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Quantitative requirements for non-life insurance under Solvency 2

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ABSTRACT

Solvency 2 is an EU directive set out to strengthen capital adequacy and risk management for insurers. The directive gives the insurers an opportunity to develop internal model in order to quantify the capital requirements. In this work we give examples on how this can be done for a non-life insurance company in the major risk areas: operational, counterparty, catastrophe, market and insurance risk. Insurance risk is usually the largest risk in non-life and we demonstrate how SAS® may be used to model this. Insurance risk is composed of underwriting risk and reserve risk, where underwriting risk is the risk arising from claims incurring in future accounting periods, and reserve risk is the risk arising from previous accounting periods. We demonstrate how these risks can be modeled and show an example on how copulas can be used to find non linear dependencies between lines of business.

INTRODUCTION

The current directive for insurers (Solvency 1) in EU is based on individual member country regulations and is the first stage in a more fundamental harmonization of the solvency requirements for EU insurers. The capital requirements for non-life insurance undertakings are basically a factor model of the written premium and technical provisions and do not necessary account for all of the real, underlying risk. The Solvency 2 directive aims at providing the insurance companies with more flexibility on how the capital requirements are calculated by allowing internal models [1].

SOLVENCY 2 OVERVIEW

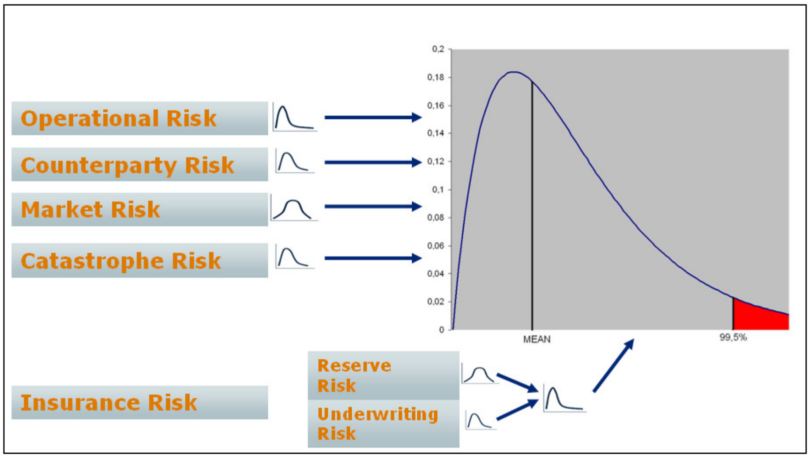
The proposed Solvency 2 framework has three major parts or pillars:

Pillar 1: quantitative requirements, like amount of capital an insurer should hold.

Pillar 2: risk management requirements.

Pillar 3: transparency requirements towards regulators.

In this work we will try to exemplify how one can calculate the value of different risk factors under pillar 1 for a non-life insurance company and how one can combine these to a single risk value. A schematic overview over the different quantitative risk factors for non-life insurance can be given as:



The goal of the model is to find distributions for each of the risk factors and combine them with an appropriate dependency. In Solvency 2 particularly, we are interested in finding the total distribution for the company and hold enough capital to cover the 99,5% quantile based on the risk for the next accounting year.

OPERATIONAL RISK

Operational risk is usually considered as risks connected to the people, systems and processes in a business. This is a very broad group of risk and includes fraud, system failures, terrorism and employee compensation claims. The model for operational risk may be very complex with detailed models for each element influencing the risk, but simplified models based on some appropriate exposure figure is also common. For non-life insurance a factor of earned premium is commonly used.

COUNTERPARTY RISK

The risk of not receiving payment as agreed is often called counterparty or credit risk and the event is often called a default. For non-life insurance the counterparty is often a reinsurer. The probability of default (PD) is often given by credit rating agencies like S&P and Moody's. The value at risk is simply the amount at stake multiplied by the probability of default given by the rating agencies. An example of rating and default probabilities are:

Rating	PD
AAA	0,010 %
AA	0,100 %
A	0,500 %
BBB	1,000 %
BB	3,000 %
B	5,000 %
…	
D	26,000 %

A further sophistication of this model is to give a distribution of the loss given default.

MARKET RISK

When modeling market risk we want to find how much the company assets may decrease due to change in the market factors. The four standard market risk factors are stock prices, interest rates, foreign exchange rates, and commodity prices [2]. The actual portfolio of the company decides which factors are to be taken into consideration. Usually one wants to model these factors as time series and a lot of work is done in this field. However, most insurance companies will rely on Economic Scenario Generators (ESG) as a part of a risk management tool rather then develop own methods for market risk.

CATASTROPHE RISK

Catastrophe modeling tries to estimate the losses that could be sustained due to a catastrophic event. Such event can be both man made and nature made. Due to the low frequency of catastrophic events, this risk is often modeled through standard formulas. What is considered a catastrophic event is often defined by the national supervisory authorities. A simplified model is based on the net written premium (P) in each line of business (t) together with a predefined constant (C) for each line of business.

$$\mathrm{NL}_{CAT} = \sqrt{\Sigma(C_t \cdot P_t)^2}$$

INSURANCE RISK

Insurance risk is the risk arising from the process of transferring risk from persons or companies to the insurance company and it is the fundamental business idea of an insurance company. It is usually divided in two main categories: underwriting risk and reserve risk. Underwriting risk is the risk arising from claims incurring in future accounting periods, while reserve risk is the risk arising from previous accounting periods. We will try to give details on how this can be calculated using SAS®.

UNDERWRITING RISK

To be able to predict the claims for a future accounting period we need to know about the claims in previous accounting periods. We will show an example on how to model next periods claims based on historical claims. To give an example we have accumulated periodical claims data from two lines of business, health insurance and workmans' compensation insurance.

Workmans' comp	Health
443	95
477	75
485	86
489	82
...	...
591	121
593	122
609	126
649	136

These time series of data will enable us to model the total underwriting risk for these two lines combined. One way of doing this is to model each of these lines separately and then combining them assuming some kind of dependency between the lines. A single line of business can easily be modeled by using PROC GENMOD. Workmans' compensation insurance is assumed to gamma distributed and the SAS® code will be:

```
PROC GENMOD DATA=Sparebank1_lob1;
         MODEL Workmans_comp= / DIST=gamma
                                LINK=log
                                TYPE1 TYPE3;
         OUTPUT OUT=pred_gamma PREDICTED=wc_pred;
RUN;
```

This gives as output the needed parameters, intercept and scale, to describe workmans' comp claims as a gamma distribution. We can then use this distribution to simulate outcomes from workmans' comp and find the desired quantiles.

Gamma Quantile	Workmans' comp
50 %	536
95 %	584
99,5 %	615

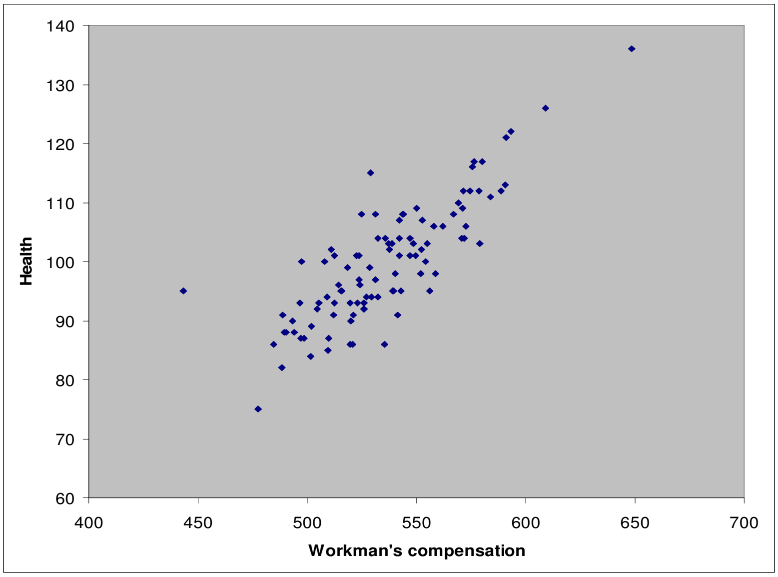
Health insurance claims are proportional with number of claims since payment pr claim is fixed. It is therefore reasonable to assume the total claim amount to be poisson distributed. We then apply the same GENMOD code but with a poisson assumption on the historical health claims.

```
PROC GENMOD DATA=Sparebank1_lob2;
         MODEL health_claim= / DIST=poisson
                               LINK=log
                               TYPE1 TYPE3;
         OUTPUT OUT=pred_poisson PREDICTED=health_pred;
RUN;
```

The output from this is the mean of the poisson distribution and the only parameter needed to simulate outcomes from this distribution. The quantiles of this distribution are:

Poisson Quantile	Health
50 %	100
95 %	117
99,5 %	127

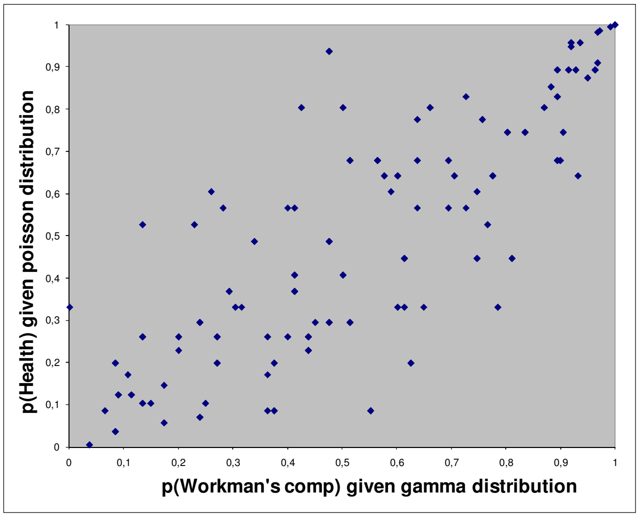
It is natural to assume that claims are somewhat correlated and if we make a plot, we observe that there is indeed some positive correlation.



By applying PROC CORR on the claims we get a Pearson correlation coefficient of 0.829. However we suspect that the larger claims are more correlated then smaller claims. This means that we need to use more complex dependency structures, so called copulas. When using copulas we use the already established marginal distributions, in our case: gamma for workman's compensation and poisson for health. Given these distributions, we can list the corresponding probability for each observed claim.

Workmans' comp	Health	p(Workmans' comp) given gamma distribution	p(Health) given poisson distribution
443	95	0,00147	0,33119
477	75	0,03754	0,00547
485	86	0,06586	0,08611
489	82	0,08501	0,03689
...	...	...	...
591	121	0,96785	0,98193
593	122	0,97190	0,98569
609	126	0,99133	0,99477
649	136	0,99978	0,99974

We may plot the corresponding probabilities in a [0,1] x [0,1] grid to get a visual impression of the dependency.



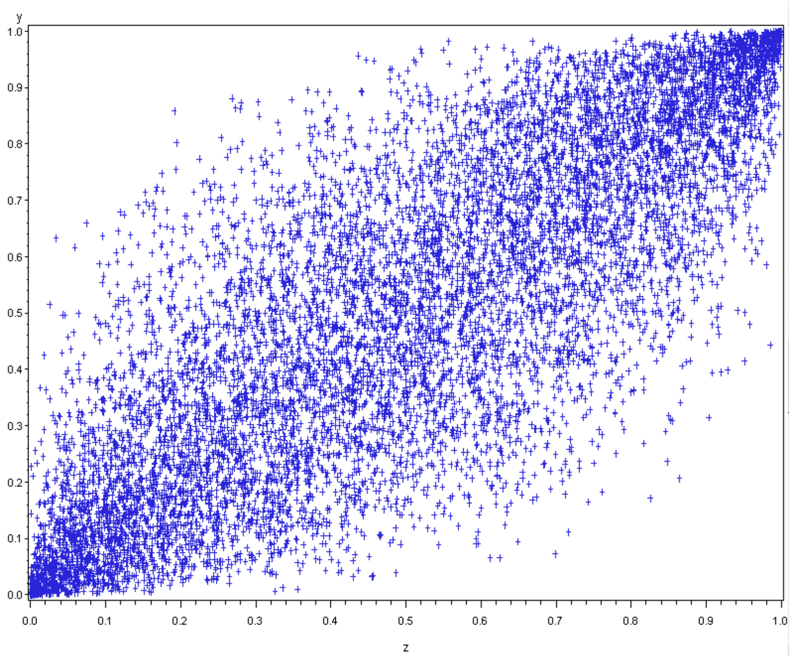
By using maximum likelihood techniques we can find a copula that fits these points much like we did when establishing which marginal distributions to use. In this case, a Gumble copula with constant 2.84 fits well. By choosing this copula we have chosen the dependency structure between the two marginal distributions. The next step is to simulate with this dependency structure together with the chosen marginal distributions and compare it with regular correlation. Regular correlation is also a copula often referred to as normal or gaussian copula. This is implemented in the PROC MODEL and is simulated as [3]:

```
DATA histdata;
y = .5; z = .5; *Initial observed data;
RUN ;

DATA sdata; *Give correlation in normal copula;
_type_ = "cov";y = 1; z = .829; _name_ = "y"; OUTPUT;
_type_ = "cov";y = .829; z = 1; _name_ = "z"; OUTPUT;
RUN;

PROC MODEL OUT=sim DATA=histdata SDATA=sdata;
y = 0; ERRORMODEL y ~ Uniform(0,1);
z = 0; ERRORMODEL z ~ Uniform(0,1);
SOLVE y z / random=10000 seed=12345 copula=(normal);
RUN;
QUIT;

PROC GPLOT DATA = sim;
PLOT y*z;
RUN;
QUIT;
```



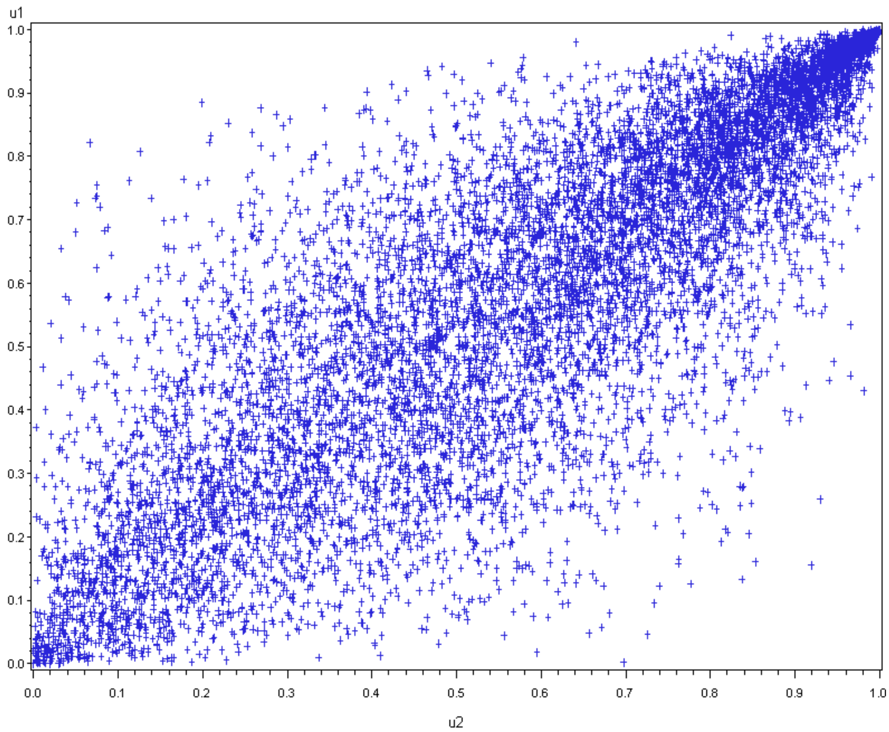
The Gumble copula is not given explicitly in PROC MODEL but can easily be simulated as:

```
DATA t1;
 DO i=1 TO 10000;
      y=UNIFORM(-1);
    OUTPUT;
 END;
RUN;
```

```
PROC MODEL DATA=t1;
      EXOGENOUS   x;
      x*(1-log(x)*(1/2.84))-y =0;
      SOLVE x/out=b;
RUN;
QUIT;

DATA c;
SET b;
v1=UNIFORM(-1);
u1=exp(log(y)*v1**(1/2.84));
u2=exp(log(y)*(1-v1)**(1/2.84));
RUN;

PROC GPLOT DATA = c;
PLOT u1*u2;
RUN;
QUIT;
```



The plot shows a higher density of points in the upper right corner and this is due to higher correlation for lager claims. The total claim for the two lines of business is then:

Quantile	Normal	Gumble
Mean	636	636
95,00 %	699	700
99,50 %	739	741

The numbers show that we do underestimate the combined risk of the two lines of business by using the normal copula instead of the more suitable Gumble copula.

RESERVE RISK

The reserve risk is often calculated based on so called payment triangles. In this triangle, rows represent the year the claim incurred and the columns represent the delay of payment in years. An example of such a triangle is given as:

YEAR of claim	d										
skaar	0	1	2	3	4	5	6	7	8	9	10
2000	72,8	110,3	279,3	252,3	306,5	244,7	162,7	96,9			
2001	39,3	134,7	223,5	291,2	228,0	230,1	136,7				
2002	40,2	102,6	203,4	228,7	199,6	186,2					
2003	32,6	94,4	157,8	243,4	223,3						
2004	19,3	112,9	162,5	222,2							
2005	57,5	131,7	158,3								
2006	33,1	109,9									
2007	43,7										

In this triangle we observe that the peak of payments is after 3 to 4 years and the line of business is long tailed, meaning that it will take many years before all the claims are settled for a given year. The future payments can be modeled with PROC GENMOD if we assume the claims to be gamma distributed and we introduce a variable that accountings for the long tail nature of the claims [4]. If we define `g=log(1+d)` we get:

```
PROC GENMOD DATA=total;
 MODEL betalt = d  g / LINK=log DIST=gamma;
 OUTPUT OUT=new P=probet3 STDRESDEV=resid U=u95 L=l95;
RUN;
```

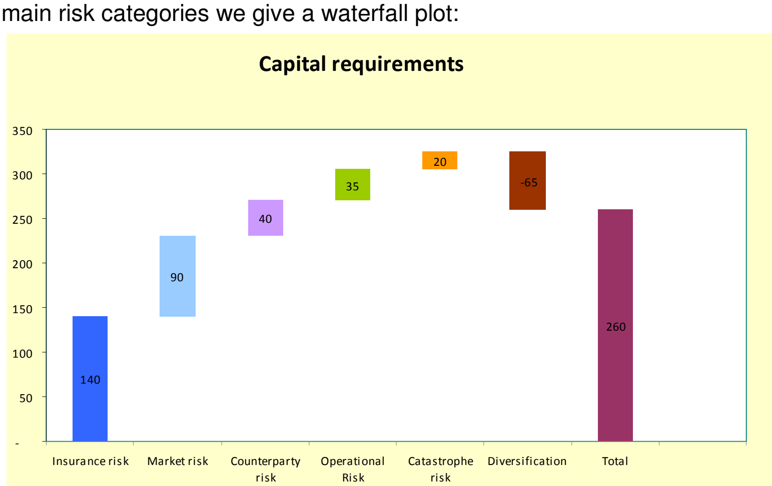
It would be natural to also include the variable `skaar` in the model but in this case it is not significant. The output from PROC GENMOD gives us a gamma distribution dependent of `d` and the resulting future payments:

YEAR of claim	d										
skaar	0	1	2	3	4	5	6	7	8	9	10
2000	72,8	110,3	279,3	252,3	306,5	244,7	162,7	96,9	96,4	70,3	49,9
2001	39,3	134,7	223,5	291,2	228,0	230,1	136,7	128,3	96,4	70,3	49,9
2002	40,2	102,6	203,4	228,7	199,6	186,2	164,1	128,3	96,4	70,3	49,9
2003	32,6	94,4	157,8	243,4	223,3	199,2	164,1	128,3	96,4	70,3	49,9
2004	19,3	112,9	162,5	222,2	225,3	199,2	164,1	128,3	96,4	70,3	49,9
2005	57,5	131,7	158,3	230,1	225,3	199,2	164,1	128,3	96,4	70,3	49,9
2006	33,1	109,9	199,8	230,1	225,3	199,2	164,1	128,3	96,4	70,3	49,9
2007	43,7	129,0	199,8	230,1	225,3	199,2	164,1	128,3	96,4	70,3	49,9

This method is an alternative to the very popular chain ladder method and will in some cases give a better reflection of the underlying risk.

TOTAL MODEL

The total model combines all of the risks in question and combines them to a total risk distribution. For all the risks the actual values need to be discounted to present value to reflect the cash flow. Once this is taken into account the Solvency 2 capital requirement is the 99,5% quantile of this distribution. To get an impression on typical values of the main risk categories we give a waterfall plot:



As the figure shows insurance risk and market risk is the larges risks in a non-life insurance company.

CONCLUSION

The opportunity Solvency 2 gives to use own developed models to determine risk for insurers is bound to increase risk modeling activity. SAS® software provides many procedures that are well suited to solve internal risk models and we have demonstrated one such model for reserve and underwriting risk using PROC GENMOD. In addition we show how PROC MODEL is used to model non linear dependency with copulas.

REFERENCES

[1] (2010), EU Solvency 2 webpage, http://ec.europa.eu/internal_market/insurance/solvency/index_en.htm
[2] (2010). Wikipedia: Market Risk.
[3] (2008). Donald J. Erdman, Arthur Sinko, Using Copulas to Model Dependency Structures in Econometrics, Paper 321-2008 SAS Global Forum 2008.
[4] (1988). Ben Zehnwirth. Age-To-Age Development Factors Versus Stochastic Models.

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