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Optimization of Gas-Injected Oil Wells

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ABSTRACT

Artificial gas injection into aging wells boosts reservoir pressures, allowing for higher production rates. For constrained gas flows and multiple wells, the solution of this problem becomes difficult and time intense. With SAS/OR® optimization techniques, a scalable (from 1 to n well) solution can be used to provide quick results. This paper provides background on artificial injection to orient the reader, theory on the mathematical formulation of the optimization, and the SAS® code with results.

INTRODUCTION

Efficient extraction of oil often requires either submersible electronic pumps or artificial gas injection. The latter is widely used in many producing wells (see Appendix A). The physical modeling of the phenomena associated with gas lift results in a complex system of partial differential equations that must be numerically solved. The equations are derived from fundamental mass and momentum balances and depend on the physical properties of the gas lift system. Solving these equations is normally accomplished in a research setting and is generally not incorporated into well operations.

A typical schematic of a continuous gas lift (CGL) oil field system is shown in **Figure 1**.

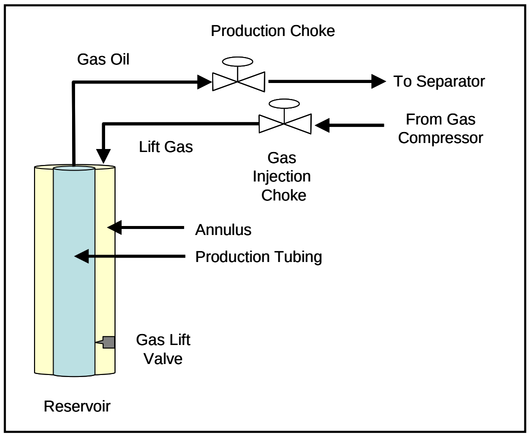

Figure 1: Continuous Gas Lift Schematic

In practical applications, field planning and operation are often complicated by the interaction of the wells in the reservoir, the gas-oil ratio of each well, the temperature of each well, the type of gas lift valves and the capacities of the surface processing facilities (compressed gas availability and gas-oil separation). For each well, there exists an unstable, optimum and stable gas injection pressure range. **Figure 2** illustrates a typical gas-lift optimization curve. The unstable region results in "heading" where wide variations in injection pressure are observed due to the physical dynamics of the fluid flow. The stable region is normally at higher injection rates. The optimal gas lift region is typically 40 percent to 60 percent of the gas injection rate at the optimal oil production point. See Appendix A for additional information on oil field production planning.

Gas lift optimization curves for each well are typically derived through measurement of gas-oil production across a range of injection pressures. With continuous instrumentation at each well, these curves can be dynamically computed and used in optimization strategies to address well interaction as well as other operating conditions.

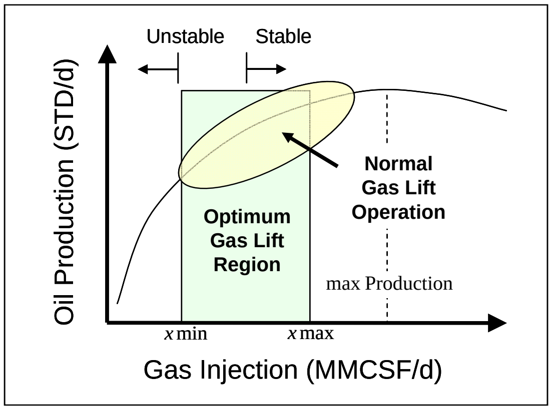

Figure 2: Gas Lift Optimization Curve

For a given set of gas lift optimization curves, computation of optimal gas injection pressures can be accomplished by various numerical techniques. Many of these techniques require significant computational resources and therefore do not lend themselves to insertion into the operation of oil wells.

To enable the well operator to optimize well production dynamically, the solution to the gas-lift optimization problem must be computationally efficient and responsive to the major operational drivers affecting production.

MODELING APPROACH

The modeling approach presented addresses the CGL alternative which is common to many fields and starts with developing a set of gas lift optimization curves from measurement data for each well in the field. These curves represent the response function between gas injection rates (the independent variable) and production (dependent variable) and are typically based on recent measurement data. Since the curves represent the actual operating conditions of each well, the phenomenon described above that affects the well's performance is embedded in the data. Data for and a graphic of a typical five well field gas-lift optimization set of curves are shown at Appendix B.

To solve the optimization problem associated with the distribution of gas flows across multiple wells, authors have provided various models and computational approaches.

Under the presented approach, the discrete gas-lift optimization data points are fitted to create a continuous function with the independent variable in gas injection flow. The optimal gas injection rates that result in the highest production flow are computed using computer-based optimization schemes.

Many forms of the gas-lift equation can be proposed. A simple polynomial in (independent variable) with varying degrees can be fitted to the data using a least squares methodology by assessing the statistical measures relating to the goodness of the best-fit polynomial can be selected for the data.

Polynomial in x:

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \quad \textbf{(Eqn 1)}$$

where:

- x is the independent variable
- a_0, a_1, a_2, a_3 and a_4 are coefficients of their respective polynomial independent variables

Fitting of polynomial equations to the general shape of the gas-lift curve (**Figure 2**) requires a higher order polynomial to approximate the shape of the curve. The fit statistics resulting from a least squares regression and the poor general shape fit indicate that a high order polynomial form is not satisfactory.

Other forms of the function can be used to fit the data and the statistical measures and resultant shape of the curve can be assessed to better match the gas-lift optimization relationship. Of several alternatives, the following seems to best fit generalized gas-lift curves.

Exponential in x :

$$f(x) = a_3 + [a_2/(a_0/a_1 - 1)] \left\{ e^{-a_1 x} - e^{-a_0 x} \right\} \quad \textbf{(Eqn 2)}$$

where:

- x is the independent variable, the gas injection rate
- a_0, a_1, a_2, and a_3 are coefficients in exponential equation

OPTIMIZE PRODUCTION FORMULATION

The formulation for the simplest constrained production optimization problem is:

Maximize

$$\textit{Production} = \sum_{\substack{i=1 \\ to\ No \\ of\ Wells}} f(x_i) = \sum_{\substack{i=1 \\ to\ No \\ of\ Wells}} a_3 + [a_2/(a_0/a_1 - 1)] \left\{ e^{-a_1 x} - e^{-a_0 x} \right\} \quad \textbf{(Eqn 3)}$$

subject to:

$$\begin{matrix} \textit{Maximum} \\ \textit{Gas Injection} \\ \textit{Volume} \end{matrix} = \sum_{\substack{i=1 \\ to\ No \\ of\ Wells}} x_i \le \begin{matrix} \textit{Total} \\ \textit{Gas} \\ \textit{Available} \end{matrix} \quad \textbf{(Eqn 4)}$$

For each well, i=1 to Number of Wells:

$x_i \geq 0$ (non-negativity of gas injection rates) **(Eqn 5)**

Additional constraint criterion can be added, such as minimum and maximum injection rates per well. This set of constraint parameters allows the optimization of each gas-lift curve in the Optimal Gas Lift region (see **Figure 2**). See **Eqn 6 and 7**.

The next step in the methodology is the computational formulation of the constrained optimization problem and its ultimate solution.

For the polynomial in X form (**Eqn 1**), a linear equation solver is adequate while the exponential form (**Eqn 2**) requires a non-linear equation solver.

Both of these solvers can be found in SAS Operations Research (SAS/OR) components[1]. For the polynomial function solver PROC GLM (Quadratic Least Squares Regression) was used and for the non-linear function solver PROC NLIN (Non-linear Least Squares) was used.

The referenced guide provides information and examples on use of these two procedures.

Results of the non-linear equation solver are presented in Appendix C for a 100 well set of data.

Since the exponential form of the gas lift curve was found best it was used to solve the constrained optimization problem. PROC OPTMODEL[2] (The NLPC Nonlinear Optimization Solver using the conjugate gradient method, a Newton-type method with line search, trust region method or quasi-Newton method) was chosen to solve this problem. The results of the optimization using **Eqn 3**, **4** and **5** are presented in Appendix D.

Adding additional constraints to **Eqn 1**, **2**, and **3** above results in the following formulation:

OPTIMIZE PRODUCTION FORMULATION FOR OPTIMAL GAS LIFT REGION

In addition to Eqn 3, 4 and 5 the below two additional constraints address the optimal gas lift region.

For each well, i=1 to number of wells:

$$x_i \leq 60\% \quad x_i \max \qquad \textbf{(Eqn 6)}$$

$$x_i \geq 40\% \quad x_i \max \qquad \textbf{(Eqn 7)}$$

Constraints **Eqn 6 and 7** ensure that the gas lift injection rate is in the optimal region (see **Figure 2**).

If the optimal gas lift region cannot be achieved (due to total gas injection capacity) for a particular well, the general practice is to set the gas lift flow rate to 0 to minimize heading effects.

OPTIMIZE PROFIT FORMULATION

Including the well head market value of produced oil, the market value of natural gas and the processing costs for gas injection (gas compression and oil/gas separation) results in the optimization of profit. **Eqn 3** is modified as follows:

Maximize **(Eqn 8)**

$$Profit = \left(\begin{array}{l} POWH * \sum\limits_{\substack{i=1 \\ \text{to No} \\ \text{of Wells}}} f(x_i) \\ - NGP * \sum\limits_{\substack{i=1 \\ \text{to No} \\ \text{of Wells}}} x_i \\ - Processing\ \ Costs \end{array} \right)$$

[1] SAS/OR® 9.2 User's Guide Mathematical Programming
[2] SAS/OR® 9.2 User's Guide Constraint Programming

where:

- POWH = Price of Oil at Wellhead
- NGP = Price of Natural Gas (if operations include gas recycling, this may be unnecessary)
- Processing Costs = Fixed and variable costs to compress gas and separate the out flow gas/oil mixture

MODIFICATION TO ALLOW SELECTION OF WHICH WELLS ARE PRODUCING

In an operating oil field, scheduled or unscheduled maintenance may require specific oil wells to be shut down. A logical extension of the above optimization would be the ability to select which wells were operating and then to optimize the gas injection rate for each operating well. This can be accomplished by including a variable (well on/off) parameter in the formulation of either **Eqn 3 or 8**.

CONCLUSION

SAS/OR optimization using PROC OPTMODEL provides a powerful and flexible approach to solve complex optimization problems. The ability to quickly translate objective functions (**Eqn 3**) into SAS Code and the efficient algorithms in PROC OPTMODEL make the choice of SAS appropriate for both small and large problems such as the optimization of a 100 well oil field.

SAS CODE

Portions of the SAS code used in this paper are shown in **Appendix E**. The code and results consider **Eqn 3, 4 and 5**. A future paper will include results using **Eqn 6, 7 and 8**.

CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the authors at:

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APPENDIX A: WELL OPERATIONS AND PLANNING

Reservoir Engineering.[3] A branch of petroleum engineering that applies scientific principles to the drainage problems arising during the development and production of oil and gas reservoirs so as to obtain a high economic recovery. The working tools of the reservoir engineer are subsurface geology, applied mathematics, and the basic laws of physics and chemistry governing the behavior of liquid and vapor phases of crude oil, natural gas, and water in reservoir rock.

Production Planning. Engineers periodically develop a production and injection plan which lists the target production of oil, gas, and water for the given period for each individual well. The injection of gas is listed for the injection wells. The duration of the plan typically is between a week and a month. The models of the processing facilities and wells/networks are used together with constraints from the reservoir planning as inputs to the planning.

Reservoir Planning. The long-term field drainage includes planning of gas and water injection. The updated reservoir model is used for finding proper draining strategies.

Strategic Planning. The production and injection plan is somehow connected to the market and the strategic considerations/policy of the company.

Well Model Updating. To help make good decisions, models may be used to develop the production plans. Typically, well tests are performed to determine the gas-oil-ratio, water cut, and production rates of each individual well. Additionally, data is collected to determine the gas lift curve. The well model is then updated based on the measurements during the test.

Processing Facility Model Updating. Typically, the processing facilities are modeled as constraints on oil, gas, and water processing capacity. This means that the model is updated whenever the capacity changes. From a gas lift optimization perspective the availability of gas compression and oil/gas/water separation facilities drive production planning.

Reservoir Model Updating. To be able to conduct the reservoir planning, a reservoir simulator may be used to evaluate different drainage strategies for the field. The simulator consists of a dynamic model of the reservoir. The state and parameters of the reservoir model must be updated by measurement data. The volumes produced and injected are important measurements used in this updating process. To ensure good accuracy of the model, its parameters may be fitted to longer series of historical measurement data.

[3] http://en.wikipedia.org/wiki/Reservoir_engineering

Gas Lift.[4] One of a number of processes used to artificially lift oil or water from wells where there is insufficient reservoir pressure to produce the well. The process involves injecting gas through the tubing-casing annulus. Injected gas aerates the fluid to reduce its density; the formation pressure is then able to lift the oil column and forces the fluid out of the wellbore. Gas may be injected continuously or intermittently, depending on the producing characteristics of the well and the arrangement of the gas lift equipment.

Gas lift is a form of artificial lift where gas bubbles lift the oil from the well. The amount of gas to be injected to maximize oil production varies based on well conditions and geometries. Too much or too little injected gas will result in less than maximum production. Generally, the optimal amount of injected gas is determined by well tests, where the rate of injection is varied and liquid production (oil and perhaps water) is measured.

Although the gas is recovered from the oil at a later separation stage, the process requires energy to drive a compressor in order to raise the pressure of the gas to a level where it can be re-injected.

The gas lift mandrel is a device installed in the tubing string of a gas lift well onto which or into which a gas lift valve is fitted. There are two common types of mandrels. In a conventional gas lift mandrel, a gas lift valve is installed as the tubing is placed in the well. Thus, to replace or repair the valve, the tubing string must be pulled. In the side-pocket mandrel, however, the valve is installed and removed by wireline while the mandrel is still in the well, eliminating the need to pull the tubing to repair or replace the valve.

A gas lift valve is a device installed on (or in) a gas lift mandrel, which in turn is put on the production tubing of a gas lift well. Tubing and casing pressures cause the valve to open and close, thus allowing gas to be injected into the fluid in the tubing to cause the fluid to rise to the surface. In the lexicon of the industry, gas lift mandrels are said to be "tubing retrievable" wherein they are deployed and retrieved attached to the production tubing. See gas lift mandrel.

Figure A-1 shows a typical oil gas separator and gas compressor configuration. **Figure A-2** shows a typical multiple-well configuration.

[4] http://en.wikipedia.org/wiki/Gas_lift

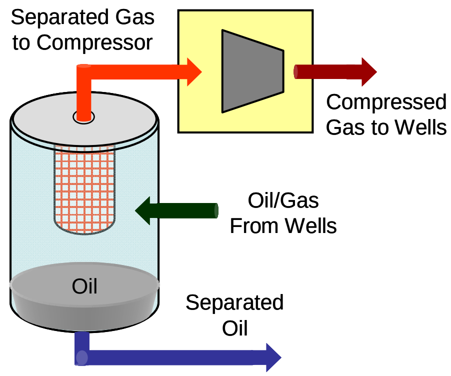

Figure A-1: Oil Gas Separator and Gas Compressor

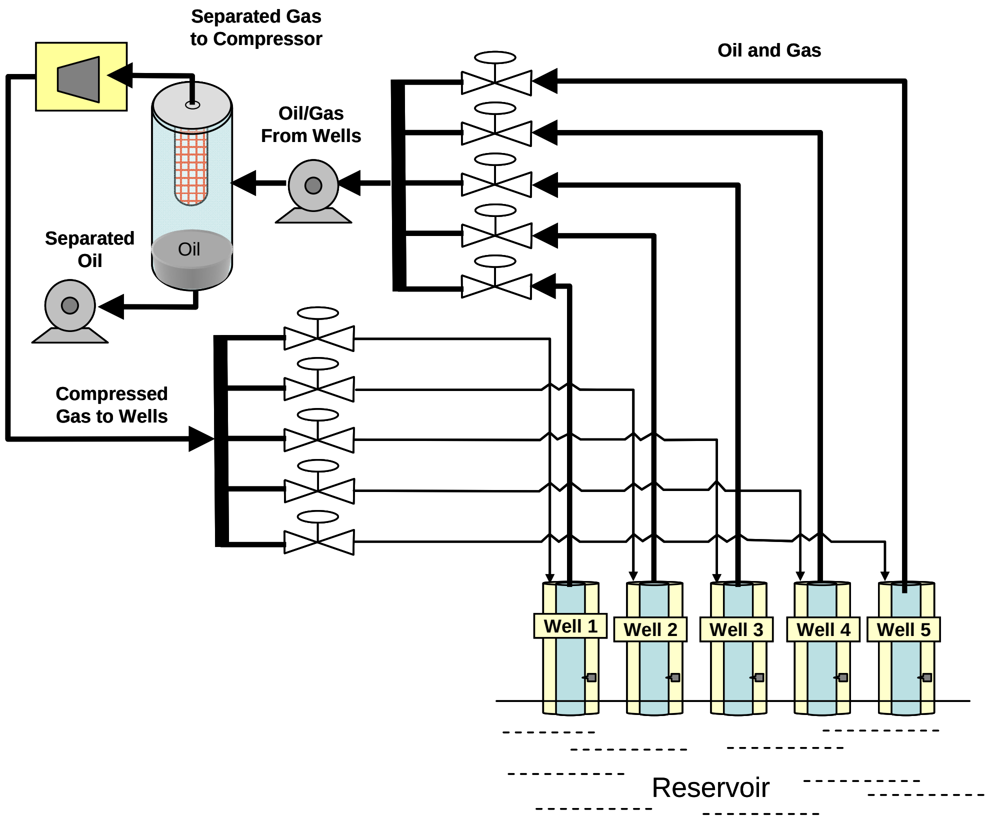

Figure A-2: Five-Well Configuration

APPENDIX B: SAMPLE GAS LIFT OPTIMIZATION CURVES

Well 1		Well 2		Well 3		Well 4		Well 5	
Qg	Qo	Qg	Qo	Qg	Qo	Qg	Qo	Qg	Qo
0	175	0	271	0	321	0	432	0	460
0.06125	187	0.2	307.2	0.2	400	0.263	500	0.3	532.7
0.125	200	0.3	325.8	0.31	425.9	0.427	540	0.444	567.3
0.1825	212	0.4	345.9	0.445	450.6	0.52	560	0.54	588.4
0.25	222	0.52	363.1	0.51	460	0.636	585.9	0.626	610
0.375	238	0.642	376.9	0.6	469	0.714	600	0.728	632.4
0.5	248	0.763	388.8	0.75	482.9	0.8	614	0.809	649.4
0.756	258.7	0.85	397.4	0.87	492	0.926	633.9	0.92	667
1	265	1	407.4	1.045	501.7	1.1	653	1	678
1.385	269.7	1.15	414.6	1.255	509.2	1.289	668.8	1.08	687
1.724	270.46	1.25	418.4	1.455	513.7	1.421	675.6	1.23	704
2	269.2	1.371	421.3	1.62	515.8	1.54	682	1.379	715.5
2.5	266	1.484	423.1	1.818	517.8	1.724	687.9	1.52	726
3	261.5	1.69	425.1	2	517.5	1.9	693	1.669	734
3.5	257	1.838	425.3	2.132	517	2.134	696.1	1.85	740
4	252.8	2.1	424.8	2.273	516.4	2.4	698.5	2	744
4.5	247.5	2.439	422.2	2.478	514.4	2.642	700	2.2	749
5	242.5	2.73	419	2.673	510.9	3.06	701.6	2.494	752.82
5.5	238	3.113	412	3.118	504.1	3.601	702.39	3	751
6	234	3.601	404	3.601	495.9	4.206	701	3.601	749
6.5	230	4	396.8	4.145	484.2	5	696	4.29	743.6
7	225	5	380	5	465	6	688	5	737
7.5	220	6	361.1	6	440	7	680	6	723.6
8	215	7	340	7	414	8	671	7	710
		8	321	8	384.9			8	695.5

Table B-1: Sample Gas Lift Curves

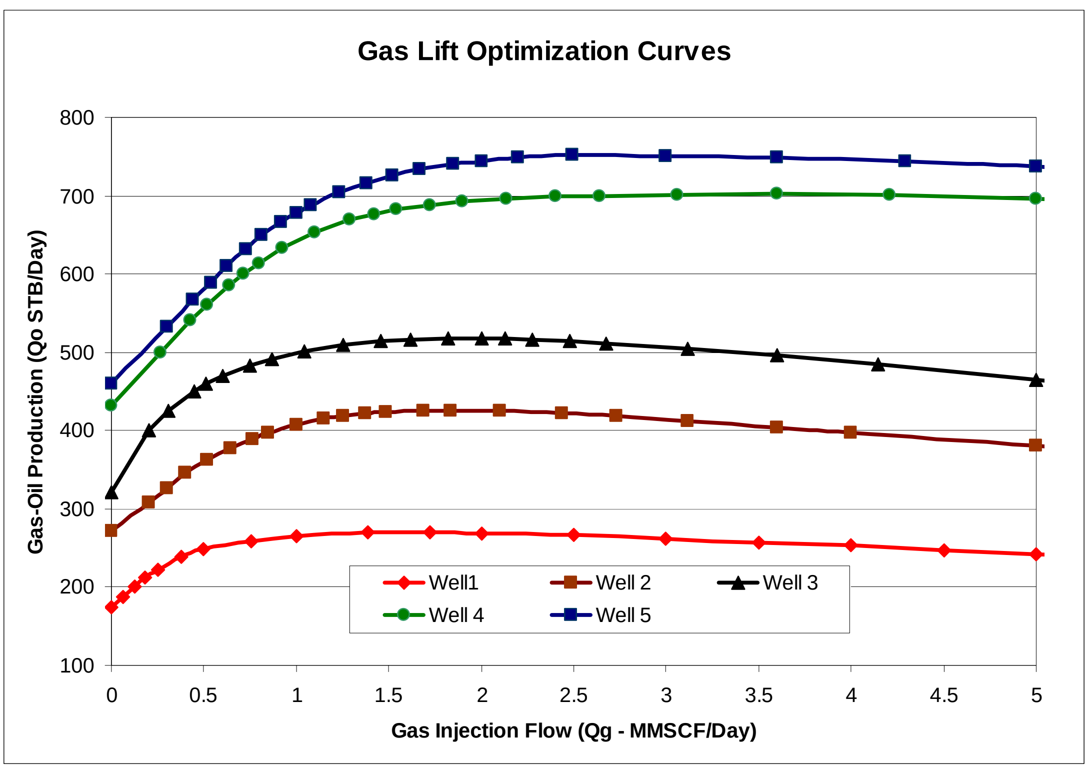

Figure B-2: Sample Gas Lift Curve

APPENDIX C: GAS LIFT CURVE CONTINUOUS FUNCTION COEFFICIENTS

COMPUTED COEFFICIENTS FOR EXPONENTIALS FOR EACH WELL

Table C-1 lists the exponential equation coefficients computed using PROC NLIN for each well. These are used in the computer optimization. These coefficients can be quickly recomputed based on streaming data on production/gas injection rates provided by Supervisory Control And Data Acquisition (SCADA) systems or otherwise collected by field data technicians. This would allow an almost real-time optimization of gas injection rates.

Well	a_0	a_1	a_2	a_3
1	0.039966	6.655841	541.225372	114.203008
2	0.029669	2.684351	868.144808	98.553974
3	0.034044	2.011141	1225.818612	179.272143
4	0.050324	4.228531	1169.122720	28.884721
5	0.236114	1.069165	2410.292270	156.828742
6	0.130546	1.972908	117.709559	175.226346
7	0.192546	1.077173	234.275314	266.932443
8	0.171796	1.377453	257.236551	330.693291
9	0.033347	1.189505	313.680046	423.355490
10	0.058388	1.002286	370.620086	446.608688
11	0.063946	12.646097	649.470446	216.985716
12	0.047471	2.952786	1736.289617	187.252551
13	0.040853	2.413370	1470.982335	304.762644
14	0.075487	6.342796	1636.771808	57.769442
15	0.425005	1.924497	2651.321497	203.877365
16	0.248038	3.353944	164.793383	350.452691
17	0.365838	1.831194	257.702845	507.171641
18	0.075936	7.987009	811.838058	125.623309
19	0.056689	1.665306	376.416055	677.368784
20	0.105098	1.804116	630.054146	759.234769
21	0.039966	6.655841	541.225372	114.203008
22	0.029669	2.684351	868.144808	98.553974
23	0.034044	2.011141	1225.818612	179.272143
24	0.050324	4.228531	1169.122720	28.884721
25	0.236114	1.069165	2410.292270	156.828742
26	0.130546	1.972908	117.709559	175.226346
27	0.192546	1.077173	234.275314	266.932443
28	0.171796	1.377453	257.236551	330.693291
29	0.033347	1.189505	313.680046	423.355490
30	0.058388	1.002286	370.620086	446.608688
31	0.079932	11.980513	865.960595	125.623309
32	0.032636	3.221221	1302.217213	197.107948
33	0.068089	2.815598	2329.055364	250.981001
34	0.090584	8.034208	1402.947264	31.773193
35	0.283337	1.710664	3856.467632	282.291736
36	0.156655	2.762071	235.419118	315.407422
37	0.308074	1.400324	445.123096	507.171641
38	0.075936	9.983761	649.470446	194.145114
39	0.040016	1.784257	595.992087	846.710980
40	0.075904	2.004573	667.116155	848.556507
41	0.250310	2.154345	351.412971	453.785153
42	0.327329	1.508042	304.557908	480.478397
43	0.385093	2.154345	421.695565	507.171641
44	0.346583	1.723476	327.985439	320.318931
45	0.288819	2.154345	374.840502	453.785153
46	0.269565	2.154345	351.412971	400.398664
47	0.250310	1.400324	304.557908	373.705420
48	0.385093	2.154345	257.702845	533.864885
49	0.250310	2.046628	351.412971	533.864885
50	0.231056	1.400324	257.702845	427.091908
51	0.044504	11.980513	865.960595	194.145114
52	0.044504	8.652593	595.347909	137.043610
53	0.050438	9.318177	757.715521	228.406017
54	0.032636	9.318177	649.470446	182.724813
55	0.038570	10.649345	920.083132	228.406017
56	0.053405	10.649345	757.715521	228.406017
57	0.041537	9.318177	865.960595	171.304512
58	0.047471	9.983761	1082.450744	194.145114
59	0.056372	7.321425	649.470446	125.623309
60	0.035603	7.987009	1082.450744	148.463911
61	0.041537	8.652593	920.083132	228.406017
62	0.032636	13.311681	1082.450744	216.985716
63	0.050438	8.652593	703.592983	159.884212
64	0.044504	11.980513	974.205669	216.985716
65	0.032636	11.314929	920.083132	171.304512
66	0.053405	11.980513	1028.328206	182.724813
67	0.059339	9.318177	703.592983	171.304512
68	0.032636	12.646097	920.083132	194.145114
69	0.032636	7.321425	811.838058	205.565415
70	0.051066	3.016712	1838.727919	250.981001
71	0.042555	2.212256	1838.727919	268.908215
72	0.044258	2.614484	1838.727919	215.126572
73	0.037449	2.212256	1593.564196	197.199358
74	0.040853	2.212256	1593.564196	242.017393
75	0.039151	2.513927	1287.109543	259.944608
76	0.035747	3.016712	1654.855127	224.090179
77	0.049364	2.111698	1287.109543	250.981001
78	0.047662	2.715041	1532.273266	206.162965
79	0.044258	2.111698	1716.146057	197.199358
80	0.051066	2.815598	1777.436988	188.235750
81	0.049364	2.413370	1654.855127	215.126572
82	0.045960	2.715041	1777.436988	250.981001
83	0.035747	2.513927	1654.855127	259.944608
84	0.037449	2.715041	1348.400474	197.199358
85	0.049364	2.916155	1287.109543	197.199358
86	0.040853	2.614484	1838.727919	197.199358
87	0.042555	2.614484	1654.855127	197.199358
88	0.040853	2.312813	1593.564196	224.090179
89	0.035747	2.111698	1838.727919	188.235750
90	0.080519	5.074237	1870.596352	31.773193
91	0.040853	2.715041	1654.855127	268.908215
92	0.090584	8.034208	1286.034992	54.880970
93	0.085551	7.611355	1753.684080	57.769442
94	0.047662	2.212256	1716.146057	233.053786
95	0.045960	3.016712	1777.436988	268.908215
96	0.044258	2.815598	1348.400474	250.981001
97	0.070454	7.611355	1636.771808	37.550137
98	0.044258	2.413370	1532.273266	206.162965
99	0.040853	3.016712	1593.564196	188.235750
100	0.065422	6.342796	1402.947264	34.661665

Table C-1: Coefficients for Gas Lift Equations for 100 Wells

APPENDIX D: CONSTRAINED GAS LIFT OPTIMIZATION RESULTS

OPTIMIZATION RESULTS USING EQUATIONS 3, 4 AND 5 – OPTIMAL GAS LIFT USAGE

A series of optimizations were completed for gas injection rates between 10 and 100 MMCF/d availability across the 100 wells. The results are shown in **Table D-1** and **Figure D-1**. At the lowest injection gas availability rate (10 MMCF/d) many wells were not allocated any gas injection. At the highest available injection rate (100 MMCF/d) the optimal production was 123,206.88 MMCF/d.

Gas Injection Rate (MMCF/d)	Total Production (STDB/d)
0	24,639.74
10	69,312.97
20	89,847.46
30	102,098.40
40	109,540.32
50	114,295.14
60	117,504.95
70	119,776.18
80	121,382.76
90	122,485.52
100	123,206.88

Table D-1: Optimization Results - Optimal Gas Usage

Figure D-1 shows the total oil production (Qo) for increasing total injection rates (Qg). As the injection rate reaches the 60 MMSCF/d range, the curve flattens, indicating that increased injection rates have little additional production benefit.

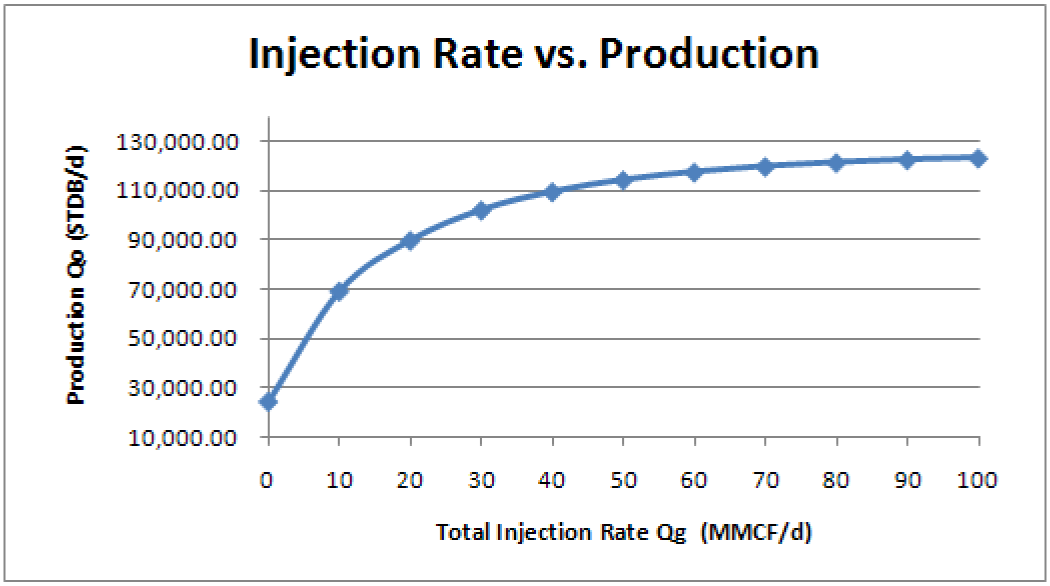

Figure D-1: Optimization Results for Variable Gas Injection Rates from 0.0 to 100.0 MMCF/d

APPENDIX E: SAS PROGRAM CODE

```
/* Filename:  Optimize_Gas_Inject_01-14-11.sas     */
/* Includes matrix (index) formulation of the optimization      */
/* Used in conjunction with Dynamic Gas Optimization using SAS  */
/* Date: 01-14-11 */
/* Author:  Robert N Hatton, SAIC & Kenneth M. Potter, SAIC     */
/* SAS 9.21_M3 (TS2M3) with SAS/OR 9.22     */

/* Reads in data for gas optimization problem for any number of different wells
   and computes the optimum solution using PROC OPTMODEL */

/* Gas lift optimization problem for multiple wells in a field uses an exponential
   approximation with four parameters for the gas lift production curve      */

/* Macro gas_opt takes two parameters (1) total gas available for injection
   and (2) file_name - contains the coefficients of the exponential equations */

 %MACRO gas_opt (Max_gas, file_name);

/* Input parameters to macro are:     */
/*  Max_gas - the maximum available injection gas rate (MMCF/d)  */
/*  file_name - file with the coefficients of the exponential equations      */

PROC OPTMODEL;
SET<num> indx ;
VAR x{indx} >= 0;
number a0{indx}, a1{indx},a2{indx},a3{indx}, xprod{indx}, xsum;

/* Read coefficients of a indx x 4 matrix corresponding to the indx well equations
*/
/* f(x) = a3 + a2*(exp(-ao*x/(a0/a1))-exp(-a0*x))/(a0/a1-1))    */
/* where x = gas pressure for an individual well   */
/*           a0 to a3 represent the coefficients for production equation    */
/* These coefficients were calculated with proc nlin from data points  */
/* on indx gas lift production curves */

READ DATA sasuser.&file_name INTO /* Assumes files are located in SASUSER    */
indx=[_N_]
a0=a0
a1=a1
a2=a2
a3=a3
;

/* a0[j]..a3[j] corresponds to the coefficents of the jth Gas_Optimization curve*/
/* Following constraint ensures sum of gas injection does not exceed capacity */

CON constr1:  SUM{i in indx}x[i] <= &Max_gas;

/*  Following is the optimization of max production across the indx wells    */

MAX prod = SUM{i in indx} (  a3[i]+ a2[i]*(exp(-a1[i]*x[i])-exp(-
a0[i]*x[i]))/(a0[i]/a1[i]-1));

SOLVE;       /*with IPNLP;   Solves with NLP Interior Point solver     */

xsum = (SUM{i in indx} x[i]) ;  /* sums the individual well injection rates  */
CREATE DATA sasuser.wells_out   /* Creates a data file in sasuser of the results
    */
FROM [i] = indx x xprod =  (  a3[i]+a2[i]*(exp(-a0[i]*x[i]/(a0[i]/a1[i]))
    -exp(-a0[i]*x[i]))/(a0[i]/a1[i]-1)) xmax =  (log(a0[i]/a1[i])/(a0[1]-a1[i]));
QUIT;
```

```
%MEND;   /* End of macro */

/* The following macro call computes the optimal solution for the available gas*/
/* (the first parameter).  The second parameter is a value used to pass a    */
/* text constant for creation of file name and titles.   */

DATA _null_;
%gas_opt(100,data_in);
RUN;

/* NOTE: See TABLE C-1 for input data set for %GAS_OPT macro */
```