is given by,

$$\begin{bmatrix} 0.95 & 0.04 & 0.01 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}.$$

Conditional on a stressed correlation at t=1 there is hence a greater likelihood of stressed correlation at t=2. The stressed regime 1 correlation matrix, $\Omega(S_1)$, and the stressed regime 2 correlation matrix, $\Omega(S_2)$, increases correlation to 0.8 and 0.99 respectively from the base case of 0.5. In addition to the correlation we will also change the common volatility for the base risk model, σ=1%, in the states to $\sigma(S_1)$=5%, and, $\sigma(S_2)$=10% respectively.

Tables 3 and 4 displays the regime switching risk models portfolio risk VaR and ES at 99% and 99.9% confidence level respectively. The risk measures are, as for the rare event model, calculated using 100,000 simulation replications. The risk, as measured by VaR and ES, is the same as for the base risk model for t=1. This is because, for both switching functions, switching at t+1 occurs based on the lagged indicator, u, at t. Subsequent risk at times t=2,...,10 is however significantly higher compared to the base risk model.

Note also that at t=2 there is the same VaR and ES for the simple and Markov switching models. This is because they have the same transition probabilities at t=2, based on the economic indicator, at t=1, to switch to the stressed parameter states. After t=2 the Markov switching model has higher risk than the simple switching model as the transition probabilities to a stressed state, given a stressed state has occurred, are much higher in the Markov switching model.

Figure 3 displays the simple switching risk model portfolio distribution at t=10 days risk horizon together with the normal quantile plot. A red line indicates the fitted normal distribution. Figure 4 displays the same portfolio distribution and normal quantile plot for the model with Markov switching. Figure 3 and figure 4 displays distributions with a much fatter left tail than right tail. This is due to the fact that we have correlated the economic indicator, u, with the portfolio positions, $P=(P_1,...,P_6)$. Hence, in states where the economic indicator has a very low value (that is, a significant downturn) it is likely that portfolio is experiencing a very large loss. This means that the switch to stressed correlation and volatility regimes will happen in states where large portfolio losses and economic downturn happens. This effect is further reinforced by the fact that the economic indicator correlation with the portfolio positions also switch to the portfolio positions stressed correlation level.

Time/Measure	VaR(99)	ES(99)	VaR(99.9)	ES(99.9)
Day 1	0.21	0.24	0.28	0.31

[2] The correlation between the economic indicator and the portfolio positions will switch regimes at the same time the correlation between positions switch regime.

Day 2	0.40	0.64	0.96	1.23
Day 3	0.56	0.90	1.38	1.73
Day 4	0.68	1.09	1.73	2.11
Day 5	0.77	1.28	2.01	2.54
Day 6	0.85	1.44	2.30	2.85
Day 7	0.94	1.55	2.41	3.01
Day 8	1.02	1.72	2.66	3.27
Day 9	1.11	1.84	2.90	3.50
Day 10	1.20	1.97	3.08	3.78

Table 3. Simple Switching Simulation Risk Model Risk Profile. VaR and ES Risk over t=1,...,10 Days

Time/Measure	VaR(99)	ES(99)	VaR(99.9)	ES(99.9)
Day 1	0.21	0.24	0.28	0.31
Day 2	0.40	0.64	0.96	1.23
Day 3	0.74	1.13	1.62	1.88
Day 4	1.06	1.51	2.03	2.38
Day 5	1.36	1.87	2.49	2.88
Day 6	1.58	2.15	2.94	3.32
Day 7	1.80	2.40	3.21	3.58
Day 8	2.02	2.68	3.48	3.99
Day 9	2.22	2.90	3.71	4.22
Day 10	2.38	3.11	4.01	4.60

Table 4. Markov Switching Simulation Risk Model Risk Profile. VaR and ES Risk over t=1,...,10 Days

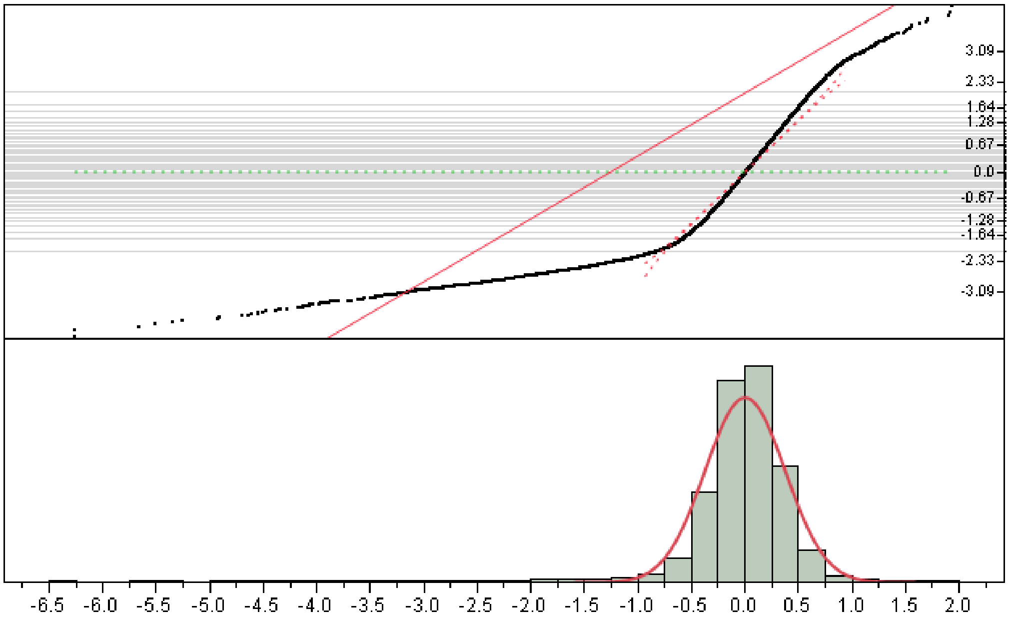

Figure 3. Terminal (t=10 days) Portfolio P/L Distribution for the Simple Regime Model

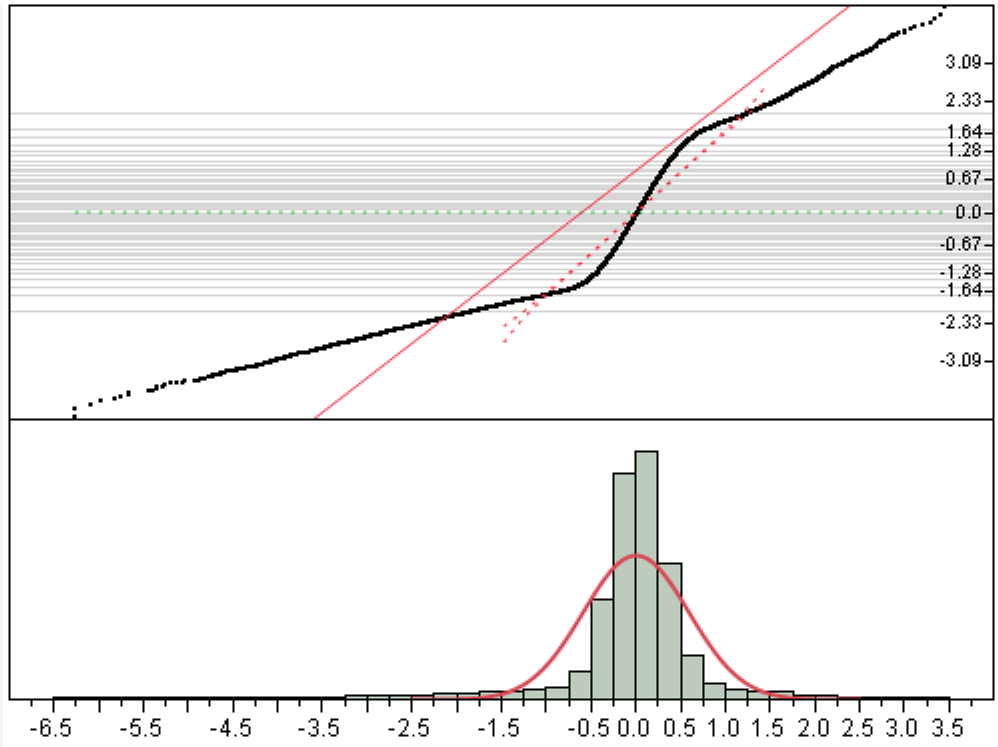

Figure 4. Terminal (t=10 days) Portfolio p/l Distribution for the Markov Regime Model

SUMMARY AND CONCLUSION

After the recent credit crisis, regulators have focused on complementing model-based risk measures with stress tests. However, it is not clear how the requirements for extended stress testing for financial institutions will eventually effect capital charges. One of the main issues is how to consistently aggregate the results of different stress tests, and, perhaps even more importantly, how to reconcile a stress test charge with a model-based VaR charge. Clearly, summing stress charges for multiple stress tests and model charges does not provide a solution[3]. In this paper we have taken the view that stress tests are complements to models. Stress tests or stress events represent forward-looking hypothetical models that complement classical risk models that are calibrated based on data. The Markov switching simulation can support the typical structure break time series model as well as many other deviations from a regular model setting. This is an important model feature as a stress test is essentially a deviation from the base model, that is , a structural break from the base risk model and its parameters implied from the historical period of model calibration. Although we make use of a structural break or switching simulation method to integrate stress views to risk models, we do not anticipate that the parameters and switching rules are estimated from historical data. Rather, they are supplied as expert forward looking views on events that can happen in the future but are not part of the historical performance used when calibrating the base risk model. The method of regime switching is appropriate for the integration of regular risk models and stress. This is because the stress events represent events in the future that typically are not counted for in the current historical data. Hence, they are regime shifts or structural breaks in comparison with the base risk model.

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[3] Rebonato (2012) consider aggregating the loss of a stress event using conditional loss events and conditional probabilities. He also proposes an event risk charge based on the maximum of conditional losses. In this setting the stress test charge is however still a stand-alone charge separated from model based risk charges.

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