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## Using PROC LOGISTIC to Estimate the Rasch Model

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### ABSTRACT

This paper describes how to use PROC LOGISTIC to estimate the Rasch model and make its estimates consistent with the results of the standard Rasch model software WINSTEPS.

### INTRODUCTION

Item Response Theory (IRT: Lord, 1980) models are widely used in educational and psychological testing. Usually the IRT models are estimated using the special computer programs, such as, WINSTEPS (Linacre, 2008), BILOG-MG (Zimowski, Muraki, Mislevy & Bock, 2003), and so on. WINSTEPS implements the joint maximum likelihood (JML: Wright & Douglas, 1977; Wright & Panchapakesan, 1969; Wright & Stone, 1979) and can only fit the Rasch model (Rasch, 1960) while BILOG-MG uses the marginal maximum likelihood (MML: Bock & Aitkin, 1981) and can estimate the Rasch model and the two- or three-parameter logistic IRT model.

Some IRT applications using SAS<sup>®</sup> are also available. Christensen and Bjorner (2003) developed a SAS macro to estimate the Rasch model using the conditional maximum likelihood method. Lee and Terry (2004) wrote a SAS macro to estimate the two- and three-parameter logistic IRT model using MML. However, Christensen and Bjorner's code is too slow for long tests, and Lee and Terry's macro is not suitable for the Rasch model. Uekawa (2005a) estimated the Rasch model using the SAS procedures, PROC LOGISTIC and PROC NL MIXED. Nord (2008) also wrote some SAS code to estimate the Rasch model based on PROC LOGISTIC and PROC NL MIXED. However, PROC NL MIXED is slow and often fails to converge. The results of their SAS methods based on PROC LOGISTIC are also inconsistent with the results of WINSTEPS. This inconsistency may cause test developers or psychometricians to set aside their results since WINSTEPS is the standard Rasch model software.

This paper develops SAS code to estimate the Rasch model using PROC LOGISTIC in order to produce results consistent or comparable with the estimates from WINSTEPS.

### THE RASCH MODEL AND LOGISTIC REGRESSION

The dichotomous logistic regression model is similar to the Rasch model since the logit link function is applied in both of them. Uekawa (2005b) showed that there exists a one-to-one correspondence between the parameters of a special logistic regression and the Rasch models. Pan (2011) used the mathematical expressions to show how their parameters are correspondingly one-to-one:

A standard Rasch model could be specified as:

$$\Pr(y_{ij} = 1) = \frac{\exp(\theta_i - b_j)}{1 + \exp(\theta_i - b_j)}$$

where  $y_{ij}$  is the score of person  $i$  on item  $j$ , person parameter  $\theta_i$  is the ability of person  $i$ , item parameter  $b_j$  is the difficulty of item  $j$ . The equation can be transformed as follows:

$$\log \frac{\Pr(y_{ij} = 1)}{1 - \Pr(y_{ij} = 1)} = \theta_i - b_j \quad (1)$$

If dummy variables  $x, z$  are used to indicate the scores of different persons on different items, then, a logistic regression model can fit the scores of all persons on all items. As Pan (2011) described, the logistic regression model is shown as follows:

$$\log \frac{\Pr(y_{ij} = 1)}{1 - \Pr(y_{ij} = 1)} = \sum_n \alpha_n x_n + \sum_m \beta_m z_m \quad (2)$$

where parameters  $\alpha$  and  $\beta$  stand for the person and item parameters, respectively. So, for any given  $i$  and  $j$ ,

$x_i=1$  and  $z_j=1$ , and all of the other dummy variables are equal to zero. Then,

$$\log \frac{\Pr(y_{ij} = 1)}{1 - \Pr(y_{ij} = 1)} = \alpha_i + \beta_j \quad (3)$$

Comparing Equation 1 with 3, it can be seen that  $\alpha_i$  and  $\beta_j$  in the logistic regression model correspond to  $\theta_i$  and  $-\beta_j$  in the Rasch model.

Due to the correspondence between the Rasch model and the logistic regression model outlined in Equations 1 through 3, PROC LOGISTIC can be used to estimate the Rasch model. PROC LOGISTIC uses maximum likelihood estimation (ML) and WINSTEPS implements JML. However, JML is essentially ML. It is called "joint" only because the method estimates person and item parameters simultaneously. Therefore, the methods should not produce meaningful differences in the parameter estimates (Pan, 2011).

## SAS CODE FOR THE RASCH MODEL

At first, a simulated data with 9 items and 20 persons was generated and entered using the following SAS code:

```
DATA test;
  INPUT score total item person $;
  CARDS;
  0 1 1 01
  0 1 2 01
  0 1 3 01
  ...
  0 2 1 02
  0 2 2 02
  1 2 3 02
  ...
  ;
```

In the above SAS data step, variable 'item' is the item sequence number, variable 'person' is the ID number of person taking the test, variable 'score' is person's score on each item and variable 'total' is the person's total score on the test.

Then, three examples of PROC LOGISTIC code and WINSTEPS were used to analyze the simulated data using the same algorithms and convergence criterion, i.e., PROC LOGISTIC model statement options, TECH=NEWTON and ABSFCNV=0.0001, were used in PROC LOGISTIC, and the WINSTEPS commands were adjusted accordingly.

### EXAMPLE 1

Nord (2008) created dummy variables for different items and persons by himself. However, the model specified in his code has an intercept while Equation 2 does not. The intercept should be excluded as shown by Uekawa (2005a). PROC LOGISTIC can create those dummy variables automatically using the parameterization option. According to Uekawa (2005a), the code estimating the Rasch model and the output are shown as follows:

```
PROC LOGISTIC DATA=test;
  CLASS item person;
  MODEL score = person item / NOINT TECH=NEWTON ABSFCNV=0.0001;
RUN;
```

Analysis of Maximum Likelihood Estimates						
Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr >	ChiSq
person 01	1	1.4798	0.8519	3.0173	0.0824	
person 02	1	0.7913	0.7836	1.0198	0.3126	
person 03	1	0.1751	0.7570	0.0535	0.8171	
person 04	1	0.7913	0.7836	1.0198	0.3126	
person 05	1	1.4798	0.8519	3.0173	0.0824	
person 06	1	0.7913	0.7836	1.0198	0.3126	
person 07	1	0.7913	0.7836	1.0198	0.3126	
person 08	1	1.4798	0.8519	3.0173	0.0824	
person 09	1	-1.0393	0.7892	1.7342	0.1879	

person	10	1	-0.4213	0.7577	0.3092	0.5782
person	11	1	-0.4213	0.7577	0.3092	0.5782
person	12	1	-0.4213	0.7577	0.3092	0.5782
person	13	1	0.1751	0.7570	0.0535	0.8171
person	14	1	0.1751	0.7570	0.0535	0.8171
person	15	1	-1.0393	0.7892	1.7342	0.1879
person	16	1	0.7913	0.7836	1.0198	0.3126
person	17	1	-1.0393	0.7892	1.7342	0.1879
person	18	1	-1.7498	0.8774	3.9766	0.0461
person	19	1	-1.7498	0.8774	3.9766	0.0461
item	1	1	-1.8184	0.5840	9.6959	0.0018
item	2	1	-0.8781	0.4992	3.0943	0.0786
item	3	1	-0.6130	0.4868	1.5853	0.2080
item	4	1	-1.4676	0.5446	7.2635	0.0070
item	5	1	0.1419	0.4773	0.0884	0.7662
item	6	1	1.2193	0.5273	5.3465	0.0208
item	7	1	0.6545	0.4916	1.7723	0.1831
item	8	1	1.2193	0.5273	5.3465	0.0208

**Output 1: Output from Example 1 PROC LOGISTIC Code****EXAMPLE 2**

By default, PROC LOGISTIC models the probability of the event, score= '0'. The Rasch model usually models the probability of the event, score= '1'. The option (EVENT= '1') should be used. Uekewa's code (Uekewa, 2005a) uses the default parameterization of the LOGISTIC procedure, namely, the effect parameterization. The parameterization puts the sum-to-zero constraint on the effects of all levels of a classification variable (SAS Institute Inc., 2008). Uekewa's SAS code (Uekewa, 2005a) sets the summations of item and person parameters respectively as zero because the effect parameterization is used. However, in the Rasch model or WINSTEPS, the constraint is only set on item parameters (Wright & Douglas, 1977; Wright & Stone, 1979). That is why the results of his code are inconsistent with WINSTEPS's.

Therefore, the effect parameterization is not appropriate. We could use GLM parameterization so the option, PARAM = GLM, is used. This parameterization specifies a model which is exactly the same as Equation 2. The second SAS code example and its output are shown below:

```
PROC LOGISTIC DATA=test;
  CLASS person item / PARAM=GLM;
  MODEL score (EVENT='1') = person item / NOINT TECH=NEWTON ABSFCNV=0.0001;
RUN;
```

Analysis of Maximum Likelihood Estimates						
Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq	
person	01	1	-3.1770	1.0940	8.4340	0.0037
person	02	1	-2.4687	1.0179	5.8823	0.0153
person	03	1	-1.8453	0.9806	3.5415	0.0599
person	04	1	-2.4687	1.0179	5.8823	0.0153
person	05	1	-3.1770	1.0940	8.4340	0.0037
person	06	1	-2.4687	1.0179	5.8823	0.0153
person	07	1	-2.4687	1.0179	5.8823	0.0153
person	08	1	-3.1770	1.0940	8.4340	0.0037
person	09	1	-0.6357	0.9690	0.4303	0.5118
person	10	1	-1.2478	0.9637	1.6765	0.1954
person	11	1	-1.2478	0.9637	1.6765	0.1954
person	12	1	-1.2478	0.9637	1.6765	0.1954
person	13	1	-1.8453	0.9806	3.5415	0.0599
person	14	1	-1.8453	0.9806	3.5415	0.0599
person	15	1	-0.6357	0.9690	0.4303	0.5118
person	16	1	-2.4687	1.0179	5.8823	0.0153
person	17	1	-0.6357	0.9690	0.4303	0.5118
person	18	1	0.0521	1.0189	0.0026	0.9592
person	19	1	0.0521	1.0189	0.0026	0.9592
person	20	1	-0.6357	0.9690	0.4303	0.5118
item	1	1	3.3571	0.8768	14.6606	0.0001
item	2	1	2.4529	0.8124	9.1169	0.0025
item	3	1	2.1918	0.8024	7.4604	0.0063
item	4	1	3.0248	0.8468	12.7587	0.0004
item	5	1	1.4385	0.7911	3.3061	0.0690

item	6	1	0.3384	0.8253	0.1681	0.6818
item	7	1	0.9194	0.7986	1.3254	0.2496
item	8	1	0.3384	0.8253	0.1681	0.6818
item	9	0	0	.	.	.

### Output 1: Output from Example 2 PROC LOGISTIC Code

Using GLM parameterization, PROC LOGISTIC arbitrarily sets the parameter of the last item as zero. In the Rasch model or WINSTEPS, the constraint is that the summation of all item parameters equals to zero. WINSTEPS does this by first estimating the item parameters then centering them to satisfy the constraint (Wright & Douglas, 1977; Wright & Stone, 1979). Therefore, item parameter estimates of PROC LOGISTIC should be consistent with WINSTEPS's after being centered. The results are displayed in Table 1.

### EXAMPLE 3

When thousands of persons take a test, the procedure takes a long time to estimate the parameters. It is well known that the Rasch model gives the same parameter estimates for each person who receives the same total score. So, variable 'person' is able to be replaced with variable 'total' when all examinees answer all items as shown by Nord (2008). After the model is fit, the estimate of the parameter for each person is equal to the estimate of the parameter of the total score corresponding to the person's total score. The third code example and its output are shown as follows:

```
PROC LOGISTIC DATA=test;
  CLASS item total / PARAM=GLM;
  MODEL score (EVENT='1') = total item / NOINT TECH=NEWTON ABSFCNV=0.0001;
RUN;
```

Analysis of Maximum Likelihood Estimates						
Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr >	ChiSq
total	2	1	-3.1770	0.8197	15.0228	0.0001
total	3	1	-2.4687	0.7172	11.8473	0.0006
total	4	1	-1.8453	0.7477	6.0902	0.0136
total	5	1	-1.2478	0.7282	2.9363	0.0866
total	6	1	-0.6357	0.6799	0.8741	0.3498
total	7	1	0.0521	0.8103	0.0041	0.9487
item	1	1	3.3571	0.8768	14.6606	0.0001
item	2	1	2.4529	0.8124	9.1169	0.0025
item	3	1	2.1918	0.8024	7.4604	0.0063
item	4	1	3.0248	0.8468	12.7587	0.0004
item	5	1	1.4385	0.7911	3.3061	0.0690
item	6	1	0.3384	0.8253	0.1681	0.6818
item	7	1	0.9194	0.7986	1.3254	0.2496
item	8	1	0.3384	0.8253	0.1681	0.6818
item	9	0	0	.	.	.

### Output 3: Output from Example 3 PROC LOGISTIC Code

Table 1 displays the estimates of the estimates of the item parameters from the three SAS code examples and WINSTEPS. The output of the Example 1 is shown in the first column of the table. Example 1 did not provide the parameter estimate for Item 9, but it was equal to the negative of the sum of the other item parameters (SAS Institute Inc., 2008). The differences between its and WINSTEPS's estimates ranged from 0.004 to 0.02. Although the signs of its item parameter estimates were the same as WINSTEPS's, the signs of the person parameter estimates were different and its person parameter estimates were much different from WINSTEPS's (see Output 1 and the Appendix).

The second and third code examples gave the same item parameter estimates (see Table 1, Outputs 2 and 3). When the results of these examples were centered, the last column of the table was obtained. Excluding the signs, the differences between the last column and the results of WINSTEPS were smaller than 0.0002. The differences were much smaller than the differences between the results of the first SAS example and WINSTEPS. Due to such small differences between the results of the second and third SAS examples and WINSTEPS, their results were consistent. The small differences between them may result from different iterative computation algorithms of the two programs. Their signs were different because item parameter  $\beta_j$  in the logistic regression model correspond to  $-b_j$  in the Rasch model as noted before.

According to these results, the item parameters of the Rasch model and the special logistic regression model (Equation 3) can be converted using the following equation:

$$b_j = \bar{\beta} - \beta_j \quad (4)$$

where  $\bar{\beta}$  is the mean of all  $\beta$ s. For the person parameters,

$$\theta_i = \alpha_i + \bar{\beta} \quad (5)$$

When the two equations were combined with Equation 1, Equations 1 and 3 are seen to be equivalent, i.e.

$$\log \frac{\Pr(y_{ij} = 1)}{1 - \Pr(y_{ij} = 1)} = \theta_i - b_j = (\alpha_i + \bar{\beta}) - (\bar{\beta} - \beta_j) = \alpha_i + \beta_j$$

The person parameter estimates of PROC LOGISTIC in Outputs 2 and 3 can be compared with those in this paper's appendix to verify the conversion equation. Pan (2011) showed the mathematical derivation of those equations.

**Table 1: Parameter Estimate Comparison of PROC LOGISTIC and WINSTEPS**

Parameter	Example 1	Example 2	Example 3	WINSTEPS	Centered SAS Results
item 1	-1.8184	3.3571	3.3571	-1.7946	1.7947
item 2	-0.8781	2.4529	2.4529	-0.8905	0.8905
item 3	-0.6130	2.1918	2.1918	-0.6294	0.6294
item 4	-1.4676	3.0248	3.0248	-1.4622	1.4624
item 5	0.1419	1.4385	1.4385	0.1239	-0.1239
item 6	1.2193	0.3384	0.3384	1.2239	-1.2240
item 7	0.6545	0.9194	0.9194	0.6429	-0.6430
item 8	1.2193	0.3384	0.3384	1.2239	-1.2240
item 9	1.5421	0	0	1.5623	-1.5624
Average	0	1.5624	1.5624	0.0000	

## DISCUSSION

The paper has shown how to use PROC LOGISTIC, especially its parameterization option, to estimate the Rasch model. We also can create the independent variables for item and person parameters by ourselves. Since WINSTEPS only puts the sum-to-zero constraint on item parameters, it actually uses the effect parameterization for classification variable 'item', and uses GLM parameterization for the classification variable 'person'. However, it is not needed to set one of them to zero because the Rasch model has no intercept. The CLASS statement is also unnecessary. If there are  $I$  persons and  $J$  items, then  $(I + J - 1)$  independent variables need to be created based on the design matrices of the two parameterizations. Then the results will be directly consistent with WINSTEPS's except for the signs. Those design matrices can be found in SAS user's manual (SAS Institute Inc., 2008).

The results of Example 2 can be converted so that they match those of WINSTEPS, but it runs too slowly for tests taken by many persons. Example 3 code only estimates parameters of examinees based on their total scores. This reduces computation time but the code is not appropriate when examinees take different item sets in a test. For example, a test has 30 items, but one examinee takes all 30 items while another takes only 20 of the 30 items. If both of them give correct responses to 10 items, WINSTEPS gives a different ability parameter estimate for each of them in terms of the Rasch theory, and so does Example 2 code. Example 3 code, however, will give the same estimates of the two examinees' abilities. One solution to this problem is to create a new variable to differentiate examinees or persons by their total scores and assigned item sets and then use the new variable to replace variable 'total' in the third SAS code example.

The three examples using PROC LOGISTIC have another potential problem. When a person or an item has zero/full response, i.e., a person answers all test items (in)correctly or all persons answer an item (in)correctly, ML fails to converge. In WINSTEPS, data are checked and then those persons/items are excluded. To use those SAS code examples, therefore, the persons/items also should be excluded. Another solution is using a penalized maximum likelihood (PML) method (Firth, 1993), which is specified by PROC LOGISTIC using the model statement option, FIRTH, in SAS 9.2. Heinze and Schemper (2002) have shown that Firth's method always yields finite estimates of parameters under complete or quasi-complete separation. All parameters of the persons and items with zero/full responses are able to be estimated using

PML. In WINSTEPS, however, an extra procedure is used to estimate parameters of the persons but the parameters of the items are still not estimable.

When compared with ML estimation, PML has another advantage. PML estimation can preventively reduce the bias of ML estimate (Firth, 1993). ML estimates of the Rasch model are also biased (Ghosh, 1995; Wright & Douglas, 1977). If the Rasch model is fit using a logistic regression framework, then the PML method is also able to reduce the bias of ML estimation (Pan, 2011). The estimates using the PML method can be converted using the equations shown above.

It is convenient for SAS users to be able to estimate the Rasch model parameters using PROC LOGISTIC. PROC LOGISTIC, however, is not able to replace WINSTEPS. Standard errors for the parameter estimates provided by PROC LOGISTIC and WINSTEPS are different since logistic regression and the Rasch model use different approaches to calculate the standard error (Pan, 2011). PROC LOGISTIC does not provide fit statistics for each item and person as WINSTEPS does. SAS users would need to write their own SAS code to calculate these statistics or use the code provided by Nord (2008).

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## APPENDIX: THE PERSON PARAMETER ESTIMATES OF WINSTEPS

Person ID	Estimates
01	-1.6145
02	-0.9063
03	-0.2829
04	-0.9063
05	-1.6145
06	-0.9063
07	-0.9063
08	-1.6145
09	0.9266
10	0.3146
11	0.3146
12	0.3146
13	-0.2829
14	-0.2829
15	0.9266
16	-0.9063
17	0.9266
18	1.6144
19	1.6144
20	0.9266

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