SAS/QC® 13.2 User's Guide

The CUSUM Procedure

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Chapter 6
The CUSUM Procedure

Contents

Introduction: CUSUM Procedure

The CUSUM procedure creates cumulative sum control charts, also known as *cusum charts*, which display cumulative sums of the deviations of measurements or subgroup means from a target value. Cusum charts are used to decide whether a process is in statistical control by detecting a shift in the process mean.

You can use the CUSUM procedure to

- apply a *one-sided cusum scheme*, also referred to as a *decision interval scheme*, which detects a shift in one direction from the target mean. You can specify the scheme with the decision interval h and the reference value k.
- apply a *two-sided cusum scheme* with a V-mask, which detects a shift in either direction from the target mean. You can specify the scheme with geometric parameters (h and k) for the V-mask or with error probabilities (α and β).
- implement cusum schemes graphically or computationally
- specify the shift to be detected as a multiple of standard error or in data units
- estimate the process standard deviation σ using a variety of methods
- compute average run lengths (ARLs)
- read raw data (actual measurements) or summarized data (subgroup means and standard deviations)
- analyze multiple process variables. If used with a BY statement, PROC CUSUM produces charts separately for groups of observations.
- save cusums and cusum scheme parameters in output data sets
- tabulate the information displayed on the chart
- read cusum scheme parameters from an input data set
- read numeric- or character-valued subgroup variables
- display subgroups with date and time formats
- enhance cusum charts with special legends and symbol markers that indicate the levels of stratification variables
- superimpose plotted points with stars (polygons) whose vertices indicate the values of multivariate data related to the process

- display a trend chart below the cusum chart that plots a systematic or fitted trend in the data
- produce charts as traditional graphics, ODS Graphics output, or legacy line printer charts. Line printer charts can use special formatting characters that improve the appearance of the chart. Traditional graphics can be annotated, saved, and replayed.

Learning about the CUSUM Procedure

If you are using the CUSUM procedure for the first time, begin by reading "PROC CUSUM Statement" on page 541 to learn about input data sets. Then turn to "Getting Started: XCHART Statement" on page 547 in "XCHART Statement: CUSUM Procedure" on page 546. This chapter also provides syntax information and advanced examples.

If you are not familiar with cusum charts, read "Formulas for Cumulative Sums" on page 577 "Defining the Decision Interval for a One-Sided Cusum Scheme" on page 579 and "Defining the V-Mask for a Two-Sided Cusum Scheme" on page 580 in the section "Details: XCHART Statement" on page 576. References lists articles and textbooks that provide more detailed information on cusum charts. The expository articles by Lucas (1976) and Goel (1982) and the textbooks by Montgomery (1996) and Ryan (1989) are recommended introductory reading.

PROC CUSUM Statement

Overview: PROC CUSUM Statement

The PROC CUSUM statement starts the CUSUM procedure and it identifies input data sets.

After the PROC CUSUM statement, you provide an XCHART statement that specifies the cusum chart you want to create and the variables in the input data set that you want to analyze. For example, the following statements request a one-sided (decision interval) cusum chart:

```
proc cusum data=values;
   xchart weight*lot / scheme = onesided
                       mu0    = 8.100
                       sigma0 = 0.050
                       delta  = 1
                       h      = 2.2
                       k      = 0.5;
run;
```

In this example, the DATA= option specifies an input data set (values) that contains the *process* measurement variable weight and the *subgroup-variable* lot.

You can use options in the PROC CUSUM statement to do the following:

- specify input data sets containing variables to be analyzed, parameters for cusum schemes, or annotation information

- specify a graphics catalog for saving traditional graphics output
- specify that line printer charts are to be produced
- define characters used for features on line printer charts

In addition to the XCHART statement, you can provide BY statements, ID statements, TITLE statements, and FOOTNOTE statements. If you are producing traditional graphics, you can also provide graphics enhancement statements, such as SYMBOL*n* statements, which are described in *SAS/GRAPH: Reference*.

See Chapter 3, "SAS/QC Graphics," for a detailed discussion of the alternatives available for producing charts with SAS/QC procedures.

NOTE: If you are using the CUSUM procedure for the first time, you should read both this chapter and the section "Getting Started: XCHART Statement" on page 547 in "XCHART Statement: CUSUM Procedure" on page 546.

Syntax: PROC CUSUM Statement

The syntax for the PROC CUSUM statement is as follows:

PROC CUSUM < *options* > **;**

The PROC CUSUM statement starts the CUSUM procedure, and it optionally identifies various data sets. You can specify the following options in the PROC CUSUM statement.

ANNOTATE=*SAS-data-set*

ANNO=*SAS-data-set*

specifies an input data set that contains appropriate annotate variables, as described in *SAS/GRAPH: Reference*. The ANNOTATE= option enables you to add features to a cusum chart (for example, labels that explain out-of-control points). The ANNOTATE= data set is used only when the chart is created as traditional graphics; it is ignored when the LINEPRINTER option is specified or ODS Graphics is enabled. The data set specified with the ANNOTATE= option in the PROC CUSUM statement is a "global" annotate data set in the sense that the information in this data set is displayed on every chart produced in the current run of the CUSUM procedure.

ANNOTATE2=*SAS-data-set*

ANNO2=*SAS-data-set*

specifies an input data set that contains appropriate annotate variables that add features to the trend chart (secondary chart) produced with the TRENDVAR= option in the XCHART statement. This option applies only when you produce traditional graphics.

DATA=*SAS-data-set*

names an input data set that contains raw data (measurements) as observations. If the values of the *subgroup-variable* are numeric, you need to sort the data set so that these values are in increasing order (within BY groups). The DATA= data set can contain more than one observation for each value of the *subgroup-variable*.

You cannot use a DATA= data set with a HISTORY= data set. If you do not specify a DATA= or HISTORY= data set, PROC CUSUM uses the most recently created data set as a DATA= data set. For more information, see "DATA= Data Set" on page 592

FORMCHAR(*index*)='*string*'

defines characters used for features on legacy line printer charts, where *index* is a list of numbers ranging from 1 to 17 and *string* is a character or hexadecimal string. This option applies only if you also specify the LINEPRINTER option.

The *index* identifies which features are controlled with the *string* characters, as described in the following table. If you specify the FORMCHAR= option and omit the *index*, the *string* controls all 17 features.

Value of *index*	Description of Character	Chart Feature
1	vertical bar	frame
2	horizontal bar	frame, central line
3	box character (upper left)	frame
4	box character (upper middle)	serifs, tick (horizontal axis)
5	box character (upper right)	frame
6	box character (middle left)	not used
7	box character (middle middle)	serifs
8	box character (middle right)	tick (vertical axis)
9	box character (lower left)	frame
10	box character (lower middle)	serifs
11	box character (lower right)	frame
12	vertical bar	control limits
13	horizontal bar	control limits
14	box character (upper right)	control limits
15	box character (lower left)	control limits
16	box character (lower right)	control limits
17	box character (upper left)	control limits

Not all printers can produce the characters in the preceding list. By default, the form character list specified by the SAS system option FORMCHAR= is used; otherwise, the default is FORMCHAR='|—-|+|—|====='. If you print to a PC screen or if your device supports the ASCII symbol set (1 or 2), the following is recommended:

```
formchar='B3,C4,DA,C2,BF,C3,C5,B4,C0,C1,D9,BA,CD,BB,C8,BC,D9'X
```

Note that you can use the FORMCHAR= option to temporarily override the values of the SAS system FORMCHAR= option. The values of the SAS system FORMCHAR= option are not altered by the FORMCHAR= option in the PROC CUSUM statement.

GOUT=*graphics-catalog*

specifies the graphics catalog for traditional graphics output from PROC CUSUM. This is useful if you want to save the output. The GOUT= option is used only when the chart is created using traditional graphics; it is ignored when the LINEPRINTER option is specified or ODS Graphics is enabled.

HISTORY=*SAS-data-set*

HIST=*SAS-data-set*

names an input data set that contains subgroup summary statistics (means, standard deviations, and sample sizes). Typically, this data set is created as an OUTHISTORY= data set in a previous run of

PROC CUSUM or PROC SHEWHART, but it can also be created with a SAS summarization procedure such as PROC MEANS.

If the values of the *subgroup-variable* are numeric, you need to sort the data set so that these values are in increasing order (within BY groups). A HISTORY= data set can contain only one observation for each value for the *subgroup-variable*.

You cannot use a HISTORY= data set together with a DATA= data set. If you do not specify a HISTORY= or DATA= data set, PROC CUSUM uses the most recently created data set as a DATA= data set. For more information on HISTORY= data sets, see "HISTORY= Data Set" on page 593.

LIMITS=*SAS-data-set*

names an input data set that contains a set of decision interval or V-mask parameters. Each observation in a LIMITS= data set contains the parameters for a *process*.

If you are using SAS 6.09 or an earlier release of SAS/QC software, you must specify the options READLIMITS or READINDEX= in the XCHART statement to read the parameters from the LIMITS= data set. In SAS 6.10 and later releases, these options are not needed.

For details about the variables needed in a LIMITS= data set, see "LIMITS= Data Set" on page 592. If you do not provide a LIMITS= data set, you must specify the parameters with options in the XCHART statement.

LINEPRINTER

requests that legacy line printer charts be produced.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC CUSUM to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

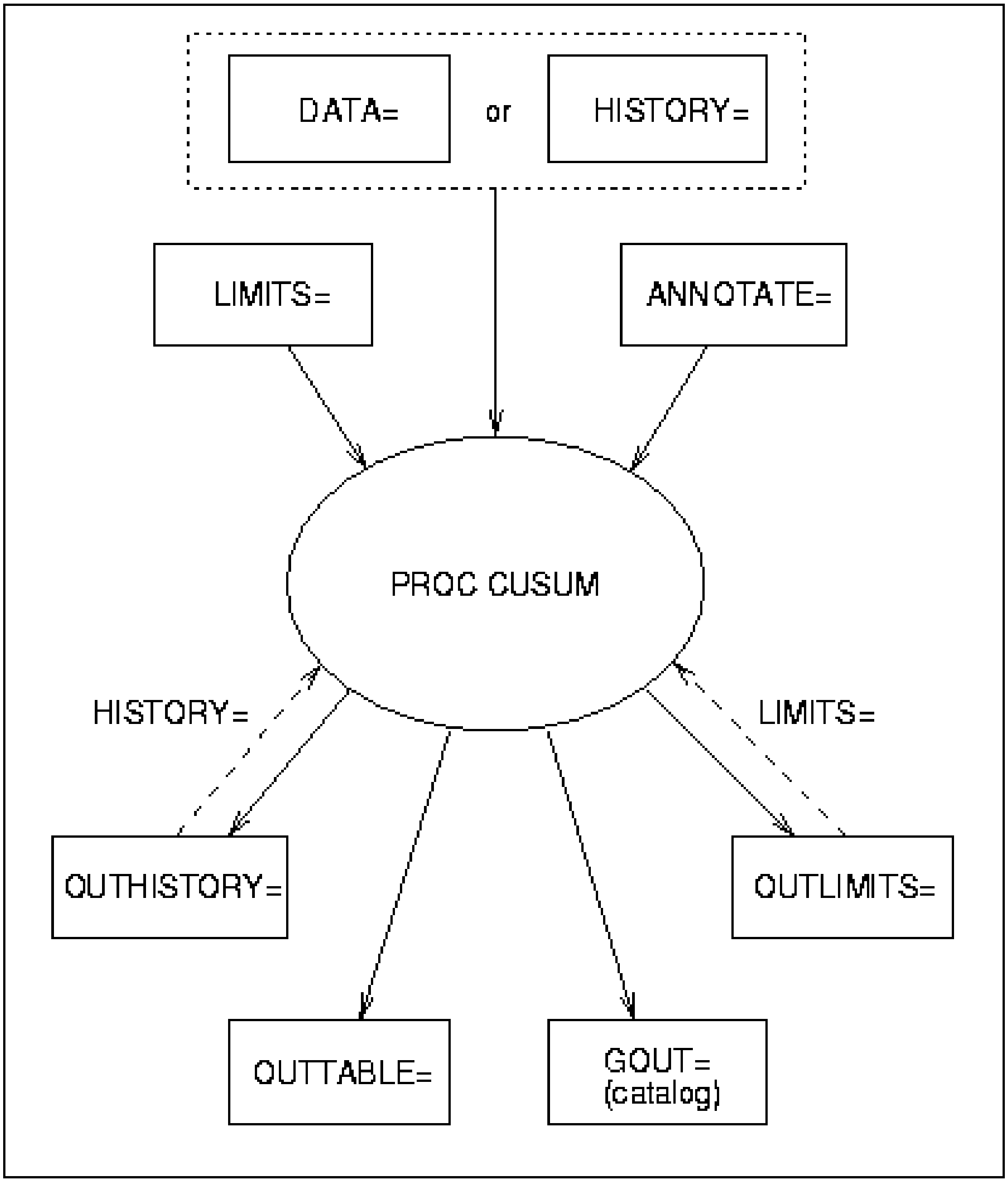
- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the CUSUM procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

Input and Output Data Sets: CUSUM Procedure

Figure 6.1 summarizes the data sets used with the CUSUM procedure.

Figure 6.1 Input and Output Data Sets in the CUSUM Procedure



XCHART Statement: CUSUM Procedure

Overview: XCHART Statement

The XCHART statement creates cumulative sum control charts from subgroup means or individual measurements. You can create these charts for one-sided cusum (decision interval) schemes or for two-sided (V-mask) schemes. A one-sided scheme is designed to detect either a positive or a negative shift from the target mean, and a two-sided scheme is designed to detect positive and negative shifts from the target mean.

You can use options in the XCHART statement to

- specify parameters for a decision interval or V-mask
- specify the shift δ to be detected
- specify the target mean μ_0
- specify a known (standard) value σ_0 for the process standard deviation or estimate the standard deviation from the data using various methods
- tabulate the information displayed on the chart
- save the information displayed on the chart in an output data set
- read parameters for the cusum scheme from a data set
- display a secondary chart that plots a time trend that has been removed from the data
- add block legends and special symbol markers to reveal stratification in process data
- superimpose stars at each point to represent related multivariate factors
- display vertical and horizontal reference lines
- modify the axis values and labels
- modify the chart layout and appearance

You have three alternatives for producing cumulative sum control charts with the XCHART statement:

- ODS Graphics output is produced if ODS Graphics is enabled, for example by specifying the ODS GRAPHICS ON statement prior to the PROC statement.
- Otherwise, traditional graphics are produced by default if SAS/GRAPH® is licensed.
- Legacy line printer charts are produced when you specify the LINEPRINTER option in the PROC statement.

See Chapter 3, “SAS/QC Graphics,” for more information about producing these different kinds of graphs.

Getting Started: XCHART Statement

This section introduces the XCHART statement with simple examples that illustrate the most commonly used options. Complete syntax for the XCHART statement is presented in the section "Syntax: XCHART Statement" on page 561, and advanced examples are given in the section "Examples: XCHART Statement" on page 594.

Creating a V-Mask Cusum Chart from Raw Data

NOTE: See *Two-sided Cusum Chart with V-Mask* in the SAS/QC Sample Library.

A machine fills eight-ounce cans of two-cycle engine oil additive. The filling process is believed to be in statistical control, and the process is set so that the average weight of a filled can is $\mu_0 = 8.100$ ounces. Previous analysis shows that the standard deviation of fill weights is $\sigma_0 = 0.050$ ounces. A two-sided cusum chart is used to detect shifts of at least one standard deviation in either the positive or negative direction from the target mean of 8.100 ounces.

Subgroup samples of four cans are selected every hour for twelve hours. The cans are weighed, and their weights are saved in a SAS data set named Oil.

```
data Oil;
   label Hour = 'Hour';
   input Hour @;
   do i=1 to 4;
      input Weight @;
      output;
   end;
   drop i;
   datalines;
 1  8.024  8.135  8.151  8.065
 2  7.971  8.165  8.077  8.157
 3  8.125  8.031  8.198  8.050
 4  8.123  8.107  8.154  8.095
 5  8.068  8.093  8.116  8.128
 6  8.177  8.011  8.102  8.030
 7  8.129  8.060  8.125  8.144
 8  8.072  8.010  8.097  8.153
 9  8.066  8.067  8.055  8.059
10  8.089  8.064  8.170  8.086
11  8.058  8.098  8.114  8.156
12  8.147  8.116  8.116  8.018
;
```

The data set Oil is partially listed in Figure 6.2.

Figure 6.2 Partial Listing of the Data Set Oil

Hour	Weight
1	8.024
1	8.135
1	8.151
1	8.065
2	7.971
2	8.165
2	8.077
2	8.157
3	8.125
3	8.031
3	8.198
3	8.050
4	8.123
4	8.107

Each observation contains one value of Weight along with its associated value of Hour, and the values of Hour are in increasing order. The CUSUM procedure assumes that DATA= input data sets are sorted in this “strung-out” form.

The following statements request a two-sided cusum chart with a V-mask for the average weights:

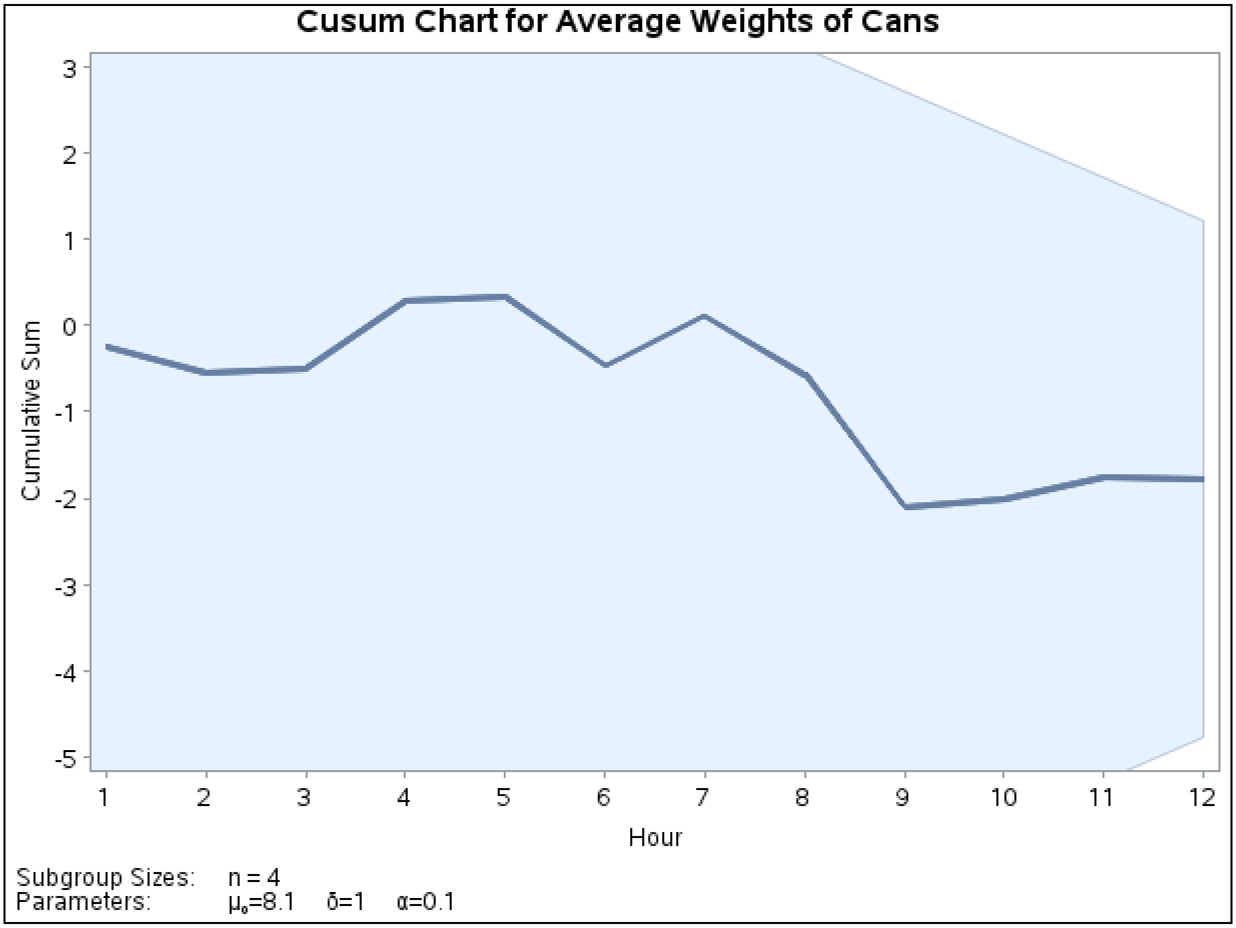
```
ods graphics off;
title 'Cusum Chart for Average Weights of Cans';
proc cusum data=Oil;
   xchart Weight*Hour /
      mu0    = 8.100          /* Target mean for process  */
      sigma0 = 0.050          /* Known standard deviation */
      delta  = 1              /* Shift to be detected     */
      alpha  = 0.10           /* Type I error probability */
      vaxis  = -5 to 3 ;
   label Weight = 'Cumulative Sum';
run;
```

The CUSUM procedure is invoked with the PROC CUSUM statement. The DATA= option in the PROC CUSUM statement specifies that the SAS data set Oil is to be read. The variables to be analyzed are specified in the XCHART statement. The process measurement variable (Weight) is specified before the asterisk (this variable is referred to more generally as a *process*). The time variable (Hour) is specified after the asterisk (this variable is referred to more generally as a *subgroup-variable* because it determines how the measurements are classified into rational subgroups).

The option ALPHA=0.10 specifies the probability of a Type 1 error for the cusum scheme (the probability of detecting a shift when none occurs).

The cusum chart is shown in Figure 6.3.

Figure 6.3 Two-Sided Cusum Chart with V-Mask



The cusum S_1 plotted at Hour=1 is simply the standardized deviation of the first subgroup mean from the target mean.

$$S_1 = \frac{8.09375 - 8.100}{0.050/\sqrt{4}} = -0.250$$

The cusum S_2 plotted at Hour=2 is S_1 plus the standardized deviation of the second subgroup mean from the target mean.

$$S_2 = S_1 + \frac{8.0925 - 8.100}{0.050/\sqrt{4}} = -0.550$$

In general, the cusum plotted at Hour=t is S_{t-1} plus the standardized deviation of the tth subgroup mean from the target mean.

$$S_t = S_{t-1} + \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}}$$

For further details, see "Two-Sided Cusum Schemes" on page 578.

You can interpret the chart by comparing the points with the V-mask whose right edge is centered at the most recent point (Hour=12). Since none of the points cross the arms of the V-mask, there is no evidence that a shift has occurred, and the fluctuations in the cusums can be attributed to chance variation. In general, crossing the lower arm is evidence of an increase in the process mean, whereas crossing the upper arm is evidence of a decrease in the mean.

Creating a V-Mask Cusum Chart from Subgroup Summary Data

NOTE: See *Two-sided Cusum Chart with V-Mask* in the SAS/QC Sample Library.

The previous example illustrates how you can create a cusum chart using raw process measurements read from a DATA= data set. In many applications, however, the data are provided in *summarized form* as subgroup means. This example illustrates the use of the XCHART statement when the input data set is a HISTORY= data set.

The following data set provides the subgroup means, standard deviations, and sample sizes corresponding to the variable Weight in the data set Oil (see the section "Creating a V-Mask Cusum Chart from Raw Data" on page 547:

```
data Oilstat;
   label Hour = 'Hour';
   input Hour WeightX WeightS WeightN;
   datalines;
 1  8.0938  0.0596  4
 2  8.0925  0.0902  4
 3  8.1010  0.0763  4
 4  8.1198  0.0256  4
 5  8.1013  0.0265  4
 6  8.0800  0.0756  4
 7  8.1145  0.0372  4
 8  8.0830  0.0593  4
 9  8.0618  0.0057  4
10  8.1023  0.0465  4
11  8.1065  0.0405  4
12  8.0993  0.0561  4
;
```

The data set Oilstat is listed in Figure 6.4.

Figure 6.4 Listing of the Data Set Oilstat

Obs	Hour	WeightX	WeightS	WeightN
1	1	8.0938	0.0596	4
2	2	8.0925	0.0902	4
3	3	8.1010	0.0763	4
4	4	8.1198	0.0256	4
5	5	8.1013	0.0265	4
6	6	8.0800	0.0756	4
7	7	8.1145	0.0372	4
8	8	8.0830	0.0593	4
9	9	8.0618	0.0057	4
10	10	8.1023	0.0465	4
11	11	8.1065	0.0405	4
12	12	8.0993	0.0561	4

Since the data set contains a subgroup variable, a mean variable, a standard deviation variable, and a sample size variable, it can be read as a HISTORY= data set. Note that the names WeightX, WeightS, and WeightN satisfy the naming conventions for summary variables since they begin with a common prefix (Weight) and end with the suffix letters *X*, *S*, and *N*.

The following statements create the cusum chart:

```
title 'Cusum Chart for Average Weights of Cans';
proc cusum history=Oilstat;
   xchart Weight*Hour /
      mu0    = 8.100          /* target mean              */
      sigma0 = 0.050          /* known standard deviation */
      delta  = 1              /* shift to be detected     */
      alpha  = 0.10           /* Type 1 error probability */
      vaxis  = -5 to 3 ;
   label WeightX = 'Cumulative Sum';
run;
```

Note that the *process* Weight specified in the XCHART statement is the prefix of the summary variable names in Oilstat. Also note that the vertical axis label is specified by associating a variable label with the subgroup mean variable (WeightX). The chart (not shown here) is identical to the one in Figure 6.2.

In general, a HISTORY= input data set used with the XRCHART statement must contain the following four variables:

- subgroup variable
- subgroup mean variable
- subgroup range variable
- subgroup sample size variable

Furthermore, the names of subgroup mean, standard deviation, and sample size variables must begin with the prefix *process* specified in the XRCHART statement and end with the special suffix characters *X*, *S*, and *N*, respectively.

Note that the interpretation of *process* depends on the input data set specified in the PROC CUSUM statement.

- If raw data are read using the DATA= option (as in the previous example), *process* is the name of the SAS variable containing the process measurements.
- If summary data are read using the HISTORY= option (as in this example), *process* is the common prefix for the names containing the summary statistics.

For more information, see "DATA= Data Set" on page 592 and "HISTORY= Data Set" on page 593.

Saving Summary Statistics

NOTE: See *Two-sided Cusum Chart with V-Mask* in the SAS/QC Sample Library.

In this example, the CUSUM procedure is used to save summary statistics and cusums in an output data set. The summary statistics can subsequently be analyzed by the CUSUM procedure (as in the preceding example). The following statements read the raw measurements from the data set Oil (see "Creating a V-Mask Cusum Chart from Raw Data" on page 547) and create a summary data set named Oilhist:

```
title 'Cusum Chart for Average Weights of Cans';
proc cusum data=Oil;
   xchart Weight*Hour /
      nochart
      outhistory = Oilhist
      mu0        = 8.100        /* Target mean for process  */
      sigma0     = 0.050        /* Known standard deviation */
      delta      = 1            /* Shift to be detected     */
      alpha      = 0.10         /* Type I error probability */
      vaxis      = -5 to 3 ;
   label Weight = 'Cumulative Sum';
run;
```

The OUTHISTORY= option names the SAS data set containing the summary information, and the NOCHART option suppresses the display of the charts (since the purpose here is simply to create an output data set). Figure 6.5 lists the data set Oilhist.

Figure 6.5 Listing of the Data Set Oilhist

Cusum Chart for Average Weights of Cans

Obs	Hour	WeightX	WeightS	WeightC	WeightN
1	1	8.0938	0.0596	-.2500	4
2	2	8.0925	0.0902	-.5500	4
3	3	8.1010	0.0763	-.5100	4
4	4	8.1198	0.0256	0.2800	4
5	5	8.1013	0.0265	0.3300	4
6	6	8.0800	0.0756	-.4700	4
7	7	8.1145	0.0372	0.1100	4
8	8	8.0830	0.0593	-.5700	4
9	9	8.0618	0.0057	-2.100	4
10	10	8.1023	0.0465	-2.010	4
11	11	8.1065	0.0405	-1.750	4
12	12	8.0993	0.0561	-1.780	4

There are five variables in the data set.

- Hour contains the subgroup index
- WeightX contains the subgroup means
- WeightS contains the subgroup standard deviations
- WeightC contains the cumulative sums

- WeightN contains the subgroup sample sizes

Note that the variables in the OUTHISTORY= data set are named by adding the suffix characters *X*, *S*, *N*, and *C* to the *process* Weight specified in the XCHART statement. In other words, the variable naming convention for OUTHISTORY= data sets is the same as for HISTORY= data sets.

For more information, see "OUTHISTORY= Data Set" on page 589.

Creating a One-Sided Cusum Chart with a Decision Interval

NOTE: See *One-sided Cusum Chart* in the SAS/QC Sample Library.

An alternative to the V-mask cusum chart is the one-sided cusum chart with a decision interval, which is sometimes referred to as the "computational form of the cusum chart." This example illustrates how you can create a one-sided cusum chart for individual measurements.

A can of oil is selected every hour for fifteen hours. The cans are weighed, and their weights are saved in a SAS data set named Cans:[1]

```
data Cans;
   length comment $16;
   label Hour = 'Hour';
   input Hour Weight comment $16. ;
   datalines;
 1  8.024
 2  7.971
 3  8.125
 4  8.123
 5  8.068
 6  8.177  Pump Adjusted
 7  8.229  Pump Adjusted
 8  8.072
 9  8.066
10  8.089
11  8.058
12  8.147
13  8.141
14  8.047
15  8.125
;
```

Suppose the problem is to detect a *positive* shift in the process mean of one standard deviation ($\delta = 1$) from the target of 8.100 ounces. Furthermore, suppose that

- a known value $\sigma_0 = 0.050$ is available for the process standard deviation
- an in-control average run length (ARL) of approximately 100 is required
- an ARL of approximately five is appropriate for detecting the shift

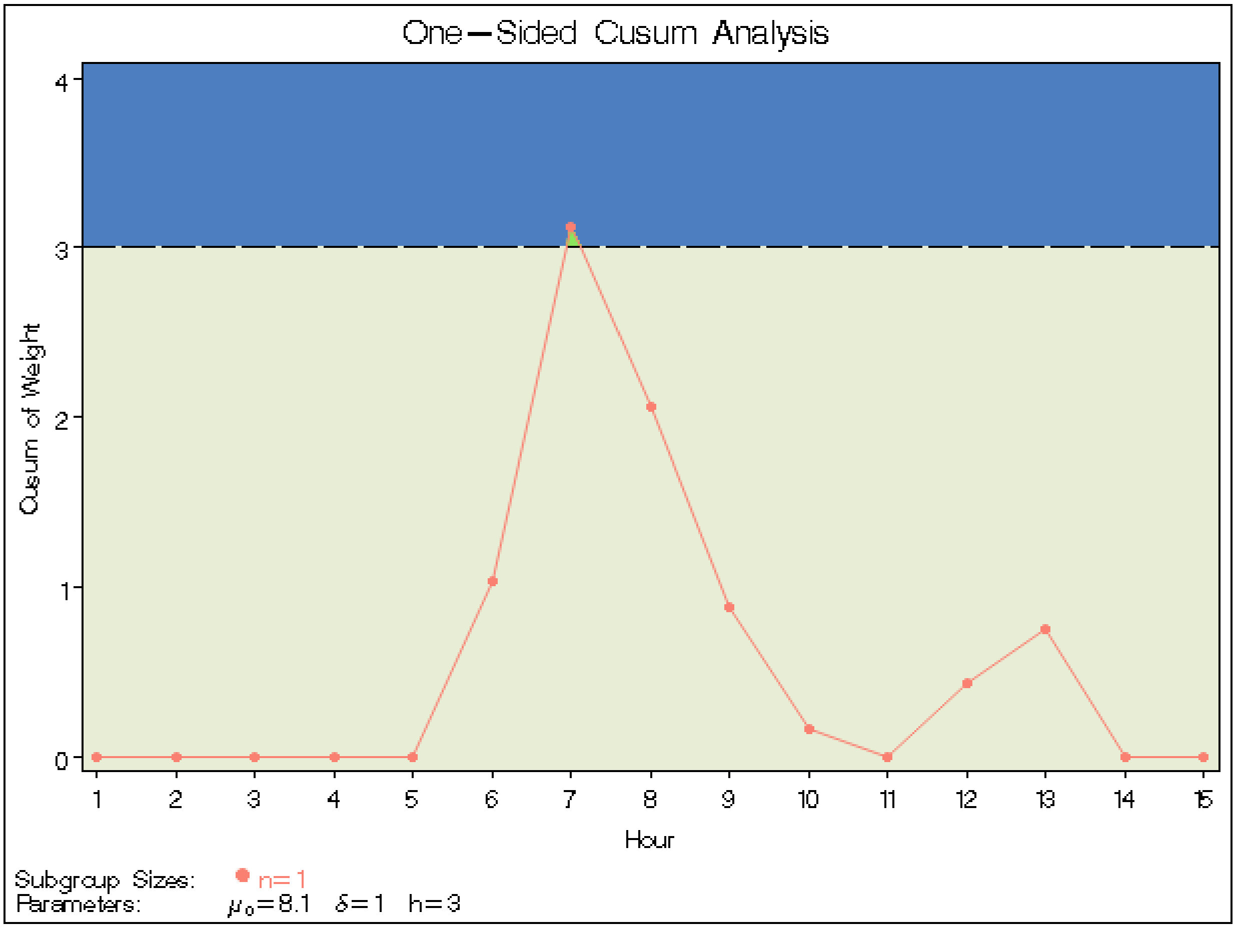
[1]This data set is used by later examples in this chapter.

Table 6.4 indicates that these ARLs can be achieved with the decision interval $h = 3$ and the reference value $k = 0.5$. The following statements use these parameters to create the chart and tabulate the cusum scheme:

```
options nogstyle;
goptions ftext=swiss;
symbol v=dot color=salmon h=1.8 pct;
title "One-Sided Cusum Analysis";
proc cusum data=Cans;
   xchart Weight*Hour /
      mu0      = 8.100     /* target mean for process     */
      sigma0   = 0.050     /* known standard deviation    */
      delta    = 1         /* shift to be detected        */
      h        = 3         /* cusum parameter h           */
      k        = 0.5       /* cusum parameter k           */
      scheme   = onesided  /* one-sided decision interval */
      tableall             /* table                       */
      cinfill  = ywh
      cframe   = bigb
      cout     = salmon
      cconnect = salmon
      climits  = black
      coutfill = bilg;
   label Weight = 'Cusum of Weight';
run;
options gstyle;
```

The NOGSTYLE system option causes ODS styles not to affect traditional graphics. Instead, the SYMBOL statement, GOPTIONS, and XCHART statement options control the appearance of the graph. The GSTYLE system option restores the use of ODS styles for traditional graphics produced subsequently. The chart is shown in Figure 6.6.

Figure 6.6 One-Sided Cusum Chart with Decision Interval



The cusum plotted at Hour=t is

$$S_t = \max(0, S_{t-1} + (z_t - k))$$

where $S_0 = 0$, and z_t is the standardized deviation of the tth measurement from the target.

$$z_t = \frac{x_t - \mu_0}{\sigma_0}$$

The cusum S_t is referred to as an *upper cumulative sum*. A shift is signaled at the seventh hour since S_7 exceeds h. For further details, see "One-Sided Cusum Schemes" on page 577.

The option TABLEALL requests the tables shown in Figure 6.7, Figure 6.8, and Figure 6.9. The table in Figure 6.7 summarizes the cusum scheme, and it confirms that an in-control ARL of 117.6 and an ARL of 6.4 at $\delta = 1$ are achieved with the specified h and k.

Figure 6.7 Summary Table

Cusum Parameters	
Process Variable	Weight (Cusum of Weight)
Subgroup Variable	Hour (Hour)
Scheme	One-Sided
Target Mean (Mu0)	8.1
Sigma0	0.05
Delta	1
Nominal Sample Size	1
h	3
k	0.5
Average Run Length (Delta)	6.40390895
Average Run Length (0)	117.595692

The table in Figure 6.8 tabulates the information displayed in Figure 6.6.

Figure 6.8 Tabulation of One-Sided Chart

One-Sided Cusum Analysis

The CUSUM Procedure

Cumulative Sum Chart Summary for Weight

Hour	Subgroup Sample Size	Individual Value	Cusum	Decision Interval	Decision Interval Exceeded
1	1	8.0240000	0.0000000	3.0000	
2	1	7.9710000	0.0000000	3.0000	
3	1	8.1250000	0.0000000	3.0000	
4	1	8.1230000	0.0000000	3.0000	
5	1	8.0680000	0.0000000	3.0000	
6	1	8.1770000	1.0400000	3.0000	
7	1	8.2290000	3.1200000	3.0000	Upper
8	1	8.0720000	2.0600000	3.0000	
9	1	8.0660000	0.8800000	3.0000	
10	1	8.0890000	0.1600000	3.0000	
11	1	8.0580000	0.0000000	3.0000	
12	1	8.1470000	0.4400000	3.0000	
13	1	8.1410000	0.7600000	3.0000	
14	1	8.0470000	0.0000000	3.0000	
15	1	8.1250000	0.0000000	3.0000	

The table in Figure 6.9 presents the computational form of the cusum scheme described by Lucas (1976).

Figure 6.9 Computational Form of Cusum Scheme

One-Sided Cusum Analysis

The CUSUM Procedure

Computational Cumulative Sum for Weight				
Hour	**Subgroup Sample Size**	**Individual Value**	**Upper Cusum**	**Number of Consecutive Upper Sums > 0**
1	1	8.0240000	0.0000000	0
2	1	7.9710000	0.0000000	0
3	1	8.1250000	0.0000000	0
4	1	8.1230000	0.0000000	0
5	1	8.0680000	0.0000000	0
6	1	8.1770000	1.0400000	1
7	1	8.2290000	3.1200000	2
8	1	8.0720000	2.0600000	3
9	1	8.0660000	0.8800000	4
10	1	8.0890000	0.1600000	5
11	1	8.0580000	0.0000000	0
12	1	8.1470000	0.4400000	1
13	1	8.1410000	0.7600000	2
14	1	8.0470000	0.0000000	0
15	1	8.1250000	0.0000000	0

Following the method of Lucas (1976), the process average at the out-of-control point (Hour=7) can be estimated as

$$\begin{aligned} \mu_0 &+ \sigma_0(N_7 k + S_7)/(N_7\sqrt{n}) \\ &= 8.10 + 0.05(2(0.5) + 3.12)/2 \\ &= 8.203 \text{ ounces} \end{aligned}$$

where $S_7 = 3.12$ is the upper sum at Hour=7, and $N_7 = 2$ is the number of successive positive upper sums at Hour=7.

Saving Cusum Scheme Parameters

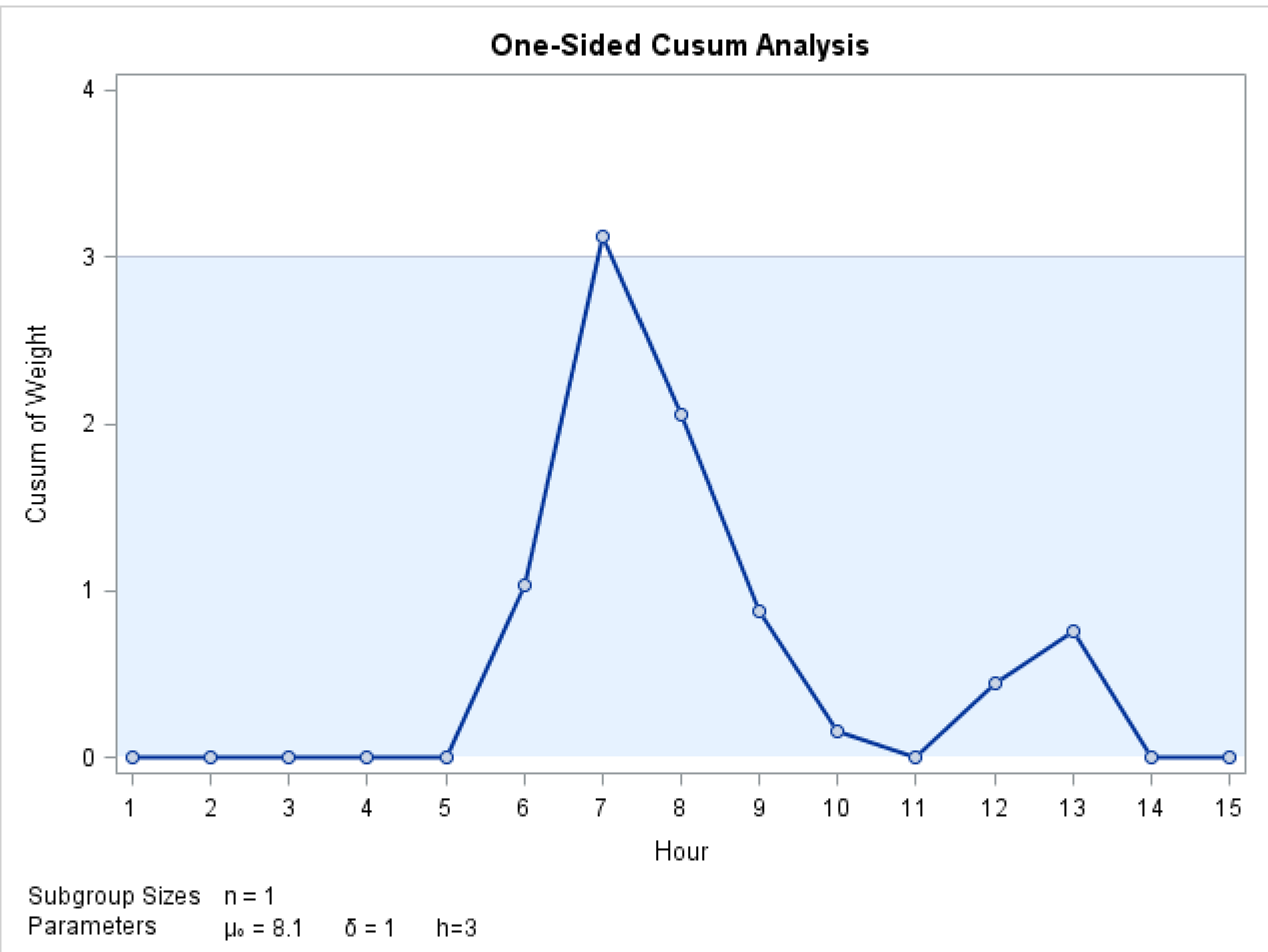
NOTE: See *One-sided Cusum Chart* in the SAS/QC Sample Library.

This example is a continuation of the previous example that illustrates how to save cusum scheme parameters in a data set specified with the OUTLIMITS= option. This enables you to apply the parameters to future data or to subsequently modify the parameters with a DATA step program.

```
ods graphics on;
title 'One-Sided Cusum Analysis';
proc cusum data=Cans;
   xchart Weight*Hour /
      mu0       = 8.100     /* target mean for process     */
      sigma0    = 0.050     /* known standard deviation    */
      delta     = 1         /* shift to be detected        */
      h         = 3         /* cusum parameter h           */
      k         = 0.5       /* cusum parameter k           */
      scheme    = onesided  /* one-sided decision interval */
      outlimits = cusparm
      odstitle  = title
      markers;
   label Weight = 'Cusum of Weight';
run;
```

The chart, shown in Figure 6.10, is similar to the one in Figure 6.6 but is created by using ODS Graphics because the ODS GRAPHICS ON statement is specified before the PROC CUSUM statement.

Figure 6.10 One-Sided Cusum Scheme with Decision Interval



The OUTLIMITS= data set is listed in Figure 6.11.

Figure 6.11 Listing of the OUTLIMITS= Data Set cusparm

One-Sided Cusum Analysis

Obs	_VAR_	_SUBGRP_	_TYPE_	_LIMITN_	_H_	_K_	_SCHEME_	_MU0_	_DELTA_
1	Weight	Hour	STANDARD	1	3	0.5	ONESIDED	8.1	1

Obs	_MEAN_	_STDDEV_	_ARLIN_	_ARLOUT_
1	8.09747	0.05	117.596	6.40391

The data set contains one observation with the parameters for *process* Weight. The variables _TYPE_, _H_, _K_, _MU0_, _DELTA_, and _STDDEV_ save the parameters specified with the options SCHEME=, H=, K=, MU0=, DELTA=, and SIGMA0=, respectively. The variable _MEAN_ saves an estimate of the process mean, and the variable _LIMITN_ saves the nominal sample size. The variables _ARLIN_ and _ARLOUT_ save the average run lengths for $\delta = 0$ and for $\delta = 1$.

The variables _VAR_ and _SUBGRP_ save the *process* and *subgroup-variable*. The variable _TYPE_ is a bookkeeping variable that indicates whether the value of _STDDEV_ is an estimate or a standard value.

For more information, see "OUTLIMITS= Data Set" on page 588.

Reading Cusum Scheme Parameters

NOTE: See *One-sided Cusum Chart* in the SAS/QC Sample Library.

This example shows how the cusum parameters saved in the previous example can be applied to new measurements saved in a data set named Cans2:

```
data Cans2;
   length pump $ 8;
   label Hour = 'Hour';
   input Hour Weight pump $ 8. ;
   datalines;
16  8.1765  Pump 3
17  8.0949  Pump 3
18  8.1393  Pump 3
19  8.1491  Pump 3
20  8.0473  Pump 1
21  8.1602  Pump 1
22  8.0633  Pump 1
23  8.0921  Pump 1
24  8.1573  Pump 1
25  8.1304  Pump 1
26  8.0979  Pump 1
27  8.2407  Pump 1
28  8.0730  Pump 1
29  8.0986  Pump 2
30  8.0785  Pump 2
31  8.2308  Pump 2
32  8.0986  Pump 2
33  8.0782  Pump 2
34  8.1435  Pump 2
35  8.0666  Pump 2
;
```

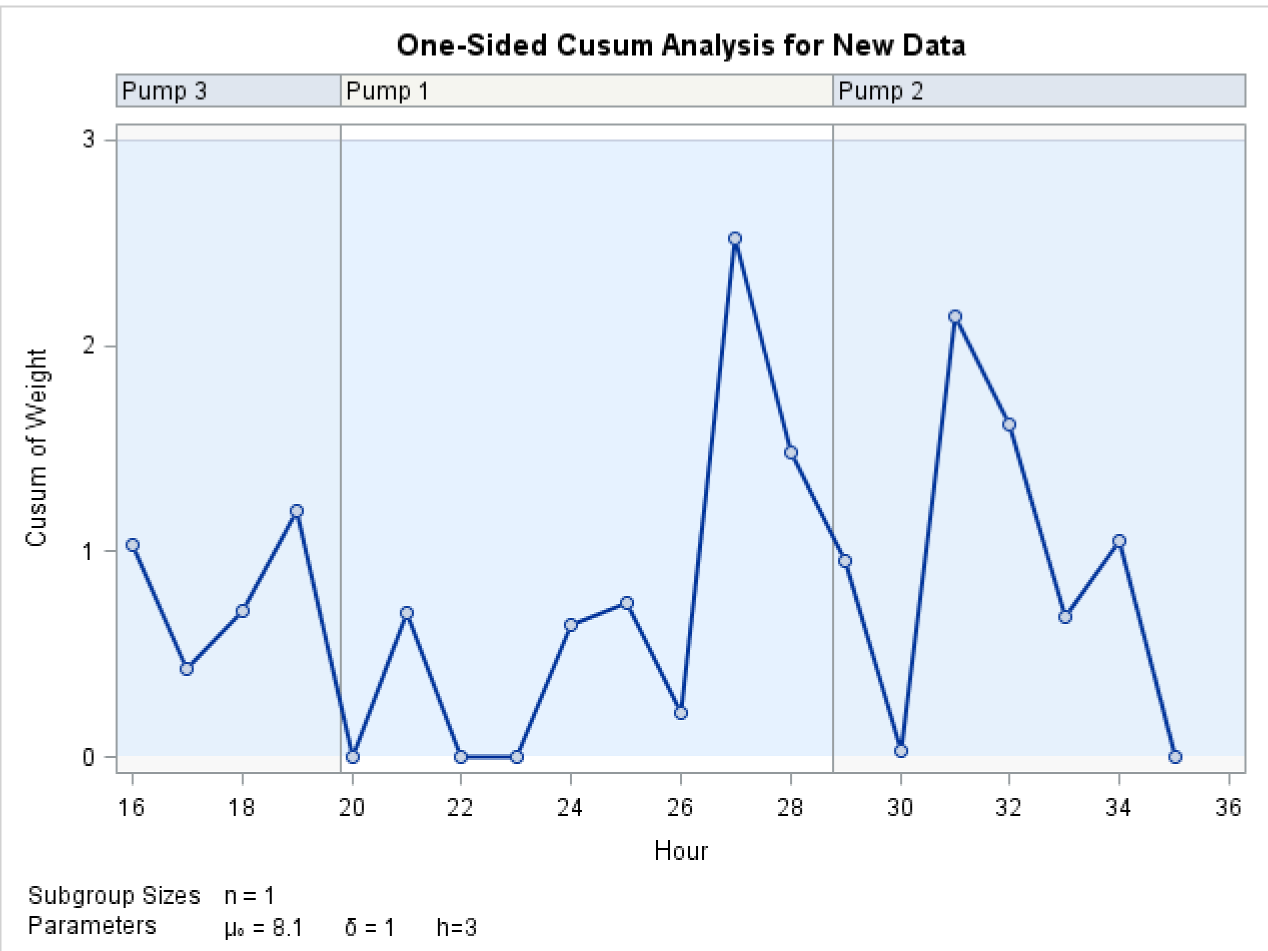
The following statements create a one-sided cusum chart for the measurements in Cans2 using the parameters in cusparm:

```
title "One-Sided Cusum Analysis for New Data";
proc cusum data=Cans2 limits=cusparm;
   xchart Weight*Hour( pump ) / odstitle=title markers;
   label Weight = 'Cusum of Weight';
run;
```

ODS Graphics remains enabled until it is disabled with the ODS GRAPHICS OFF statement, so this cusum chart is also created using ODS Graphics.

The LIMITS= option in the PROC CUSUM statement specifies the data set containing preestablished cusum parameters. The chart, shown in Figure 6.12, indicates that the process is in control. Levels of the variable pump (referred to as a *block-variable*) do not enter into the analysis but are displayed in a block legend across the top of the chart. See Block Variable Legend Options.

Figure 6.12 Cusum Chart with Decision Interval for New Data



In general, the parameters for a specified *process* and *subgroup-variable* are read from the first observation in the LIMITS= data set for which

- the value of _VAR_ matches the *process* (in this case, Weight)
- the value of _SUBGRP_ matches the *subgroup-variable* name (in this case, Hour)

If you are maintaining more than one set of cusum parameters for a particular *process*, you will find it convenient to include a special identifier variable named _INDEX_ in the LIMITS= data set. This must be a character variable of length 16. Then, if you specify READINDEX=*'value'* in the XCHART statement, the parameters for a specified *process* and *subgroup-variable* are read from the first observation in the LIMITS= data set for which

- the value of _VAR_ matches *process*
- the value of _SUBGRP_ matches the *subgroup-variable* name
- the value of _INDEX_ matches *value*

In this example, the LIMITS= data set was created in a previous run of the CUSUM procedure. You can also create a LIMITS= data set with the DATA step. See "LIMITS= Data Set" on page 592 for details concerning the variables that you must provide.

Syntax: XCHART Statement

The basic syntax for a *one-sided (decision interval) scheme* using the XCHART statement is as follows:

XCHART *process* ∗ *subgroup-variable* / **SCHEME=ONESIDED** **MU0=***target* **DELTA=***shift* **H=***h* < *options* > **;**

The general form of this syntax is as follows:

XCHART *processes* ∗ *subgroup-variable* < *(block-variables)* > < **=***symbol-variable* **=**'*character*' > / **SCHEME=ONESIDED** **MU0=***target* **DELTA=***shift* **H=***h* < *options* > **;**

Note that the options SCHEME=ONESIDED, MU0=, DELTA=, and H= are required unless their values are read from a LIMITS= data set.

The basic syntax for a *two-sided (V-mask) scheme* is as follows:

XCHART *process* ∗ *subgroup-variable* / **MU0=***target* **DELTA=***shift* **ALPHA=***alpha* **H=***h* < *options* > **;**

The general form of this syntax is as follows:

XCHART *processes* ∗ *subgroup-variable* < *(block-variables)* > < **=***symbol-variable* | **=** '*character*' > / **MU0=***target* **DELTA=***shift* **ALPHA=***alpha* **H=***h* < *options* > **;**

Note that the options MU0=, DELTA=, and either ALPHA= or H= are required unless their values are read from a LIMITS= data set.

You can use any number of XCHART statements in the CUSUM procedure. The components of the XCHART statement are described as follows.

process

processes

identify one or more processes to be analyzed. The specification of *process* depends on the input data set specified in the PROC CUSUM statement.

- If raw data are read from a DATA= data set, *process* must be the name of the variable containing the raw measurements. For an example, see "Creating a V-Mask Cusum Chart from Raw Data" on page 547.
- If summary data are read from a HISTORY= data set, *process* must be the common prefix of the summary variables in the HISTORY= data set. For an example, see "Creating a V-Mask Cusum Chart from Subgroup Summary Data" on page 550.

A *process* is required. If more than one *process* is specified, enclose the list in parentheses. The parameters specified in the XCHART statement are applied to all of the *processes*.[2]

subgroup-variable

is the variable that classifies the data into subgroups. The *subgroup-variable* is required. In the examples "Creating a V-Mask Cusum Chart from Raw Data" on page 547 and "Creating a V-Mask Cusum Chart from Subgroup Summary Data" on page 550, Hour is the subgroup variable.

block-variables

are optionally specified variables that group the data into blocks of consecutive subgroups. The blocks are labeled in a legend, and each *block-variable* provides one level of labels in the legend. See Figure 6.12 for an example.

symbol-variable

is an optionally specified variable whose levels (unique values) determine the plotting character or symbol marker used to plot the cusums.

- If you produce a line printer chart, an 'A' marks points corresponding to the first level of the *symbol-variable*, a 'B' marks points corresponding to the second level, and so on.
- If you produce traditional graphics, distinct symbol markers are displayed for points corresponding to the various levels of the *symbol-variable*. You can specify the symbol markers with SYMBOL*n* statements.

character

specifies a plotting character for line printer charts. See Figure 6.10 for an example.

options

specify optional cusum parameters, enhance the appearance of the chart, request additional analyses, save results in data sets, and so on. The section "Summary of Options", which follows, lists all options by function.

[2]For this reason, it may be preferable to read distinct cusum parameters for each *process* from a LIMITS= data set.

Summary of Options

The following tables list the XCHART statement options by function. Options unique to the CUSUM procedure are listed in Table 6.1, and are described in detail in the section "Dictionary of Special Options" on page 570. Options that are common to both the CUSUM and SHEWHART procedures are listed in Table 6.2. They are described in detail in "Dictionary of Options: SHEWHART Procedure" on page 1946.

Table 6.1 XCHART Statement Special Options

Option	Description
Options for Specifying a One-Sided (Decision Interval) Cusum Scheme	
DELTA=	specifies shift to be detected as a multiple of standard error
H=	specifies decision interval h ($h > 0$) as a multiple of standard error
HEADSTART=	specifies headstart value S_0 as a multiple of standard error
K=	specifies reference value k ($k > 0$)
LIMITN=	specifies fixed nominal sample size for cusum scheme
LIMITN=	specifies that cusums are to be computed for all subgroups regardless of sample size
MU0=	specifies target μ_0 for mean
NOREADLIMITS	specifies that cusum parameters are not to be read from LIMITS= data set (SAS 6.10 and later releases)
READINDEX=	reads cusum scheme parameters from a LIMITS= data set
READLIMITS	specifies that cusum parameters are to be read from LIMITS= data set (SAS 6.09 and earlier releases)
SCHEME=ONESIDED	specifies a one-sided scheme
SHIFT=	specifies shift to be detected in data units
SIGMA0=	specifies standard (known) value σ_0 for process standard deviation
Options for Specifying a Two-Sided (V-Mask) Cusum Scheme	
ALPHA=	specifies probability of Type 1 error
BETA=	specifies probability of Type 2 error
H=	specifies vertical distance between V-mask origin and upper (or lower) arm
K=	specifies slope of lower arm of V-mask
LIMITN=	specifies fixed nominal sample size for cusum scheme
LIMITN=	specifies that cusums are to be computed for all subgroups regardless of sample size
NOREADLIMITS	specifies that cusum parameters are not to be read from LIMITS= data set (SAS 6.10 and later releases)
READINDEX=	reads cusum scheme parameters from a LIMITS= data set
READLIMITS	specifies that cusum parameters are to be read from LIMITS= data set (SAS 6.09 and earlier releases)
READSIGMAS	reads _SIGMAS_ instead of _ALPHA_ from LIMITS= data set when both variables are available
SIGMAS=	specifies probability of Type 1 error as probability that standard normally distributed variable exceeds *value* in absolute value

Table 6.1 *continued*

Option	Description
Options for Estimating Process Standard Deviation	
SMETHOD=	specifies method for estimating process standard deviation σ
TYPE=	identifies whether _STDDEV_ in OUTLIMITS= data set is an estimate or standard, and specifies value of _TYPE_ in OUTLIMITS= data set
Options for Displaying Decision Interval or V-Mask	
CINFILL=	specifies color for area under decision interval line or inside V-mask
CLIMITS=	specifies color of decision interval line
CMASK=	specifies color of V-mask outline
LLIMITS=	specifies line type for decision interval
LMASK=	specifies line type for V-mask arms
NOMASK	suppresses display of V-mask
ORIGIN=	specifies value of *subgroup-variable* locating origin of V-mask
WLIMITS=	specifies line width for decision interval
WMASK=	specifies line width for V-mask
Tabulation Options	
TABLEALL	specifies the options TABLECHART, TABLECOMP, TABLEID, TABLEOUT, and TABLESUMMARY
TABLECHART	tabulates the information displayed in the cusum chart
TABLECOMP	tabulates the computational form of the cusum scheme as described by Lucas (1976) and Lucas and Crosier (1982)
TABLEID	augments TABLECHART and TABLECOMP tables with columns for ID variables
TABLEOUT	augments TABLECHART table with a column indicating if the decision interval or V-mask was exceeded
TABLESUMMARY	tabulates the parameters for the cusum scheme and the average run lengths corresponding to shifts of zero and δ

Table 6.2 XCHART Statement General Options

Option	Description
Options for Plotting and Labeling Points	
ALLLABEL=	labels every point on cusum chart
ALLLABEL2=	labels every point on trend chart
CLABEL=	specifies color for labels
CCONNECT=	specifies color for line segments that connect points on chart
CFRAMELAB=	specifies fill color for frame around labeled points
COUT=	specifies color for portions of line segments that connect points outside control limits
COUTFILL=	specifies color for shading areas between the connected points and control limits outside the limits
LABELANGLE=	specifies angle at which labels are drawn

Table 6.2 *continued*

Option	Description
LABELFONT=	specifies software font for labels
LABELHEIGHT=	specifies height of labels
NOCONNECT	suppresses line segments that connect points on chart
NOTRENDCONNECT	suppresses line segments that connect points on trend chart
OUTLABEL=	labels points outside control limits
SYMBOLLEGEND=	specifies LEGEND statement for levels of *symbol-variable*
SYMBOLORDER=	specifies order in which symbols are assigned for levels of *symbol-variable*
TURNALL|TURNOUT	turns point labels so that they are strung out vertically
Axis and Axis Label Options	
CAXIS=	specifies color for axis lines and tick marks
CFRAME=	specifies fill colors for frame for plot area
CTEXT=	specifies color for tick mark values and axis labels
DISCRETE	produces horizontal axis for discrete numeric group values
HAXIS=	specifies major tick mark values for horizontal axis
HEIGHT=	specifies height of axis label and axis legend text
HMINOR=	specifies number of minor tick marks between major tick marks on horizontal axis
HOFFSET=	specifies length of offset at both ends of horizontal axis
INTSTART=	specifies first major tick mark value on horizontal axis when a date, time, or datetime format is associated with numeric subgroup variable
NOHLABEL	suppresses label for horizontal axis
NOTICKREP	specifies that only the first occurrence of repeated, adjacent subgroup values is to be labeled on horizontal axis
NOVANGLE	requests vertical axis labels that are strung out vertically
SKIPHLABELS=	specifies thinning factor for tick mark labels on horizontal axis
SPLIT=	specifies splitting character for axis labels
TURNHLABELS	requests horizontal axis labels that are strung out vertically
VAXIS=	specifies major tick mark values for vertical axis of cusum chart
VAXIS2=	specifies major tick mark values for vertical axis of trend chart
VFORMAT=	specifies format for primary vertical axis tick mark labels
VFORMAT2=	specifies format for secondary vertical axis tick mark labels
VMINOR=	specifies number of minor tick marks between major tick marks on vertical axis
VOFFSET=	specifies length of offset at both ends of vertical axis

Table 6.2 *continued*

Option	Description
WAXIS=	specifies width of axis lines
Plot Layout Options	
ALLN	plots means for all subgroups
BILEVEL	creates control charts using half-screens and half-pages
EXCHART	creates control charts for a process only when exceptions occur
INTERVAL=	natural time interval between consecutive subgroup positions when time, date, or datetime format is associated with a numeric subgroup variable
MAXPANELS=	maximum number of pages or screens for chart
NMARKERS	requests special markers for points corresponding to sample sizes not equal to nominal sample size for fixed control limits
NOCHART	suppresses creation of chart
NOFRAME	suppresses frame for plot area
NOLEGEND	suppresses legend for subgroup sample sizes
NPANELPOS=	specifies number of subgroup positions per panel on each chart
REPEAT	repeats last subgroup position on panel as first subgroup position of next panel
TOTPANELS=	specifies number of pages or screens to be used to display chart
TRENDVAR=	specifies list of trend variables
YPCT1=	specifies length of vertical axis on cusum chart as a percentage of sum of lengths of vertical axes for cusum and trend charts
Reference Line Options	
CHREF=	specifies color for lines requested by HREF= and HREF2= options
CVREF=	specifies color for lines requested by VREF= and VREF2= options
HREF=	specifies position of reference lines perpendicular to horizontal axis on cusum chart
HREF2=	specifies position of reference lines perpendicular to horizontal axis on trend chart
HREFDATA=	specifies position of reference lines perpendicular to horizontal axis on cusum chart
HREF2DATA=	specifies position of reference lines perpendicular to horizontal axis on trend chart
HREFLABELS=	specifies labels for HREF= lines
HREF2LABELS=	specifies labels for HREF2= lines
HREFLABPOS=	specifies position of HREFLABELS= and HREF2LABELS= labels
LHREF=	specifies line type for HREF= and HREF2= lines

Table 6.2 *continued*

Option	Description
LVREF=	specifies line type for VREF= and VREF2= lines
NOBYREF	specifies that reference line information in a data set applies uniformly to charts created for all BY groups
VREF=	specifies position of reference lines perpendicular to vertical axis on cusum chart
VREF2=	specifies position of reference lines perpendicular to vertical axis on trend chart
VREFLABELS=	specifies labels for VREF= lines
VREF2LABELS=	specifies labels for VREF2= lines
VREFLABPOS=	position of VREFLABELS= and VREF2LABELS= labels
Grid Options	
CGRID=	specifies color for grid requested with GRID or ENDGRID option
ENDGRID	adds grid after last plotted point
GRID	adds grid to control chart
LENDGRID=	specifies line type for grid requested with the ENDGRID option
LGRID=	specifies line type for grid requested with the GRID option
WGRID=	specifies width of grid lines
Graphical Enhancement Options	
ANNOTATE=	specifies annotate data set that adds features to cusum chart
ANNOTATE2=	specifies annotate data set that adds features to trend chart
DESCRIPTION=	specifies description of cusum chart's GRSEG catalog entry
FONT=	specifies software font for labels and legends on charts
NAME=	specifies name of cusum chart's GRSEG catalog entry
PAGENUM=	specifies the form of the label used in pagination
PAGENUMPOS=	specifies the position of the page number requested with the PAGENUM= option
WTREND=	specifies width of line segments connecting points on trend chart
Options for Producing Graphs Using ODS Styles	
BLOCKVAR=	specifies one or more variables whose values define colors for filling background of *block-variable* legend
CFRAMELAB	draws a frame around labeled points
COUT	draw portions of line segments that connect points outside control limits in a contrasting color
CSTAROUT	specifies that portions of stars exceeding inner or outer circles are drawn using a different color

Table 6.2 *continued*

Option	Description
OUTFILL	shades areas between control limits and connected points lying outside the limits
STARFILL=	specifies a variable identfying groups of stars filled with different colors
STARS=	specifies a variable identfying groups of stars whose outlines are drawn with different colors
Options for ODS Graphics	
BLOCKREFTRANSPARENCY=	specifies the wall fill transparency for blocks and phases
INFILLTRANSPARENCY=	specifies the control limit infill transparency
MARKERS	plots subgroup points with markers
NOBLOCKREF	suppresses block and phase reference lines
NOBLOCKREFFILL	suppresses block and phase wall fills
NOFILLLEGEND	suppresses legend for levels of a STARFILL= variable
NOPHASEREF	suppresses block and phase reference lines
NOPHASEREFFILL	suppresses block and phase wall fills
NOREF	suppresses block and phase reference lines
NOREFFILL	suppresses block and phase wall fills
NOSTARFILLLEGEND	suppresses legend for levels of a STARFILL= variable
NOTRANSPARENCY	disables transparency in ODS Graphics output
ODSFOOTNOTE=	specifies a graph footnote
ODSFOOTNOTE2=	specifies a secondary graph footnote
ODSLEGENDEXPAND	specifies that legend entries contain all levels observed in the data
ODSTITLE=	specifies a graph title
ODSTITLE2=	specifies a secondary graph title
OUTFILLTRANSPARENCY=	specifies control limit outfill transparency
OVERLAYURL=	specifies URLs to associate with overlay points
OVERLAY2URL=	specifies URLs to associate with overlay points on secondary chart
PHASEPOS=	specifies vertical position of phase legend
PHASEREFLEVEL=	associates phase and block reference lines with either innermost or the outermost level
PHASEREFTRANSPARENCY=	specifies the wall fill transparency for blocks and phases
REFFILLTRANSPARENCY=	specifies the wall fill transparency for blocks and phases
SIMULATEQCFONT	draws central line labels using a simulated software font
STARTRANSPARENCY=	specifies star fill transparency
URL=	specifies a variable whose values are URLs to be associated with subgroups
URL2=	specifies a variable whose values are URLs to be associated with subgroups on secondary chart
Input Data Set Options	
MISSBREAK	specifies that observations with missing values are not to be processed

Table 6.2 *continued*

Option	Description
Output Data Set Options	
OUTHISTORY=	creates output data set containing subgroup summary statistics
OUTINDEX=	specifies value of _INDEX_ in the OUTLIMITS= data set
OUTLIMITS=	creates output data set containing control limits
OUTTABLE=	creates output data set containing subgroup summary statistics and control limits
Specification Limit Options	
CIINDICES	specifies α value and type for computing capability index confidence limits
LSL=	specifies list of lower specification limits
TARGET=	specifies list of target values
USL=	specifies list of upper specification limits
Block Variable Legend Options	
BLOCKLABELPOS=	specifies position of label for *block-variable* legend
BLOCKLABTYPE=	specifies text size of *block-variable* legend
BLOCKPOS=	specifies vertical position of *block-variable* legend
BLOCKREP	repeats identical consecutive labels in *block-variable* legend
CBLOCKLAB=	specifies fill colors for frames enclosing variable labels in *block-variable* legend
CBLOCKVAR=	specifies one or more variables whose values are colors for filling background of *block-variable* legend
Phase Options	
CPHASELEG=	specifies text color for *phase* legend
OUTPHASE=	specifies value of _PHASE_ in the OUTHISTORY= data set
PHASEBREAK	disconnects last point in a *phase* from first point in next *phase*
PHASELABTYPE=	specifies text size of *phase* legend
PHASELEGEND	displays *phase* labels in a legend across top of chart
PHASELIMITS	labels control limits for each phase, provided they are constant within that phase
PHASEREF	delineates *phases* with vertical reference lines
READPHASES=	specifies *phases* to be read from an input data set
Star Options	
CSTARCIRCLES=	specifies color for STARCIRCLES= circles
CSTARFILL=	specifies color for filling stars
CSTAROUT=	specifies outline color for stars exceeding inner or outer circles
CSTARS=	specifies color for outlines of stars
LSTARCIRCLES=	specifies line types for STARCIRCLES= circles

Table 6.2 *continued*

Option	Description
LSTARS=	specifies line types for outlines of STARVERTICES= stars
STARBDRADIUS=	specifies radius of outer bound circle for vertices of stars
STARCIRCLES=	specifies reference circles for stars
STARINRADIUS=	specifies inner radius of stars
STARLABEL=	specifies vertices to be labeled
STARLEGEND=	specifies style of legend for star vertices
STARLEGENDLAB=	specifies label for STARLEGEND= legend
STAROUTRADIUS=	specifies outer radius of stars
STARSPECS=	specifies method used to standardize vertex variables
STARSTART=	specifies angle for first vertex
STARTYPE=	specifies graphical style of star
STARVERTICES=	superimposes star at each point on cusum chart
WSTARCIRCLES=	specifies width of STARCIRCLES= circles
WSTARS=	specifies width of STARVERTICES= stars
Options for Interactive Control Charts	
HTML=	specifies a variable whose values create links to be associated with subgroups
HTML2=	specifies variable whose values create links to be associated with subgroups on secondary chart
HTML_LEGEND=	specifies a variable whose values create links to be associated with symbols in the symbol legend
WEBOUT=	creates an OUTTABLE= data set with additional graphics coordinate data
Options for Line Printer Charts	
CONNECTCHAR=	specifies character used to form line segments that connect points on chart
HREFCHAR=	specifies line character for HREF= and HREF2= lines
SYMBOLCHARS=	specifies characters indicating *symbol-variable*
VREFCHAR=	specifies line character for VREF= and VREF2= lines

Dictionary of Special Options

General Options

You can specify the following *options* when you use either ODS Graphics or traditional graphics:

ALPHA=*value*

specifies the probability α of incorrectly deciding that a shift has occurred when the process mean is equal to the target mean. This is known as the probability of a Type 1 error. The *value* must be between zero and one, and it is typically set at 0.05 or 0.10. If you specify the ALPHA= option, the error probability approach is used to determine the V-mask. For details, see "Defining the V-Mask for a Two-Sided Cusum Scheme" on page 580.

The ALPHA= option is applicable only with two-sided cusum schemes. As an alternative to the ALPHA= *value*, you can specify the percentile $z_{1-\alpha/2}$ from a standard normal distribution with the

SIGMAS= option. As a second alternative, you can specify the geometric parameter h for the V-mask (in standard error units) with the H= option.

In addition to the ALPHA= option, you can optionally specify the probability of a Type 2 error with the BETA= option.

BETA=*value*

specifies the probability β of failing to discover that the specified shift has occurred. This is known as the probability of a Type 2 error. The *value* must be between zero and one. The BETA= option is used in conjunction with either the ALPHA= option or the SIGMAS= option.

The interpretation of β is based on the analogy between cusum charts and sequential probability ratio tests, and it is inexact since the cusum chart does not provide an acceptance region. Refer to Johnson (1961) and van Dobben de Bruyn (1968) for further details.

DATAUNITS

computes cumulative sums without standardizing the subgroup means or individual measurements. As a result, the vertical axis of the cusum chart is scaled in the same units as the data.

The DATAUNITS option requires constant subgroup sample sizes. If your data do not have constant subgroup sample sizes, you need to specify a constant nominal sample size n for the V-mask or decision interval with the LIMITN= option or with the variable _LIMITN_ in the LIMITS= data set.

DELTA=*value*

specifies the absolute value of the smallest shift to be detected as a multiple δ of the process standard deviation σ or the standard error $\sigma_{\bar{X}}$, depending on whether δ is viewed as a shift in the population mean or a shift in the sampling distribution of the subgroup mean $\bar{X}$, respectively.

If you specify SCHEME=ONESIDED (see the SCHEME= option later in this list) and the *value* is positive, a shift above the process mean is to be detected, whereas if the *value* is negative, a shift below the process mean is to be detected.

As an alternative to specifying the DELTA= option, you can specify the shift in the same units as the data with the SHIFT= option.

H=*value*

specifies the decision interval h for a one-sided cusum scheme. This type of scheme is completely specified by the parameters h and k (see the K= option later in this list). You can also specify the H= option as an alternative to the ALPHA= or SIGMAS= options for a two-sided cusum scheme with a V-mask. In this case, the H= option specifies the vertical distance h between the origin for the V-mask and the upper or lower arm of the V-mask. In either case, the H=*value* must be positive and must be expressed as a multiple of standard error.

You can use a table of average run lengths to choose h (this is typically between zero and 10). See Table 6.4 and Table 6.5

HEADSTART=*value*

HSTART=*value*

specifies a headstart value S_0 for a one-sided cusum scheme. The value must be expressed as a multiple of standard error. See the section "Headstart Values" on page 578,and refer to Lucas and Crosier (1982), Ryan (1989), and Montgomery (1996).

K=*value*

specifies the reference value k for a one-sided (decision interval) cusum scheme. This type of scheme is completely specified by the parameters k and h (see the H= option earlier in this list). You can also specify the K= and H= options as geometric parameters for a two-sided cusum scheme with a V-mask . In this case, the K= option specifies the slope of the lower arm of the V-mask, and the K= and H= options together are alternatives to the error probability options ALPHA=, SIGMAS=, and BETA=. In either case, the K= *value* must be positive and must be expressed as a multiple of standard error.

You can use a table of average run lengths to choose k and h (k is typically between zero and two). See Table 6.4 and Table 6.5.

For a one-sided scheme, the default K= *value* is $\delta/2$, which is referred to as the *central reference value*. For a two-sided scheme where the V-mask is specified geometrically with the H= option, the default K= *value* is $\delta/2$. If, however, the V-mask is specified by an error probability with the ALPHA= option, then the K= option should not be specified.

CAUTION: The interpretation of the K= *value* depends on the *subgroup-variable* and the interval between subgroups that is specified with the INTERVAL= option. For a two-sided scheme, the *value* is the increase in the lower V-mask arm per unit change on the subgroup axis, so the *value* depends on how the *subgroup-variable* is scaled.

- If integer values are assigned to the *subgroup-variable*, then a unit change is defined as one.
- If the *subgroup-variable* has character values, then a unit change is defined as the increment between adjacent values of the *subgroup-variable*.
- If the *subgroup-variable* is numeric and is formatted with a SAS date or time format, then a unit change is defined as the default value for the INTERVAL= option. For example, if a DATE7. format is associated with the *subgroup-variable*, then a unit change is defined as one day.

You can use the INTERVAL= option to modify the definition of a unit change. For example, if a DATE7. format is associated with the *subgroup-variable* but subgroups are collected hourly, then INTERVAL=HOUR defines a unit change as one hour rather than one day.

LIMITN=*n*

LIMITN=VARYING

specifies either a fixed or varying nominal sample size for the control limits. If you specify LIMITN=*n*, cusums are calculated and displayed only for those subgroups with a sample size equal to *n*, although you can specify the ALLN option to force all cusums to be plotted. If you specify LIMITN=VARYING, cusums are calculated and displayed for all subgroups, regardless of sample size.

MU0=*value*

specifies the target mean μ_0 for the process. The target mean must be scaled in the same units as the data.

NOARL

suppresses calculation of average run lengths. By default, this calculation is performed if you specify the TABLESUMMARY option or an OUTLIMITS= data set.

NOMASK

suppresses the display of the V-mask on charts for two-sided schemes. This option does not affect computations of cusums or V-mask parameters.

NOREADLIMITS

specifies that the cusum scheme parameters for each *process* listed in the chart statement are *not* to be read from the LIMITS= data set specified in the PROC CUSUM statement. The NOREADLIMITS option is available only in SAS 6.10 and later releases. See the READLIMITS option later in this list.

ORIGIN=*value*

specifies the origin of the V-mask, which is defined as the horizontal coordinate of the right edge of the V-mask. If a date, time, or datetime format is associated with the *subgroup-variable*, you must specify the *value* as a date, time, or datetime constant, respectively. If the subgroup variable is character, you must specify the *value* as a quoted string. The default *value* is the last (most recent) value of the *subgroup-variable*.

Note that estimates for the process mean and standard deviation are calculated only from subgroups up to and including the origin subgroup.

READINDEX=*'value'*

reads cusum scheme parameters from a LIMITS= data set (specified in the PROC CUSUM statement) for each *process* listed in the chart statement. The *i*th set of control limits for a particular *process* is read from the first observation in the LIMITS= data set for which

- the value of _VAR_ matches *process*
- the value of _SUBGRP_ matches the *subgroup-variable*
- the value of _INDEX_ matches *value*

The *value* can be up to 16 characters and must be enclosed in quotes.

READLIMITS

specifies that cusum scheme parameters are to be read from a LIMITS= data set specified in the PROC CUSUM statement. The parameters for a particular *process* are read from the first observation in the LIMITS= data set for which

- the value of _VAR_ matches *process*
- the value of _SUBGRP_ matches the *subgroup variable*

The use of the READLIMITS option depends on which release of SAS/QC software you are using.

- **In SAS 6.10 and later releases, the READLIMITS option is not necessary.** To read cusum scheme parameters as described previously, you simply specify a LIMITS= data set. However, even though the READLIMITS option is redundant, it continues to function as in earlier releases.
- **In SAS 6.09 and earlier releases, you must specify the READLIMITS option to read cusum scheme parameters as described previously.** If you specify a LIMITS= data set without specifying the READLIMITS option (or the READINDEX= option), the cusum scheme parameters are computed from the data.

READSIGMAS

specifies that the variable _SIGMAS_ (instead of _ALPHA_) is to be read from a LIMITS= data set that contains both variables. The variables _SIGMAS_ and _ALPHA_ provide the same parameters as the SIGMAS= and ALPHA= options. By default, _ALPHA_ is read from the LIMITS= data set.

SCHEME=ONESIDED

SCHEME=TWOSIDED

indicates whether the cusum scheme is a one-sided (decision interval) scheme or a two-sided scheme with a V-mask. By default, SCHEME=TWOSIDED.

SHIFT=*value*

specifies the shift to be detected in the same units as the data. The *value* is interpreted as the shift in the mean of the sampling distribution of the subgroup mean. The SHIFT= option is an alternative to the DELTA= option. To specify the SHIFT= option, one of the following must be true:

- The subgroup sample sizes are constant.
- A constant nominal sample size *n* is provided for the cusum scheme with the LIMITN= option or the _LIMITN_ variable in a LIMITS= data set.

The relationship between the SHIFT= *value* (denoted by Δ) and the DELTA= value (denoted by δ) is $\delta = \Delta/(\sigma/\sqrt{n})$, where σ is the process standard deviation.

SIGMA0=*value*

specifies a known standard deviation σ_0 for the process standard deviation σ. The *value* must be positive. By default, PROC CUSUM estimates σ from the data using the formulas given in "Methods for Estimating the Standard Deviation" on page 586. You can use the variable _STDDEV_ in a LIMITS= data set as an alternative to the SIGMA0= option.

SIGMAS=*value*

specifies the probability α of false detection for a two-sided cusum scheme with a V-mask as the probability that the absolute value of a standard normally distributed variable is greater than the *value*. For example, SIGMAS=3 corresponds to the probability α =0.0027. The *value* must be positive. The SIGMAS= option is an alternative to the ALPHA= and H= options, and only one of these three options can be specified.

The SIGMAS= option is useful for defining cusum charts that correspond to Shewhart charts whose control limits are defined with the same *value* as the multiple of σ. Refer to Johnson and Leone (1962, 1974).

SMETHOD=NOWEIGHT | MVLUE | RMSDF

specifies a method for estimating the process standard deviation from subgroup observations, σ, as summarized by the following table.

Keyword	Method for Estimating Standard Deviation
NOWEIGHT	estimates σ as an unweighted average of unbiased subgroup estimates of σ
MVLUE	calculates a minimum variance linear unbiased estimate for σ
RMSDF	calculates a root-mean square estimate for σ

For formulas, see "Methods for Estimating the Standard Deviation" on page 586.

TABLEALL

requests all the tables specified by the options TABLECHART, TABLECOMP, TABLEID, TABLEOUT, and TABLESUMMARY.

TABLECHART < (EXCEPTIONS) >

creates a table of the subgroup variable, the subgroup sample sizes, the subgroup means, the cumulative sums, and the decision interval or V-mask limits. A table is produced for each *process* specified in the XCHART statement. The keyword EXCEPTIONS (enclosed in parentheses) is optional and restricts the tabulation to those subgroups for which the decision interval or V-mask values are exceeded.

TABLECOMP

tabulates the computational form of the cusum scheme as described by Lucas (1976) and Lucas and Crosier (1982). Upper or lower cumulative sums (or both) are tabulated for each *process* given in the XCHART statement. See "Formulas for Cumulative Sums" on page 577 for more information.

TABLEID

augments the tables specified by the TABLECHART and TABLECOMP options with a column for each of the ID variables.

TABLEOUT

augments the table specified by the TABLECHART option with a column indicating whether the decision interval or V-mask values are exceeded.

TABLESUMMARY

produces a table that summarizes the cusum scheme. The table lists the parameters of the scheme and the average run lengths corresponding to shifts of zero and δ. The average run lengths are computed using the method of Goel and Wu (1971). A table is produced for each *process*. You can save the summary in a data set by specifying the OUTLIMITS= option. See "OUTLIMITS= Data Set" on page 588 for details.

TYPE=ESTIMATE

TYPE=STANDARD

specifies the value of _TYPE_ in an OUTLIMITS= data set. The variable _TYPE_ indicates whether the variable _STDDEV_ in the OUTLIMITS= data set represents an estimate or a standard (known) value. The default is 'STANDARD' if the SIGMA0= option is specified; otherwise, the default is 'ESTIMATE'.

Options for Traditional Graphics

You can specify the following *options* when you produce traditional graphics:

CINFILL=*color*

specifies the color for the area under the decision interval or inside the V-mask arms. See also the COUTFILL= option.

CLIMITS=*color*

specifies the color for the decision interval line.

CMASK=*color*
specifies the color for the V-mask arms.

LLIMITS=*linetype*
specifies the line type for the decision interval.

LMASK=*linetype*
specifies the line type for the V-mask arms.

WLIMITS=*linetype*
specifies the width (in pixels) of the decision interval line.

WMASK=*linetype*
specifies the width (in pixels) of the V-mask arms.

Details: XCHART Statement

Basic Notation for Cusum Charts

The following notation is used in this chapter:

μ denotes the mean of the population, also referred to as the *process mean* or the *process level*.

μ_0 denotes the target mean (goal) for the population. Goel and Wu (1971) refer to μ_0 as the "acceptable quality level" and use the symbol μ_a instead. The symbol $\bar{X}_0$ is used for μ_0 in *Glossary and Tables for Statistical Quality Control*. You can provide μ_0 with the MU0= option or with the variable _MU0_ in a LIMITS= data set.

σ denotes the population standard deviation. You can provide σ with the variable _STDDEV_ in a LIMITS= data set (where _TYPE_='STANDARD').

σ_0 denotes a known standard deviation. You can provide σ_0 with the SIGMA0= option or the variable _STDDEV_ in a LIMITS= data set.

$\hat{\sigma}$ denotes an estimate of σ. You can provide $\hat{\sigma}$ with the SIGMA0= option or the variable _STDDEV_ in a LIMITS= data set. To identify this value as an estimate, specify TYPE=ESTIMATE or assign the value 'ESTIMATE' to the variable _TYPE_ in a LIMITS= data set.

n denotes the nominal sample size for the cusum scheme. You can provide n with the LIMITN= option or the variable _LIMITN_ in a LIMITS= data set.

δ denotes the shift in μ to be detected, expressed as a multiple of the standard deviation. You can provide δ with the DELTA= option or the variable _DELTA_ in a LIMITS= data set.

Δ denotes the shift in μ to be detected, expressed in data units. If the sample size n is constant across subgroups, then $\Delta = \delta\sigma_{\bar{X}} = (\delta\sigma)/\sqrt{n}$. Some authors use the symbol D instead of Δ; for example, refer to Johnson and Leone (1962, 1974) and Wadsworth, Stephens, and Godfrey (1986). You can provide Δ with the SHIFT= option. Although it may be more natural to specify the shift in data units, it is preferable to specify the shift as δ, since this generalizes to data with unequal subgroup sample sizes.

Formulas for Cumulative Sums

One-Sided Cusum Schemes

Positive Shifts If the shift δ to be detected is positive, the cusum computed for the tth subgroup is

$$S_t = \max(0, S_{t-1} + (z_t - k))$$

for t=1, 2, . . . , n, where S_0=0, z_t is defined as for two-sided schemes, and the parameter k, termed the *reference value*, is positive. The cusum S_t is referred to as an *upper cumulative sum*. Since S_t can be written as

$$\max\left(0, S_{t-1} + \frac{\bar{X}_i - (\mu_0 + k\sigma_{\bar{X}_i})}{\sigma_{\bar{X}_i}}\right)$$

the sequence S_t cumulates deviations in the subgroup means greater than k standard errors from μ_0. If S_t exceeds a positive value h (referred to as the *decision interval*), a shift or out-of-control condition is signaled. This formulation follows that of Lucas (1976), Lucas and Crosier (1982), and Montgomery (1996).

Negative Shifts If the shift δ to be detected is negative, the cusum computed for the tth subgroup is

$$S_t = \max(0, S_{t-1} - (z_t + k))$$

for t=1, 2, . . . , n, where S_0=0, z_t is defined as for two-sided cusum schemes, and the parameter k, termed the *reference value*, is positive. The cusum S_t is referred to as a *lower cumulative sum*. Since S_t can be written as

$$\max\left(0, S_{t-1} - \frac{\bar{X}_i - (\mu_0 - k\sigma_{\bar{X}_i})}{\sigma_{\bar{X}_i}}\right)$$

the sequence S_t cumulates the absolute value of deviations in the subgroup means less than k standard errors from μ_0. If S_t exceeds a positive value h (referred to as the *decision interval*), a shift or out-of-control condition is signaled.

This formulation follows that of Lucas (1976), Lucas and Crosier (1982), and Montgomery (1996). Note that S_t is always positive and h is always positive, regardless of whether δ is positive or negative. For schemes designed to detect a negative shift, some authors, including van Dobben de Bruyn (1968) and Wadsworth, Stephens, and Godfrey (1986), define a reflected version of S_t for which a shift is signaled when S_t is less than a negative limit.

Headstart Values Lucas and Crosier (1982) describe the properties of a fast initial response (FIR) feature for cusum schemes in which the initial cusum S_0 is set to a "headstart" value. Average run length calculations given by Lucas and Crosier (1982) show that the FIR feature has little effect when the process is in control and that it leads to a faster response to an initial out-of-control condition than a standard cusum scheme. You can provide headstart value S_0 with the HEADSTART= option or the variable _HSTART_ in a LIMITS= data set.

Constant Sample Sizes When the subgroup sample sizes are constant (= n), it may be preferable to compute cusums that are scaled in the same units as the data. Refer to Montgomery (1996) and Wadsworth, Stephens, and Godfrey (1986). To request this, specify the DATAUNITS option. Cusums are then computed as

$$S_t = \max(0, S_{t-1} + (\bar{X}_i - (\mu_0 + k\sigma/\sqrt{n})))$$

for δ >0 and the equation

$$S_t = \max(0, S_{t-1} - (\bar{X}_i - (\mu_0 - k\sigma/\sqrt{n})))$$

for $\delta < 0$. In either case, a shift is signaled if S_t exceeds $h' = h\sigma/\sqrt{n}$. Wadsworth, Stephens, and Godfrey (1986) use the symbol H for h'.

If the subgroup sample sizes are not constant, you can specify a constant nominal sample size n with the LIMITN= option or the variable _LIMITN_ in a LIMITS= data set. In this case, only those subgroups with sample size n are analyzed unless you also specify the option ALLN. You can further specify the option NMARKERS to request special symbol markers for points corresponding to sample sizes not equal to n.

Two-Sided Cusum Schemes

If the cusum scheme is two-sided, the cumulative sum S_t plotted for the tth subgroup is

$$S_t = S_{t-1} + z_t$$

for t=1, 2, . . . , n. Here S_0=0, and the term z_t is calculated as

$$z_t = (\bar{X}_t - \mu_0)/(\sigma/\sqrt{n_t})$$

where $\bar{X}_t$ is the tth subgroup average, and n_t is the tth subgroup sample size. If the subgroup samples consist of individual measurements x_t, the term z_t simplifies to

$$z_t = (x_t - \mu_0)/\sigma$$

Since the first equation can be rewritten as

$$S_t = \sum_{i=1}^{t} z_i = \sum_{i=1}^{t} (\bar{X}_i - \mu_0)/\sigma_{\bar{X}_i}$$

the sequence S_t cumulates standardized deviations of the subgroup averages from the target mean μ_0.

In many applications, the subgroup sample sizes n_i are constant ($n_i = n$), and the equation for S_t can be simplified.

$$S_t = (1/\sigma_{\bar{X}}) \sum_{i=1}^{t} (\bar{X}_i - \mu_0) = (\sqrt{n}/\sigma) \sum_{i=1}^{t} (\bar{X}_i - \mu_0)$$

In some applications, it may be preferable to compute S_t as

$$S_t = \sum_{i=1}^{t} (\bar{X}_i - \mu_0)$$

which is scaled in the same units as the data. Refer to Montgomery (1996), Wadsworth, Stephens, and Godfrey (1986), and American Society for Quality Control (1983). If the subgroup sample sizes are constant (= n) and if you specify the DATAUNITS option in the XCHART statement, the CUSUM procedure computes cusums using the final equation above. In this case, the procedure rescales the V-mask parameters h and k to $h' = h\sigma/\sqrt{n}$ and $k' = k\sigma/\sqrt{n}$, respectively. Wadsworth, Stephens, and Godfrey (1986) use the symbols F for k' and H for h'.

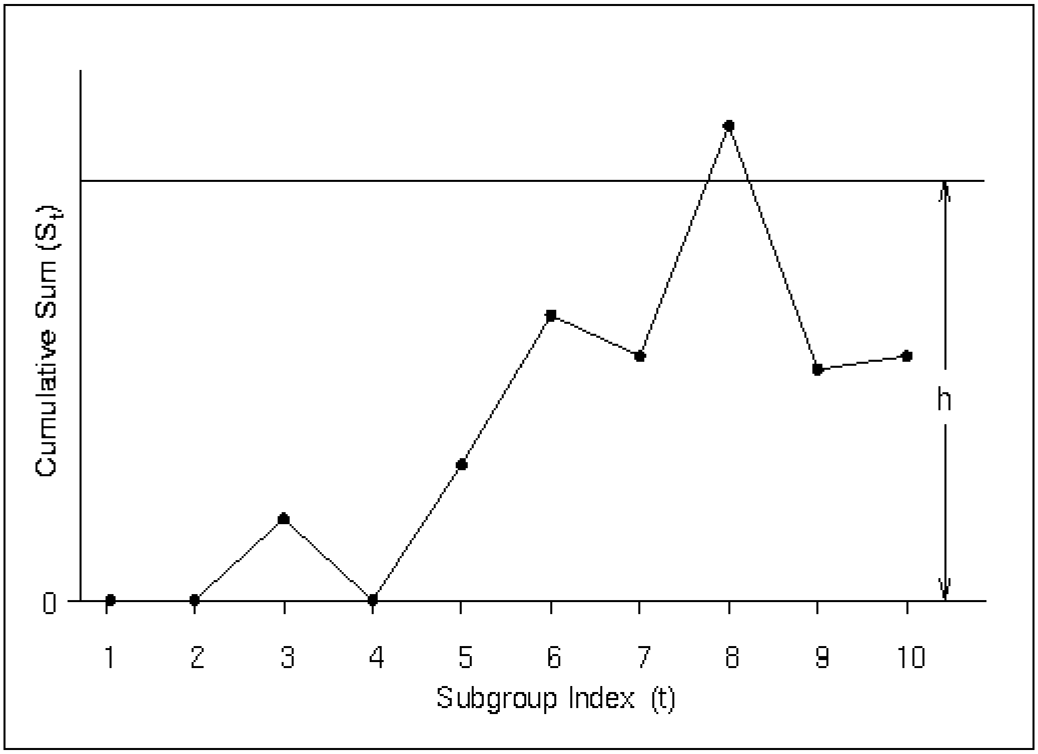
If the subgroup sample sizes are not constant, you can specify a constant nominal sample size n with the LIMITN= option or with the variable _LIMITN_ in a LIMITS= data set. In this case, only those subgroups with sample size n are analyzed unless you also specify the option ALLN. You can further specify the option NMARKERS to request special symbol markers for points corresponding to sample sizes not equal to n.

If the process is in control and the mean μ is at or near the target μ_0, the points will not exhibit a trend since positive and negative displacements from μ_0 tend to cancel each other. If μ shifts in the positive direction, the points exhibit an upward trend, and if μ shifts in the negative direction, the points exhibit a downward trend.

Defining the Decision Interval for a One-Sided Cusum Scheme

The height of the decision interval is h, expressed as a multiple of the standard error of the subgroup mean. You can specify h with the H= option in the XCHART statement or with the variable _H_ in a LIMITS= data set. The decision interval is displayed as a horizontal line on the cusum chart, as illustrated in Figure 6.13.

Figure 6.13 Decision Interval



Interpreting One-Sided Cusum Charts

A shift or out-of-control condition is signaled at time t if the cusum S_t plotted at time t exceeds the decision interval line.

Defining the V-Mask for a Two-Sided Cusum Scheme

The dimensions of the V-mask can be specified using two distinct sets of two parameters.

- θ, defined as half of the angle formed by the V-mask arms, and d, the distance between the origin and the vertex, as shown in Figure 6.14. This parameterization is used by many authors, including Johnson and Leone (1962, 1974) and Montgomery (1996).
- h, the vertical distance between the origin and the upper (or lower) V-mask arm, and k, the rise (drop) in the lower (upper) arm corresponding to an interval of one subgroup unit on the horizontal axis. You can specify the definition of an interval with the INTERVAL= option. This parameterization is used by Lucas (1976) and Wadsworth, Stephens, and Godfrey (1986). Lucas (1976) uses the symbols h^* for h and k^* for k, and Wadsworth, Stephens, and Godfrey (1986) use the symbol f in place of k.

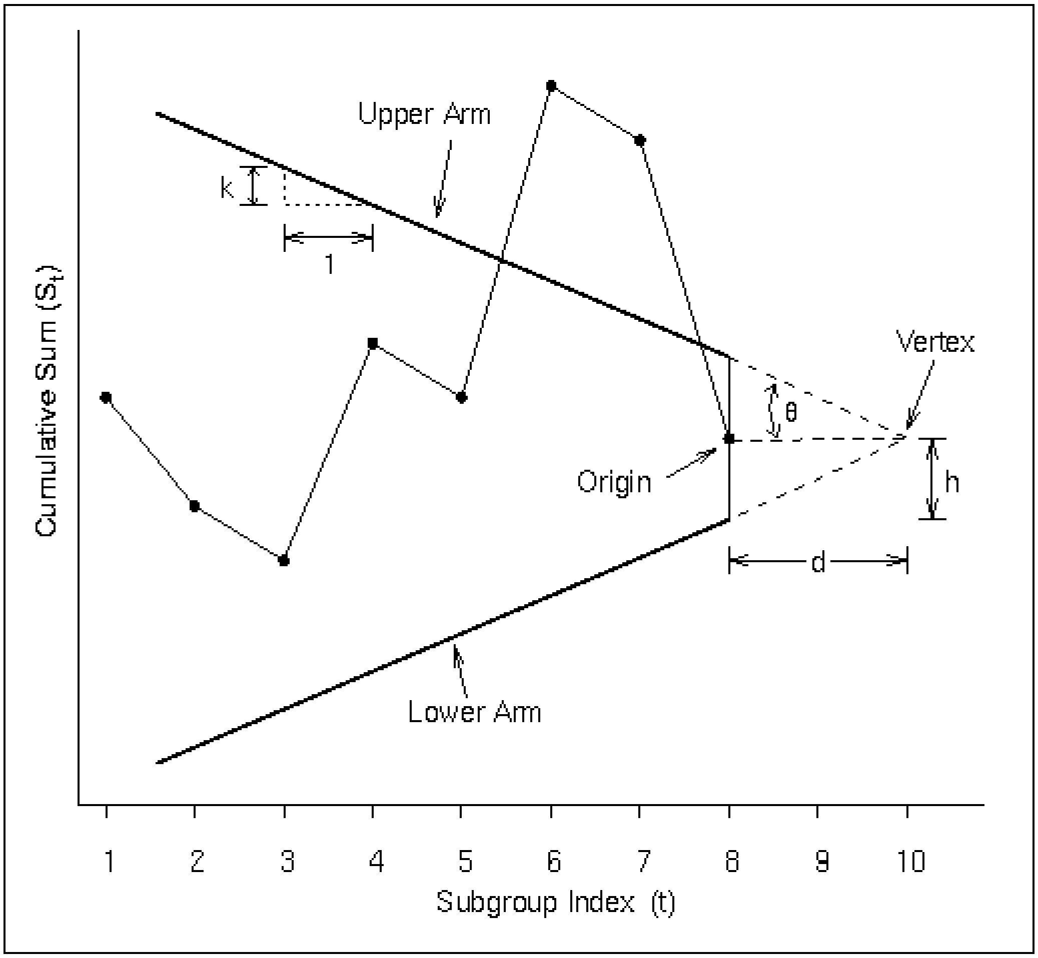
The two parameterizations are related by the equations

$$\theta = \arctan(k/a)$$

$$d = h/k$$

where the aspect ratio a is the number of units on the vertical axis corresponding to one unit on the horizontal axis. The CUSUM procedure uses the h and k parameterization because it eliminates the need for working with aspect ratios. Furthermore, h and k are also useful for average run length computations and for parameterizing one-sided cusum schemes.

Figure 6.14 V-Mask Parameters



You can specify the V-mask in two ways:

- geometrically, by providing h and k (or simply h) with the H= and K= options or with the variables _H_ and _K_ in a LIMITS= data set
- in terms of error probabilities, by providing α and β (or simply α) with the ALPHA= and BETA= options or with the variables _ALPHA_ and _BETA_ in a LIMITS= data set. The SIGMAS= option is an alternative to the ALPHA= option, and the variable _SIGMAS_ is an alternative to the variable _ALPHA_ (if the READSIGMAS option is specified).

If you provide α and β, h and k are computed using the formulas

$$h = |\delta|^{-1} \log((1 - \beta)/(\alpha/2))$$

$$k = |\delta|/2$$

If you provide α but not β, h and k are computed using the formulas

$$h = -|\delta|^{-1} \log(\alpha/2)$$

$$k = |\delta|/2$$

In the preceding equations, the error probability α is divided by two because two-sided deviations from the target mean are detected. Refer to Johnson and Leone (1962, 1974).

Interpreting Two-Sided Cusum Charts

The origin of the V-mask is located at the most recently plotted point, as illustrated in Figure 6.14. As additional data are collected and the cumulative sum sequence is updated, the origin is relocated at the newest point. A shift or out-of-control condition is signaled at time t if one or more of the points plotted up to time t cross an arm of the V-mask. An upward shift is signaled by points crossing the lower arm, and a downward shift is signaled by points crossing the upper arm. The time at which the shift occurred corresponds to the time at which a distinct change is observed in the slope of the plotted points.

Designing a Cusum Scheme

There are three main methods for designing a cusum scheme: the *average run length (ARL) approach*, the *error probability approach*, and the *economic design approach*.

Average Run Length (ARL) Approach

With the ARL approach, the parameters h and k are chosen to yield desired average run lengths when the process is operating at the target mean and when a shift of magnitude δ has occurred. The average run length is the expected number of samples taken before an out-of-control condition is signaled. Ideally, the ARL should be long when $\mu = \mu_0$ and short when μ shifts away from μ_0.

The ARL method typically involves the use of a table or nomogram. Refer to Kemp (1961), van Dobben de Bruyn (1968), Goel and Wu (1971), Duncan (1974), Lucas (1976), Montgomery (1996), and Wadsworth, Stephens, and Godfrey (1986).

For one-sided charts, average run lengths are tabulated as a function of h, k, and δ in Table 6.4. No headstart is assumed in this table. For two-sided charts, average run lengths are tabulated as a function of h, k, and δ in Table 6.5, which is formatted similarly to Table 2 given by Lucas (1976).

The ARLs in Table 6.4 and Table 6.5 were calculated with the DATA step function CUSUMARL (see the section "CUSUMARL Function" on page 2176). This function uses the method of Goel and Wu (1971). You can use this function to generate more detailed, interpolated versions of the tables or to compute ARLs with headstart values.

It can be shown that the two-sided (V-mask) cusum scheme parameterized by h and k is equivalent to two simultaneously operating one-sided cusum schemes, one that computes an upper cusum and one that computes a lower cusum. Both one-sided schemes use the same parameters h and k.

You can specify h, k, and δ with the options H=, K=, and DELTA= or with the variables _H_, _K_, and _DELTA_ in a LIMITS= data set. The reference value k is optional, and its default value is $k = |\delta|/2$, referred to as the *central reference value*.

Error Probability Approach

This approach is available only for two-sided cusum schemes. Values of α (the probability of incorrectly signaling the occurrence of a shift) and β (the probability of failing to detect a shift) are specified, and h and k are computed from α and β as described in "Defining the V-Mask for a Two-Sided Cusum Scheme" on page 580. The error probability approach interprets the cusum as a sequence of reversed sequential probability ratio tests. Refer to Johnson (1961), Johnson and Leone (1962, 1974), van Dobben de Bruyn (1968), Montgomery (1996), and Wadsworth, Stephens, and Godfrey (1986).

Although the error probability method is intuitively appealing, the actual error probabilities achieved may not be close to those specified since the V-mask does not provide for an acceptance region. This has been pointed out by various authors, including Johnson (1961) and van Dobben de Bruyn (1968). If you follow this approach, it is recommended that you examine the average run lengths for the cusum scheme (these are tabulated by the TABLESUMMARY option and are saved in OUTLIMITS= data sets).

You can specify α and β with the ALPHA= and BETA= options or with the variables _ALPHA_ and _BETA_ in a LIMITS= data set. It is not necessary to specify β, and the interpretation of β is somewhat questionable. The SIGMAS= option is an alternative to the ALPHA= option, and the variable _SIGMAS_ is an alternative to the variable _ALPHA_ (if you specify the READSIGMAS option).

Economic Design

The parameters n, h, and k are chosen so that the long-run average cost of the cusum scheme is minimized. Refer to Chiu (1974), Montgomery (1980), Svoboda (1991), and Ho and Case (1994) for reviews of the literature on economic design. This approach typically requires numerical optimization techniques, which are available in SAS/IML software and in the NLP procedure in SAS/OR software.

You can pass the optimal parameters to the CUSUM procedure as values of the variables _LIMITN_, _H_, and _K_ in a LIMITS= data set.

Table 6.4 Average Run Lengths for One-Sided V-Mask Cusum Charts as a Function of h, k, and δ.

Parameters		δ (shift in mean)										
h	**k**	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	4.00	5.00
2.50	0.25	27.27	13.43	7.96	5.42	4.06	2.71	2.06	1.68	1.42	1.11	1.01
4.00	0.25	77.08	26.68	13.29	8.38	6.06	3.91	2.93	2.38	2.05	1.61	1.23
6.00	0.25	350.80	51.34	20.90	12.37	8.73	5.51	4.07	3.26	2.74	2.13	1.90
8.00	0.25	736.78	84.00	28.76	16.37	11.39	7.11	5.21	4.15	3.48	2.67	2.14
10.00	0.25	2071.51	124.66	36.71	20.37	14.06	8.71	6.36	5.04	4.20	3.20	2.65
2.00	0.50	38.55	18.19	10.00	6.32	4.45	2.74	1.99	1.58	1.32	1.07	1.01
3.00	0.50	117.60	39.47	17.35	9.68	6.40	3.75	2.68	2.12	1.77	1.31	1.07
4.00	0.50	335.37	77.08	26.68	13.29	8.38	4.75	3.34	2.62	2.19	1.71	1.31
5.00	0.50	930.89	141.69	38.01	17.05	10.38	5.75	4.01	3.11	2.57	2.01	1.69
6.00	0.50	2553.11	250.80	51.34	20.90	12.37	6.75	4.68	3.62	2.98	2.24	1.95
1.50	0.75	42.57	21.09	11.59	7.09	4.78	2.73	1.90	1.48	1.24	1.04	1.00
2.25	0.75	139.71	51.46	22.38	11.66	7.13	3.73	2.51	1.91	1.56	1.16	1.02
3.00	0.75	442.80	117.60	39.47	17.35	9.68	4.73	3.12	2.36	1.93	1.41	1.11
3.75	0.75	1375.71	258.96	65.65	24.16	12.37	5.73	3.71	2.79	2.27	1.72	1.31
4.50	0.75	4251.69	559.95	105.12	32.09	15.15	6.73	4.31	3.21	2.59	1.97	1.60
1.00	1.00	35.29	19.22	11.21	7.03	4.75	2.63	1.78	1.38	1.17	1.02	1.00
1.50	1.00	93.85	42.57	21.09	11.59	7.09	3.50	2.24	1.66	1.34	1.07	1.01
2.00	1.00	258.67	94.34	38.55	18.19	10.00	4.45	2.74	1.99	1.58	1.16	1.02
2.50	1.00	716.00	205.97	68.19	27.27	13.43	5.42	3.25	2.34	1.85	1.31	1.07
3.00	1.00	1962.79	442.80	117.60	39.47	17.35	6.40	3.75	2.68	2.12	1.52	1.16
3.50	1.00	5341.40	943.73	199.57	55.69	21.76	7.39	4.25	3.01	2.37	1.73	1.31
0.70	1.50	67.72	36.03	20.26	12.07	7.63	3.66	2.18	1.55	1.25	1.04	1.00
1.10	1.50	184.28	86.36	42.72	22.50	12.74	5.17	2.80	1.86	1.43	1.08	1.01
1.50	1.50	549.69	221.49	93.85	42.57	21.09	7.09	3.50	2.24	1.66	1.16	1.02
1.90	1.50	1762.09	595.61	210.95	80.54	34.26	9.38	4.26	2.64	1.92	1.29	1.05
2.30	1.50	5897.30	1638.15	476.90	151.04	54.47	12.00	5.03	3.04	2.20	1.45	1.12

Table 6.5 Average Run Lengths for Two-Sided V-Mask Cusum Charts as a Function of h, k, and δ.

Parameters		**δ (shift in mean)**										
h	**k**	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	4.00	5.00
2.50	0.25	13.64	11.22	7.67	5.38	4.06	2.71	2.06	1.68	1.42	1.11	1.01
4.00	0.25	38.54	24.71	13.20	8.38	6.06	3.91	2.93	2.38	2.05	1.61	1.23
6.00	0.25	125.40	50.33	20.89	12.37	8.73	5.51	4.07	3.26	2.74	2.13	1.90
8.00	0.25	368.39	83.63	28.76	16.37	11.39	7.11	5.21	4.15	3.48	2.67	2.14
10.00	0.25	1035.75	124.55	36.71	20.37	14.06	8.71	6.36	5.04	4.20	3.20	2.65
2.00	0.50	19.27	15.25	9.63	6.27	4.44	2.74	1.99	1.58	1.32	1.07	1.01
3.00	0.50	58.80	36.24	17.20	9.67	6.40	3.75	2.68	2.12	1.77	1.31	1.07
4.00	0.50	167.68	74.22	26.63	13.29	8.38	4.75	3.34	2.62	2.19	1.71	1.31
5.00	0.50	465.44	139.49	38.00	17.05	10.38	5.75	4.01	3.11	2.57	2.01	1.69
6.00	0.50	1276.55	249.26	51.34	20.90	12.37	6.75	4.68	3.62	2.98	2.24	1.95
1.50	0.75	21.28	17.22	11.01	7.00	4.77	2.73	1.90	1.48	1.24	1.04	1.00
2.25	0.75	69.85	45.97	22.04	11.63	7.13	3.73	2.51	1.91	1.56	1.16	1.02
3.00	0.75	221.40	110.95	39.31	17.34	9.68	4.73	3.12	2.36	1.93	1.41	1.11
3.75	0.75	687.85	251.56	65.58	24.16	12.37	5.73	3.71	2.79	2.27	1.72	1.31
4.50	0.75	2125.85	552.11	105.09	32.09	15.15	6.73	4.31	3.21	2.59	1.97	1.60
1.00	1.00	17.65	15.03	10.39	6.88	4.72	2.63	1.78	1.38	1.17	1.02	1.00
1.50	1.00	46.92	35.70	20.31	11.49	7.07	3.50	2.24	1.66	1.34	1.07	1.01
2.00	1.00	129.34	84.00	37.93	18.14	10.00	4.45	2.74	1.99	1.58	1.16	1.02
2.50	1.00	358.00	191.48	67.76	27.25	13.43	5.42	3.25	2.34	1.85	1.31	1.07
3.00	1.00	981.39	423.29	117.32	39.47	17.35	6.40	3.75	2.68	2.12	1.52	1.16
3.50	1.00	2670.70	917.89	199.40	55.69	21.76	7.39	4.25	3.01	2.37	1.73	1.31
0.70	1.50	33.86	28.41	18.90	11.84	7.59	3.66	2.18	1.55	1.25	1.04	1.00
1.10	1.50	92.14	71.41	40.91	22.29	12.71	5.17	2.80	1.86	1.43	1.08	1.01
1.50	1.50	274.84	191.58	91.58	42.39	21.07	7.09	3.50	2.24	1.66	1.16	1.02
1.90	1.50	881.05	536.07	208.31	80.41	34.25	9.38	4.26	2.64	1.92	1.29	1.05
2.30	1.50	2948.65	1523.15	474.09	150.96	54.47	12.00	5.03	3.04	2.20	1.45	1.12

Cusum Charts Compared with Shewhart Charts

Although cusum charts and Shewhart charts are both used to detect shifts in the process mean, there are important differences in the two methods.

- Each point on a Shewhart chart is based on information for a single subgroup sample or measurement. Each point on a cusum chart is based on information from all samples (measurements) up to and including the current sample (measurement).
- On a Shewhart chart, upper and lower control limits are used to decide whether a point signals an out-of-control condition. On a cusum chart, the limits take the form of a decision interval or a V-mask.
- On a Shewhart chart, the control limits are commonly computed as 3σ limits. On a cusum chart, the limits are determined from average run length specifications, specified error probabilities, or an economic design.

A cusum chart offers several advantages over a Shewhart chart.

- A cusum chart is more efficient for detecting small shifts in the process mean, in particular, shifts of 0.5 to 2 standard deviations from the target mean (refer to Montgomery 1996). Lucas (1976) noted that "a V-mask designed to detect a 1σ shift will detect it about four times as fast as a competing Shewhart chart."
- Shifts in the process mean are visually easy to detect on a cusum chart since they produce a change in the slope of the plotted points. The point at which the slope changes is the point at which the shift has occurred.

These advantages are not as pronounced if the Shewhart chart is augmented by the tests for special causes described by Nelson (1984, 1985). Also see "Tests for Special Causes: SHEWHART Procedure" on page 2073. Moreover,

- cusum schemes are more complicated to design.
- a cusum chart can be slower to detect large shifts in the process mean.
- it can be difficult to interpret point patterns on a cusum chart since the cusums are correlated.

Methods for Estimating the Standard Deviation

It is recommended practice to provide a stable estimate or standard value for σ with either the SIGMA0= option or the variable _STDDEV_ in a LIMITS= data set. However, if such a value is not available, you can compute an estimate $\hat{\sigma}$ from the data, as described in this section.

This section provides formulas for various methods used to estimate the standard deviation σ. One method is applicable with individual measurements, and three are applicable with subgrouped data. The methods can be requested with the SMETHOD= option.

Method for Individual Measurements

When the cumulative sums are calculated from individual observations

$$x_1, x_2, \ldots, x_N$$

rather than subgroup samples of two or more observations, the CUSUM procedure estimates σ as $\sqrt{\hat{\sigma}^2}$, where

$$\hat{\sigma}^2 = \frac{1}{2(N-1)} \sum_{i=1}^{N-1} (x_{i+1} - x_i)^2$$

where N is the number of observations. Wetherill (1977) states that the estimate of the variance is biased if the measurements are autocorrelated.

Note that you can compute alternative estimates (for instance, robust estimates or estimates based on variance components models) by analyzing the data with SAS modeling procedures or your own DATA step program. Such estimates can be passed to the CUSUM procedure as values of the variable _STDDEV_ in a LIMITS= data set.

NOWEIGHT Method for Subgroup Samples

This method is the default for cusum charts for subgrouped data. The estimate is

$$\hat{\sigma} = \frac{(s_1/c_4(n_1)) + \cdots + (s_N/c_4(n_N))}{N}$$

where n_i is the sample size of the ith subgroup, N is the number of subgroups for which $n_i \geq 2$, s_i is the sample standard deviation of the observations $x_{i1}, \ldots, x_{in_i}$ in the ith subgroup.

$$s_i = \sqrt{(1/(n_i - 1)) \sum_{j=1}^{n_i} (x_{ij} - \bar{X}_i)^2}$$

and

$$c_4(n_i) = \frac{\Gamma(n_i/2)\sqrt{2/(n_i-1)}}{\Gamma((n_i-1)/2)}$$

where $\Gamma(\cdot)$ denotes the gamma function, and $\bar{X}_i$ denotes the ith subgroup mean. A subgroup standard deviation s_i is included in the calculation only if $n_i \geq 2$. If the observations are normally distributed, then the expected value of s_i is

$$\mathrm{E}(s_i) = c_4(n_i)\sigma$$

Thus, $\hat{\sigma}$ is the unweighted average of N unbiased estimates of σ. This method is described in the *ASTM Manual on Presentation of Data and Control Chart Analysis*.

MVLUE Method for Subgroup Samples

If you specify SMETHOD=MVLUE, a minimum variance linear unbiased estimate (MVLUE) is computed, as introduced by Burr (1969, 1976). This estimate is a weighted average of unbiased estimates of σ of the form

$$s_i/c_4(n_i)$$

where

s_i	is the standard deviation of the ith subgroup.
$c_4(n_i)$	is the unbiasing factor defined previously.
n_i	is the ith subgroup sample size, $i = 1, 2, \ldots, N$.
N	is the number of subgroups for which $n_i \geq 2$.

The estimate is

$$\hat{\sigma} = \frac{h_1 s_1 / c_4(n_1) + \cdots + h_N s_N / c_4(n_N)}{h_1 + \cdots + h_N}$$

where $h_i = c_4^2(n_i)/(1 - c_4^2(n_i))$. A subgroup standard deviation s_i is included in the calculation only if $n_i \geq 2$.

The MVLUE assigns greater weight to estimates of σ from subgroups with larger sample sizes and is intended for situations where the subgroup sample sizes vary. If the subgroup sample sizes are constant, the MVLUE reduces to the default estimate (NOWEIGHT).

RMSDF Method for Subgroup Samples

If you specify SMETHOD=RMSDF, a weighted root-mean-square estimate is computed:

$$\hat{\sigma} = \frac{\sqrt{(n_1 - 1)s_1^2 + \cdots + (n_N - 1)s_N^2}}{c_4(n)\sqrt{n_1 + \cdots + n_N - N}}$$

where

n_i	is the sample size of the ith subgroup.
N	is the number of subgroups for which $n_i \geq 2$.
s_i	is the sample standard deviation of the ith subgroup.
$c_4(n_i)$	is the unbiasing factor defined previously.
n	is equal to $(n_1 + \cdots + n_N) - (N - 1)$.

The weights in the root-mean-square expression are the degrees of freedom $n_i - 1$. A subgroup standard deviation s_i is included in the calculation only if $n_i \geq 2$.

If the unknown standard deviation σ is constant across subgroups, the root-mean-square estimate is more efficient than the minimum variance linear unbiased estimate. However, as noted by Burr (1969), "the constancy of σ is the very thing under test," and if σ varies across subgroups, the root-mean-square estimate tends to be more inflated than the MVLUE.

Output Data Sets

OUTLIMITS= Data Set

When you save the parameters for the cusum scheme in an OUTLIMITS= data set, the following variables are included:

Table 6.6 OUTLIMITS= Data Set

Variable	Description
ALPHA	probability (α) of Type 1 error
ARLIN	average run length for zero shift
ARLOUT	average run length for shift of δ
BETA	probability (β) of Type 2 error
DELTA	shift (δ) to be detected
_H_	decision interval h for one-sided scheme; distance h between origin and upper arm V-mask for two-sided scheme
HSTART	headstart value
INDEX	optional identifier for cusum parameters (if the OUTINDEX= option is specified)
_K_	reference value k for one-sided scheme; slope of lower V-mask arm for two-sided scheme
LIMITN	nominal sample size for cusum scheme
MEAN	estimated process mean ($\overline{\overline{X}}$)
_MU0_	target mean μ_0
ORIGIN	origin of V-mask
SCHEME	type of scheme ('ONESIDED' or 'TWOSIDED')
SIGMAS	$z_{1-\alpha/2}$
STDDEV	estimated or known standard deviation ($\hat{\sigma}$ or σ_0)
SUBGRP	*subgroup-variable* specified in XCHART statement
TYPE	type ('ESTIMATE' or 'STANDARD') of _STDDEV_
VAR	*process* specified in XCHART statement

Notes:

1. If the subgroup sample sizes vary, the special missing value V is assigned to the variable _LIMITN_.
2. If a V-mask is specified with SIGMAS=k, _ALPHA_ is computed as $\alpha = 2(1 - \Phi(k))$, where $\Phi(\cdot)$ is the standard normal distribution function.
3. If a V-mask is specified with ALPHA=α, _SIGMAS_ is computed as $k = \Phi^{-1}(1 - \alpha/2)$, where Φ^{-1} is the inverse standard normal distribution function.
4. BY variables are saved in the OUTLIMITS= data set.

The OUTLIMITS= data set contains one observation for each *process* specified in the XCHART statement. For an example, see "Saving Cusum Scheme Parameters" on page 557.

OUTHISTORY= Data Set

When you save subgroup summary statistics in an OUTHISTORY= data set, the following variables are included:

- the *subgroup-variable*
- a subgroup mean variable named by *process* suffixed with *X*

- a subgroup sample size variable named by *process* suffixed with *N*
- a subgroup standard deviation variable named by *process* suffixed with *S*
- a cusum variable named by *process* suffixed with *C*

Given a *process* name that contains 32 characters, the procedure first shortens the name to its first 16 characters and its last 15 characters, and then it adds the suffix.

Variables containing subgroup summary statistics are created for each *process* specified in the XCHART statement. For example, consider the following statements:

```
proc cusum data=Steel limits=Stparm;
   xchart (Width Diameter)*Lot / outhistory=Summary;
run;
```

The data set Summary would contain nine variables named Lot, WidthX, WidthS, WidthN, WidthC, DiameterX, DiameterS, DiameterN, and DiameterC.

Additionally, if specified, the following variables are included:

- BY variables
- *block-variables*
- *symbol-variable*
- ID variables
- _PHASE_ (if the OUTPHASE= option is specified)

For an example creating an OUTHISTORY= data set, see "Saving Summary Statistics" on page 552.

OUTTABLE= Data Set

The OUTTABLE= data set saves subgroup means, subgroup sample sizes, cusums, and cusum limits. The following variables are included:

Variable	Description
CUSUM	cumulative sum
EXLIM	decision interval or V-mask arm exceeded
_H_	decision interval
MASKL	lower arm of V-mask
MASKU	upper arm of V-mask
subgroup	values of the subgroup variable
SUBN	subgroup sample size
SUBX	subgroup mean
SUBS	subgroup standard deviation
VAR	*process* specified in XCHART statement

In addition, the following variables are saved if specified:

- BY variables
- *block-variables*
- ID variables
- _PHASE_ (if the READPHASES= option is specified)
- _TREND_ (if the TRENDVAR= option is specified)
- *symbol-variable*

Note that the variables _VAR_ and _EXLIM_ are character variables of length eight. The variable _PHASE_ is a character variable of length 16.

ODS Tables

The following table summarizes the ODS tables that you can request with the XCHART statement.

Table 6.7 ODS Tables Produced with the XCHART Statement

Table Name	Description	Options
CompCusum	computational form of the cusum scheme	TABLEALL, TABLECOMP
Parameters	cusum parameters and computed average run lengths	TABLEALL, TABLESUMMARY
XCHART	cusum chart summary statistics	TABLEALL, TABLECHART, TABLEOUT

ODS Graphics

Before you create ODS Graphics output, ODS Graphics must be enabled (for example, by using the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" (Chapter 21, *SAS/STAT User's Guide*). **NOTE:** In SAS/QC 13.1 the CUSUM procedure does not support the creation of graphs that are editable with the ODS Graphics Editor.

The appearance of a graph produced with ODS Graphics is determined by the style associated with the ODS destination where the graph is produced. XCHART options used to control the appearance of traditional graphics are ignored for ODS Graphics output. Options for Producing Graphs Using ODS Styles lists options that can be used to control the appearance of graphs produced with ODS Graphics or with traditional graphics using ODS styles. Options for ODS Graphics lists options to be used exclusively with ODS Graphics. Detailed descriptions of these options are provided in "Dictionary of Options: SHEWHART Procedure" on page 1946.

When ODS Graphics is in effect, the XCHART statement assigns a name to the graph it creates. You can use this name to reference the graph when using ODS. The name is listed in Table 6.8.

Table 6.8 ODS Graphics Produced by the XCHART Statement

ODS Graph Name	Plot Description
XChart	cusum chart

See Chapter 3, "SAS/QC Graphics," for more information about ODS Graphics and other methods for producing charts.

Input Data Sets

DATA= Data Set

You can read raw data (measurements) from a DATA= data set specified in the PROC CUSUM statement. Each *process* specified in the XCHART statement must be a SAS variable in the DATA= data set. The values of this variable are typically measurements of a quality characteristic taken on items in subgroup samples indexed by the values of the subgroup variable. The *subgroup-variable* specified in the XCHART statement must also be a SAS variable in the DATA= data set. Other variables that can be read from a DATA= data set include

- _PHASE_ (if the READPHASES= option is specified)
- *block-variables*
- *symbol-variable*
- BY variables
- ID variables

Each observation in a DATA= data set should contain a raw measurement for each *process* and a value for the subgroup variable. If the *i*th subgroup contains n_i items, there should be n_i consecutive observations for which the value of the subgroup variable is the index of the *i*th subgroup. For example, if each of 30 subgroup samples contains five items, the DATA= data set should contain 150 observations.

By default, the CUSUM procedure reads all of the observations in a DATA= data set. However, if the DATA= data set includes the variable _PHASE_, you can read selected groups of observations (referred to as *phases*) by specifying the READPHASES= option in the XCHART statement.

For an example of a DATA= data set, see "Creating a V-Mask Cusum Chart from Raw Data" on page 547.

LIMITS= Data Set

You can read cusum scheme parameters from a LIMITS= data set specified in the PROC CUSUM statement. As an alternative to specifying the parameters with options, a LIMITS= data set provides the following advantages: it facilitates reusing a permanently saved set of parameters, reading a distinct set of parameters for each *process* specified in the XCHART statement, and keeping track of multiple sets of parameters for the same *process* over time.

The LIMITS= data set can be an OUTLIMITS= data set that was created in a previous run of the CUSUM procedure. Such data sets always contain the variables required for a LIMITS= data set; consequently, this is the easiest way to construct a LIMITS= data set.

A LIMITS= data set can also be created directly using a DATA step. The variables required for the data set depend on the type of cusum scheme and how the scheme is specified. The following restrictions apply:

- The variables _VAR_, _SUBGRP_, _DELTA_, and _MU0_ are required.
- For a one-sided cusum scheme, _H_ is required.
- For a two-sided cusum scheme, one of the following three variables is required: _ALPHA_, _H_, or _SIGMAS_.
- If you plan to use the READINDEX= option, the variable _INDEX_ is required; otherwise, it is optional.
- For a one-sided scheme, the variable _SCHEME_ is required; otherwise, it is optional.
- If you want to provide a value for the process standard deviation σ, the variable _STDDEV_ is required; otherwise, it is optional.

Variable names in a LIMITS= data set are predefined; the procedure reads only variables with these predefined names. With the exception of BY variables, all names start and end with an underscore. In addition, note the following:

- The variables _VAR_, _SUBGRP_, _TYPE_, and _SCHEME_ must be character variables of length eight. The variable _INDEX_ must be a character variable of length 16.
- The variable _TYPE_ is a bookkeeping variable that uses the values 'ESTIMATE' and 'STANDARD' to record whether the value of _STDDEV_ represents an estimate or standard (known) value.
- BY variables are required if specified with a BY statement.

For an example of reading control limit information from a LIMITS= data set, see "Reading Cusum Scheme Parameters" on page 559.

HISTORY= Data Set

Instead of reading raw data from a DATA= data set, you can read subgroup summary statistics from a HISTORY= data set specified in the PROC CUSUM statement. This enables you to reuse OUTHISTORY= data sets that have been created in previous runs of the CUSUM, MACONTROL, or SHEWHART procedures or to read output data sets created with SAS summarization procedures such as PROC MEANS. A HISTORY= data set must contain the following variables:

- *subgroup-variable*
- subgroup mean variable for each *process*
- subgroup standard deviation variable for each *process*
- subgroup sample size variable for each *process*

The names of the subgroup mean, subgroup standard deviation, and subgroup sample size variables must be the *process* concatenated with the special suffix characters *X*, *S*, and *N* respectively.

For example, consider the following statements:

```
proc cusum history=Steel limits=Steelparm;
   xchart (Weight Yieldstrength)*Batch;
run;
```

The data set Steel must contain the variables Batch, WeightX, WeightS, WeightN, YieldstrengthX, YieldstrengthS, and YieldstrengthN.

Note that if you specify a *process* name that contains 32 characters, the names of the summary variables must be formed from the first 16 characters and the last 15 characters of the *process* name, suffixed with the appropriate character.

Other variables that can be read from a HISTORY= data set include

- _PHASE_ (if the READPHASES= option is specified)
- *block-variables*
- *symbol-variable*
- BY variables
- ID variables

By default, the CUSUM procedure reads all of the observations in a HISTORY= data set. However, if the HISTORY= data set includes the variable _PHASE_, you can read selected groups of observations (referred to as phases) by specifying the READPHASES= option.

For an example of reading summary information from a HISTORY= data set, see "Creating a V-Mask Cusum Chart from Subgroup Summary Data" on page 550.

Missing Values

An observation read from a DATA= or HISTORY= data set is not analyzed if the value of the subgroup variable is missing. For a particular process variable, an observation read from a DATA= data set is not analyzed if the value of the process variable is missing. Missing values of process variables generally lead to unequal subgroup sample sizes. For a particular process variable, an observation read from a HISTORY= data set is not analyzed if the values of any of the corresponding summary variables are missing.

Examples: XCHART Statement

This section provides advanced examples of the XCHART statement.

Example 6.1: Cusum and Standard Deviation Charts

NOTE: See *Cusum and Standard Deviation Charts* in the SAS/QC Sample Library.

When you are working with subgrouped data, it can be helpful to accompany a cusum chart for means with a Shewhart *s* chart for monitoring the variability of the process. This example creates this combination for the variable Weight in the data set Oil (see "Creating a V-Mask Cusum Chart from Raw Data" on page 547).

The first step is to create a one-sided cusum chart for means that detects a shift of one standard error ($\delta = -1$) below the target mean.

```
proc cusum data=Oil;
   xchart Weight*Hour /
      nochart
      mu0=8.100        /* target mean for process  */
      sigma0=0.050     /* known standard deviation */
      delta=-1         /* shift to be detected     */
      h=3              /* cusum parameter h        */
      k=0.5            /* cusum parameter k        */
      scheme=onesided
      outtable = Tabcusum
         ( drop   = _var_ _subn_ _subx_ _exlim_
           rename = ( _cusum_ = _subx_ _h_ = _uclx_ ) )
      ;
run;
```

The results are saved in an OUTTABLE= data set named Tabcusum. The cusum variable (_CUSUM_) and the decision interval variable (_H_) are renamed to _SUBX_ and _LCLX_ so that they can later be read by the SHEWHART procedure.

The next step is to construct a Shewhart $\bar{X}$ and s chart for Weight and save the results in a data set named Tabxscht.

```
proc shewhart data=Oil;
   xschart Weight*Hour /
      nochart
      outtable = Tabxscht
         ( drop = _subx_  _uclx_ );
run;
```

Note that the variables _SUBX_ and _UCLX_ are dropped from Tabxscht.

The third step is to merge the data sets Tabcusum and Tabxscht.

```
data taball;
   merge Tabxscht Tabcusum; by Hour;
   _mean_ = _uclx_ * 0.5;
   _lclx_ = 0.0;
run;
```

The variable _LCLX_ is assigned the role of the lower limit for the cusums, and the variable _MEAN_ is assigned a dummy value. Now, TABALL, which is listed in Output 6.1.1, has the structure required for a TABLE= data set used with the XSCHART statement in the SHEWHART procedure (see “TABLE= Data Set” on page 1917).

Output 6.1.1 Listing of the Data Set TABALL

Obs	_VAR_	Hour	_SIGMAS_	_LIMITN_	_SUBN_	_LCLX_	_MEAN_	_STDDEV_	_EXLIM_	_LCLS_
1	Weight	1	3	4	4	0	1.5	0.05		0
2	Weight	2	3	4	4	0	1.5	0.05		0
3	Weight	3	3	4	4	0	1.5	0.05		0
4	Weight	4	3	4	4	0	1.5	0.05		0
5	Weight	5	3	4	4	0	1.5	0.05		0
6	Weight	6	3	4	4	0	1.5	0.05		0
7	Weight	7	3	4	4	0	1.5	0.05		0
8	Weight	8	3	4	4	0	1.5	0.05		0
9	Weight	9	3	4	4	0	1.5	0.05		0
10	Weight	10	3	4	4	0	1.5	0.05		0
11	Weight	11	3	4	4	0	1.5	0.05		0
12	Weight	12	3	4	4	0	1.5	0.05		0

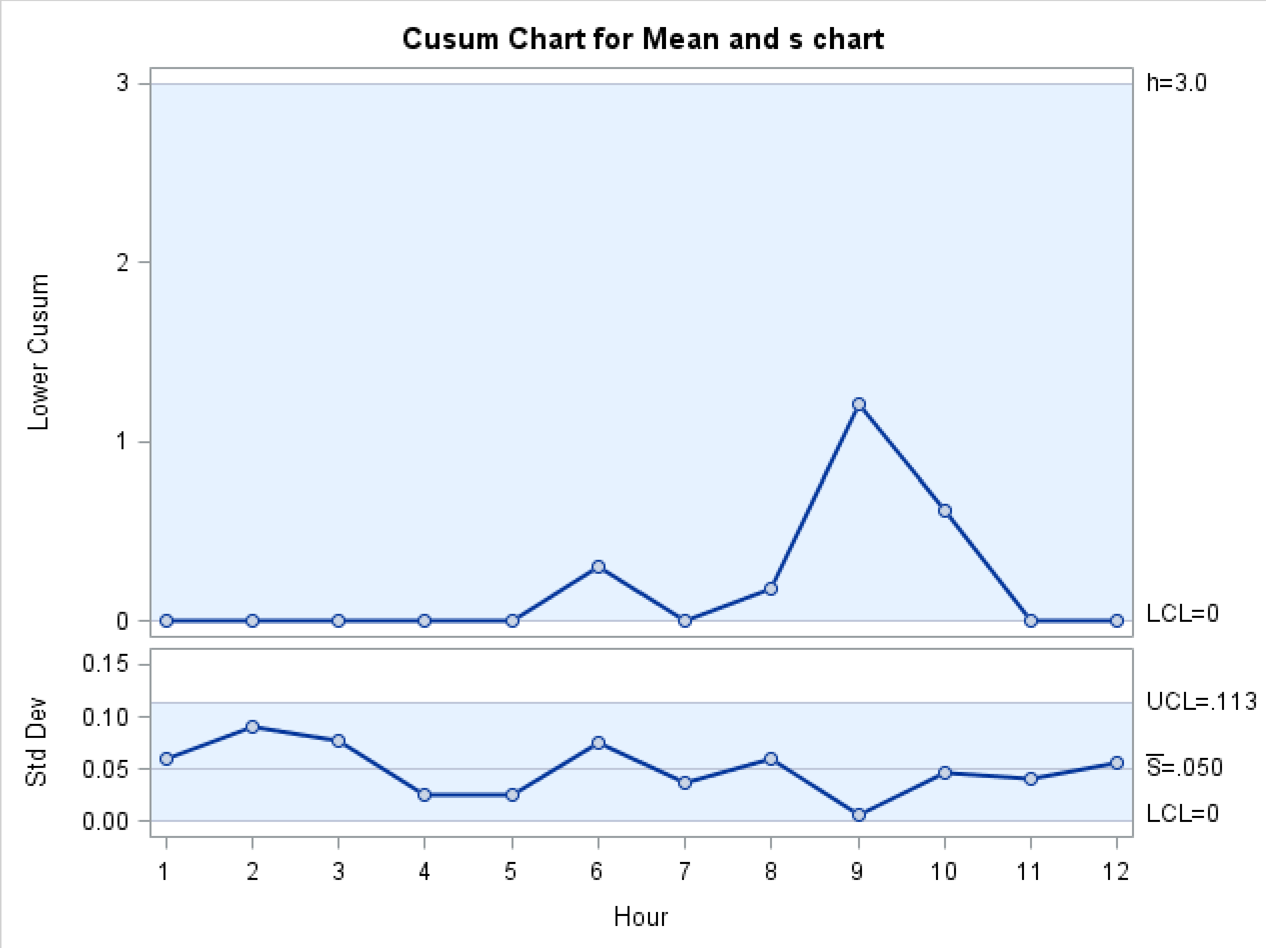
Obs	_SUBS_	_S_	_UCLS_	_EXLIMS_	_subx_	_uclx_
1	0.059640	0.049943	0.11317		0.00	3
2	0.090220	0.049943	0.11317		0.00	3
3	0.076346	0.049943	0.11317		0.00	3
4	0.025552	0.049943	0.11317		0.00	3
5	0.026500	0.049943	0.11317		0.00	3
6	0.075617	0.049943	0.11317		0.30	3
7	0.037242	0.049943	0.11317		0.00	3
8	0.059290	0.049943	0.11317		0.18	3
9	0.005737	0.049943	0.11317		1.21	3
10	0.046522	0.049943	0.11317		0.62	3
11	0.040542	0.049943	0.11317		0.00	3
12	0.056103	0.049943	0.11317		0.00	3

The final step is to use the SHEWHART procedure to read TABALL as a TABLE= data set and to display the cusum and *s* charts.

```
ods graphics on;
title 'Cusum Chart for Mean and s chart';
proc shewhart table=taball;
   xschart Weight * Hour /
      nolimitslegend
      ucllabel = 'h=3.0'
      odstitle = title
      markers
      noctl
      split = '/'
      nolegend ;
   label _subx_ = 'Lower Cusum/Std Dev';
run;
```

The central line for the primary (cusum) chart is suppressed with the NOCTL option, and the default *3σ Limits* legend is suppressed with the NOLIMITLEGEND option. The charts are shown in Output 6.1.2.

Output 6.1.2 Combined Cusum Chart and s Chart



The process variability is stable, and there is no signal of a downward shift in the process mean.

Example 6.2: Upper and Lower One-Sided Cusum Charts

NOTE: See *Upper and Lower One-Sided Cusum Charts* in the SAS/QC Sample Library.

This example illustrates how to combine upper and lower one-sided cusum charts for means in the same display. As in the preceding example, OUTTABLE= data sets are created with the CUSUM procedure, and the display is created with the SHEWHART procedure.

The following statements analyze the variable Weight in the data set Oil (see "Creating a V-Mask Cusum Chart from Raw Data" on page 547). The first step is to compute and save upper and lower one-sided cusums for shifts of one standard error in the positive and negative directions.

```
proc cusum data=Oil;
   xchart Weight*Hour /
      nochart
      mu0=8.100        /* target mean for process  */
      sigma0=0.050     /* known standard deviation */
```

```
      delta=1          /* shift to be detected     */
      h=3              /* cusum parameter h        */
      k=0.5            /* cusum parameter k        */
      scheme=onesided
      outtable = tabupper
         ( drop   = _subx_ _subs_ _exlim_
           rename = ( _cusum_ = _subx_ _h_ = _uclx_ ) )
      ;
   xchart Weight*Hour /
      nochart
      mu0=8.100        /* target mean for process  */
      sigma0=0.050     /* known standard deviation */
      delta=-1         /* shift to be detected     */
      h=3              /* cusum parameter h        */
      k=0.5            /* cusum parameter k        */
      scheme=onesided
      outtable = tablower
         ( drop   = _var_ _subn_ _subx_ _subs_ _exlim_
           rename = ( _cusum_ = _subs_ _h_ = _ucls_ ) )
      ;
run;
```

Next, the OUTTABLE= data sets are merged.

```
data Tabboth;
   merge tabupper tablower; by Hour;
   _mean_   = _uclx_ * 0.5;
   _s_      = _ucls_ * 0.5;
   _lclx_   = 0.0;
   _lcls_   = 0.0;
run;
```

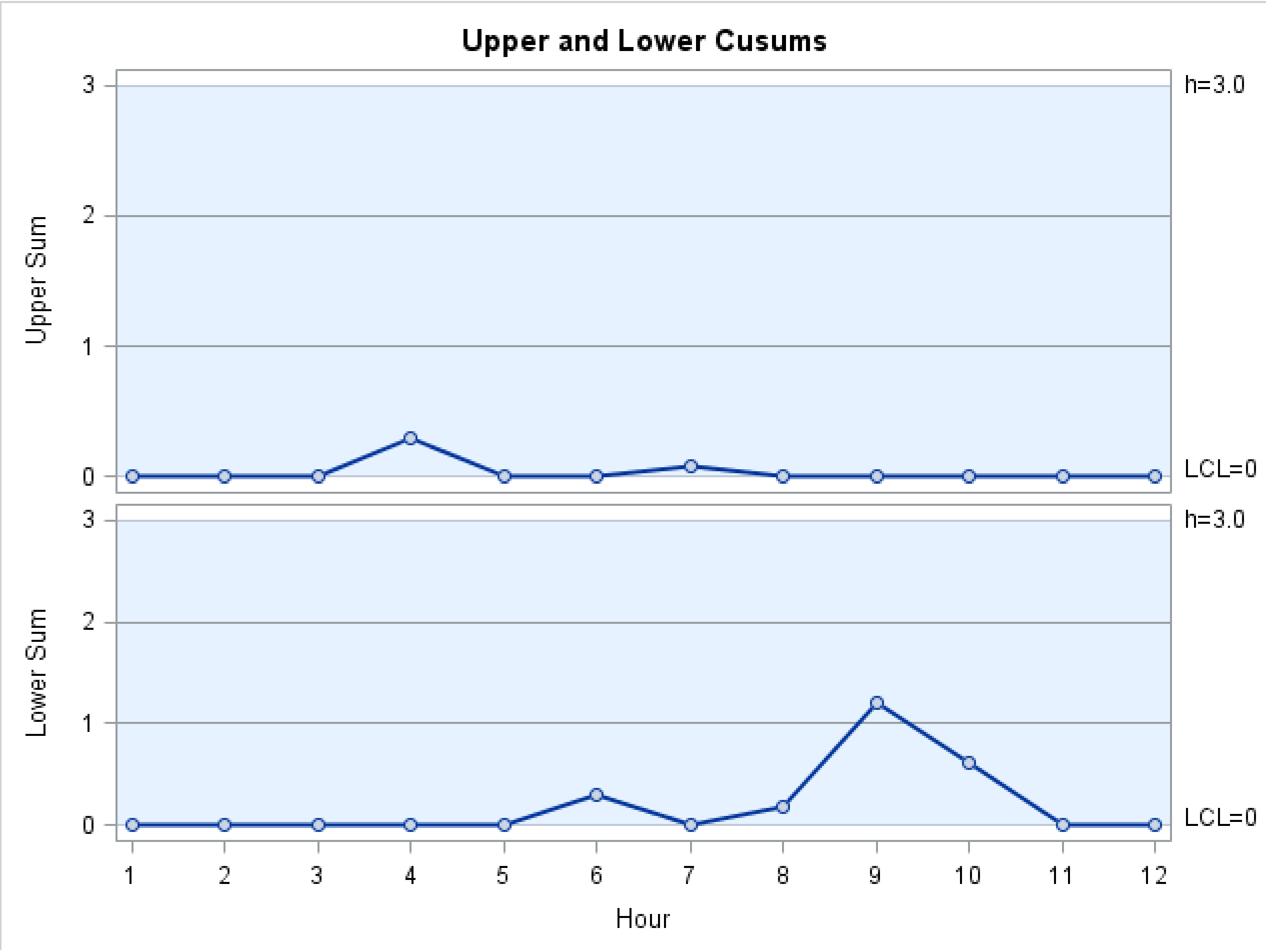
The variables _LCLX_ and _UCLX_ are assigned lower limits of zero for the cusums, and the variables _MEAN_ and _S_ are assigned dummy values. Now, Tabboth has the structure required for a TABLE= data set used with the XSCHART statement in the SHEWHART procedure (see “TABLE= Data Set” on page 1917).

The final step is to read Tabboth as a TABLE= data set with the SHEWHART procedure.

```
ods graphics on;
title 'Upper and Lower Cusums';
proc shewhart table=Tabboth;
   xschart Weight * Hour /
      nolimitslegend
      markers
      odstitle  = title
      ucllabel  = 'h=3.0'
      ucllabel2 = 'h=3.0'
      ypct1     = 50
      vref      = 1 2
      vref2     = 1 2
      noctl
      noctl2
      split = '/'
      nolegend ;
   label _subx_ = 'Upper Sum/Lower Sum';
run;
```

The combined display is shown in Output 6.2.1. There is no evidence of a shift in either direction.

Output 6.2.1 Upper and Lower One-Sided Cusum Charts



Example 6.3: Combined Shewhart–Cusum Scheme

NOTE: See *Combined Shewhart-Cusum Scheme* in the SAS/QC Sample Library.

Lucas and Crosier (1982) introduced a combined Shewhart-cusum scheme that is illustrated in this example. Also refer to Ryan (1989). The data set used here is Cans, which is created in "Creating a One-Sided Cusum Chart with a Decision Interval" on page 553.

The first step is to compute and save one-sided cusums to detect a positive shift from the mean.

```
proc cusum data=Cans;
   xchart Weight*Hour /
      nochart
      mu0    = 8.100     /* target mean for process   */
      sigma0 = 0.050     /* known standard deviation */
      delta  = 1         /* shift to be detected      */
      h      = 3         /* cusum parameter h         */
```

```
      k      = 0.5       /* cusum parameter k        */
      scheme = onesided
      outtable = Tabcus
         ( drop   = _var_ _subn_ _exlim_
           rename = ( _cusum_ = _subr_ _h_ = _uclr_ ) )
      ;
run;
```

Note that a headstart value is not used here but can be specified with the HSTART= option. Several variables in the OUTTABLE= data set are dropped or renamed so that they can later be read by the SHEWHART procedure.

The next step is to construct a Shewhart chart (not shown) for individual measurements.

```
proc shewhart data=Cans;
   irchart Weight*Hour /
      nochart
      mu0      = 8.100
      sigma0   = 0.050
      outtable = Tabx
         ( drop   = _subr_  _lclr_ _r_ _uclr_ );
   id comment;
run;
```

By default, 3σ limits are computed, but the multiple of σ can be modified with the SIGMAS= option. As before, the results are saved in an OUTTABLE= data set.

Next, the two OUTTABLE= data sets are merged.

```
data Combine;
   merge Tabx Tabcus; by Hour;
   _lclr_ = 0.0;
   _r_    = 0.5 * _uclr_;
run;
```

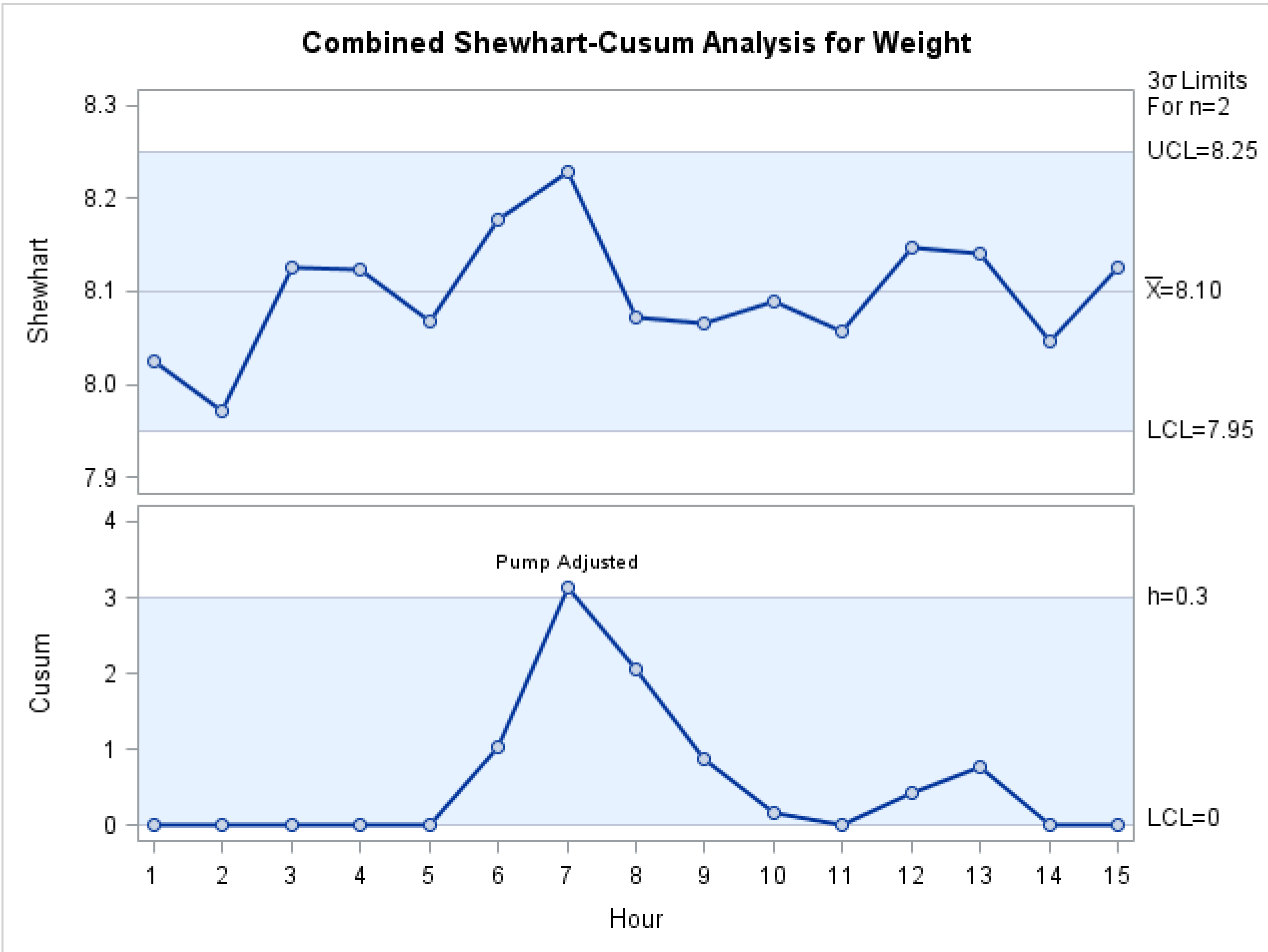
The data set Combine has the structure required for a TABLE= data set used with the IRCHART statement in the SHEWHART procedure (see the section “TABLE= Data Set” on page 1503).

Finally, the combined scheme is displayed with the SHEWHART procedure.

```
ods graphics on;
title "Combined Shewhart-Cusum Analysis for Weight";
proc shewhart table=Combine;
   irchart Weight*Hour /
      odstitle  = title
      ypct1     = 50
      noctl2
      markers
      ucllabel2 = 'h=0.3'
      outlabel  = ( comment )
      outlabel2 = ( comment )
      split     = '/';
   label _subi_ = 'Shewhart/Cusum';
run;
```

The chart is shown in Output 6.3.1.

Output 6.3.1 Combined Shewhart–Cusum Scheme



Note that a shift is detected by the cusum scheme but not by the Shewhart chart. The point exceeding the decision interval is labeled with the variable comment created in the data set Cans.

Lucas and Crosier (1982) tabulates average run lengths for combined Shewhart-cusum schemes. The scheme used here has an ARL of 111.1 for $\delta = 0$ and an ARL of 6.322 for $\delta = 1$.

INSET Statement: CUSUM Procedure

Overview: INSET Statement

The INSET statement enables you to enhance a cusum chart by adding a box or table (referred to as an *inset*) of summary statistics directly to the graph. A possible application of an inset is to present cusum parameters on the chart rather than displaying them in a legend. An inset can also display arbitrary values provided in a SAS data set.

Note that the INSET statement by itself does not produce a display but must be used in conjunction with an XCHART statement. Insets are not available with line printer charts, so the INSET statement is not applicable when the LINEPRINTER option is specified in the PROC CUSUM statement.

You can use options in the INSET statement to do the following:

- specify the position of the inset
- specify a header for the inset table
- specify graphical enhancements, such as background colors, text colors, text height, text font, and drop shadows

Getting Started: INSET Statement

This section introduces the INSET statement with a basic example showing how it is used. See the section "INSET and INSET2 Statements: SHEWHART Procedure" on page 1928 in Chapter 17, "The SHEWHART Procedure," for a complete description of the INSET statement.

This example is based on the same scenario as the first example in the "Getting Started" subsection of "XCHART Statement: CUSUM Procedure" on page 546. A machine fills cans with oil additive and a two-sided cusum chart is used to detect shifts from the target mean of 8.100 ounces. The following statements create the data set Oil and request a two-sided cusum chart with an inset:

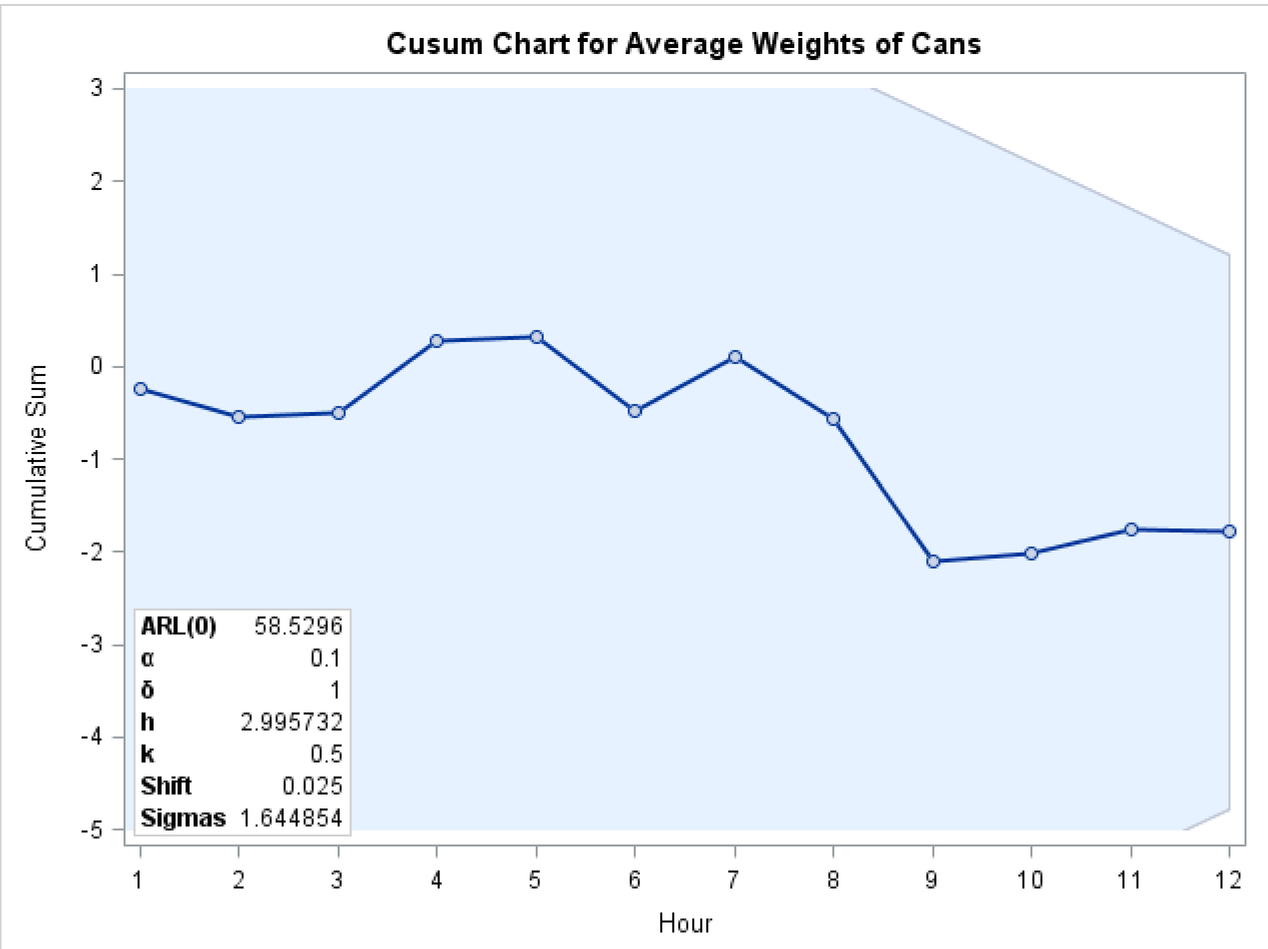
```
data Oil;
   label Hour = 'Hour';
   input Hour @;
   do i=1 to 4;
      input Weight @;
      output;
   end;
   drop i;
   datalines;
 1  8.024  8.135  8.151  8.065
 2  7.971  8.165  8.077  8.157
 3  8.125  8.031  8.198  8.050
 4  8.123  8.107  8.154  8.095
 5  8.068  8.093  8.116  8.128
 6  8.177  8.011  8.102  8.030
 7  8.129  8.060  8.125  8.144
 8  8.072  8.010  8.097  8.153
 9  8.066  8.067  8.055  8.059
10  8.089  8.064  8.170  8.086
11  8.058  8.098  8.114  8.156
12  8.147  8.116  8.116  8.018
;
```

```
title 'Cusum Chart for Average Weights of Cans';
ods graphics on;
proc cusum data=Oil;
   xchart Weight*Hour /
      mu0       = 8.100           /* Target mean for process   */
      sigma0    = 0.050           /* Known standard deviation */
      delta     = 1               /* Shift to be detected      */
      alpha     = 0.10            /* Type I error probability */
      vaxis     = -5 to 3
      odstitle = title
      markers
      nolegend;
   label Weight = 'Cumulative Sum';
   inset arl0 ualpha udelta h k shift sigmas / pos = sw;
run;
```

The ODS GRAPHICS ON statement specified before the PROC CUSUM statement enables ODS Graphics, so the cusum chart is created using ODS Graphics instead of traditional graphics.

The resulting cusum chart is shown in Figure 6.3.2.

Output 6.3.2 Two-Sided Cusum Chart with an Inset



Syntax: INSET Statement

The syntax for the INSET statement is as follows:

INSET *keyword-list* < */ options* > **;**

You can use any number of INSET statements in the CUSUM procedure. However, when ODS Graphics is enabled, at most two insets are displayed inside the plot area and at most two are displayed in the chart margins. Each INSET statement produces a separate inset and must follow an XCHART statement. The inset appears on every panel (page) produced by the last XCHART statement preceding it.

Keywords specify the statistics to be displayed in an inset; options control the inset's location and appearance. A complete description of the INSET statement syntax is given in the section "Syntax: INSET and INSET2 Statements" on page 1934 of Chapter 17, "The SHEWHART Procedure." The INSET statement options are identical in the CUSUM and SHEWHART procedures, but the available keywords are different. The options are listed in Table 17.87. The keywords available with the CUSUM procedure are listed in Table 6.9 to Table 6.12.

Table 6.9 Summary Statistics

Keyword	Description
ARL0	average run length for zero shift
ARLDELTA	average run length for shift of δ
DATA=	arbitrary values from *SAS-data-set*
N	nominal subgroup size
NMIN	minimum subgroup size
NMAX	maximum subgroup size

Table 6.10 Parameters for One-Sided (Decision Interval) Cusum Scheme

Keyword	Description
DELTA	shift to be detected as multiple of standard error
H	decision interval h as a multiple of standard error
HEADSTART	headstart value S_0 as a multiple of standard error
K	reference value k
MU0	target mean μ_0
SHIFT	shift to be detected in data units
STDDEV	estimated or specified process standard deviation

Table 6.11 Parameters for Two-Sided (V-Mask) Cusum Scheme

Keyword	Description
ALPHA	probability of Type 1 error
BETA	probability of Type 2 error
H	vertical distance between V-mask origin and upper (or lower) arm
K	slope of lower arm of V-mask
SIGMAS	probability of Type 1 error as probability that standard normally distributed variable exceeds a specified value in absolute value

You can use the keywords in Table 6.12 only when producing ODS Graphics output. The labels for the statistics use Greek letters.

Table 6.12 Keywords Specific to ODS Graphics Output

Keyword	Description
UALPHA	probability of Type 1 error
UARLDELTA	average run length for shift of δ
UBETA	probability of Type 2 error
UDELTA	shift to be detected as multiple of standard error
UMU0	target mean μ_0
USIGMA	estimated or specified process standard deviation

References

American Society for Quality Control (1983), *ASQC Glossary and Tables for Statistical Quality Control*, Milwaukee: ASQC.

Burr, I. W. (1969), “Control Charts for Measurements with Varying Sample Sizes,” *Journal of Quality Technology*, 1, 163–167.

Burr, I. W. (1976), *Statistical Quality Control Methods*, New York: Marcel Dekker.

Chiu, W. K. (1974), “The Economic Design of Cusum Charts for Controlling Normal Means,” *Applied Statistics*, 23, 420–433.

Duncan, A. J. (1974), *Quality Control and Industrial Statistics*, 4th Edition, Homewood, IL: Richard D. Irwin.

Goel, A. L. (1982), “Cumulative Sum Control Charts,” in S. Kotz, N. L. Johnson, and C. B. Read, eds., *Encyclopedia of Statistical Sciences*, volume 2, 233–241, New York: John Wiley & Sons.

Goel, A. L. and Wu, S. M. (1971), "Determination of A.R.L. and a Contour Nomogram for Cusum Charts to Control Normal Mean," *Technometrics*, 13, 221–230.

Ho, C. and Case, K. E. (1994), "Economic Design of Control Charts: A Literature Review for 1981–1991," *Journal of Quality Technology*, 26, 39–53.

Johnson, N. L. (1961), "A Simple Theoretical Approach to Cumulative Sum Control Chart," *Journal of the American Statistical Association*, 56, 835–840.

Johnson, N. L. and Leone, F. C. (1962), "Cumulative Sum Control Charts: Mathematical Principles Applied to Their Construction and Use," *Industrial Quality Control*, 18, June, 15–21; July, 29–36; August, 22–28.

Johnson, N. L. and Leone, F. C. (1974), *Statistics and Experimental Design*, volume 1, 2nd Edition, New York: John Wiley & Sons.

Kemp, K. W. (1961), "The Average Run Length of the Cumulative Sum Control Chart When a 'V' Mask Is Used," *Journal of the Royal Statistical Society, Series B*, 23, 149–153.

Lucas, J. M. (1976), "The Design and Use of V-Mask Control Schemes," *Journal of Quality Technology*, 8, 1–12.

Lucas, J. M. and Crosier, R. B. (1982), "Fast Initial Response for CUSUM Quality Control Schemes: Give Your CUSUM a Head Start," *Technometrics*, 24, 199–205.

Montgomery, D. C. (1980), "The Economic Design of Control Charts: A Review and Literature," *Journal of Quality Technology*, 12, 75–87.

Montgomery, D. C. (1996), *Introduction to Statistical Quality Control*, 3rd Edition, New York: John Wiley & Sons.

Nelson, L. S. (1984), "The Shewhart Control Chart—Tests for Special Causes," *Journal of Quality Technology*, 15, 237–239.

Nelson, L. S. (1985), "Interpreting Shewhart $\bar{X}$ Control Charts," *Journal of Quality Technology*, 17, 114–116.

Ryan, T. P. (1989), *Statistical Methods for Quality Improvement*, New York: John Wiley & Sons.

Svoboda, L. (1991), "Economic Design of Control Charts: A Review and Literature Survey (1979–1989)," in J. B. Keats and D. C. Montgomery, eds., *Statistical Process Control in Manufacturing*, New York: Marcel Dekker.

van Dobben de Bruyn, C. S. (1968), *Cumulative Sum Tests: Theory and Practice, Griffin's Statistical Monographs & Courses, No. 24*, New York: Hafner Publishing.

Wadsworth, H. M., Stephens, K. S., and Godfrey, A. B. (1986), *Modern Methods for Quality Control and Improvement*, New York: John Wiley & Sons.

Wetherill, G. B. (1977), *Sampling Inspection and Quality Control*, 2nd Edition, New York: Chapman & Hall.

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