

# SAS/ETS<sup>®</sup> 15.1 User's Guide The SYSLIN Procedure

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# SAS/ETS® 15.1 User's Guide

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# Chapter 35 The SYSLIN Procedure

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# **Overview: SYSLIN Procedure**

The SYSLIN procedure estimates parameters in an interdependent system of linear regression equations.

Ordinary least squares (OLS) estimates are biased and inconsistent when current period endogenous variables appear as regressors in other equations in the system. The errors of a set of related regression equations are often correlated, and the efficiency of the estimates can be improved by taking these correlations into account. The SYSLIN procedure provides several techniques that produce consistent and asymptotically efficient estimates for systems of regression equations.

The SYSLIN procedure provides the following estimation methods:

- ordinary least squares (OLS)
- two-stage least squares (2SLS)
- limited information maximum likelihood (LIML)
- K-class
- seemingly unrelated regressions (SUR)
- iterated seemingly unrelated regressions (ITSUR)
- three-stage least squares (3SLS)
- iterated three-stage least squares (IT3SLS)
- full information maximum likelihood (FIML)
- minimum expected loss (MELO)

Other features of the SYSLIN procedure enable you to:

• impose linear restrictions on the parameter estimates

- test linear hypotheses about the parameters
- write predicted and residual values to an output SAS data set
- write parameter estimates to an output SAS data set
- write the crossproducts matrix (SSCP) to an output SAS data set
- use raw data, correlations, covariances, or cross products as input

# **Getting Started: SYSLIN Procedure**

This section introduces the use of the SYSLIN procedure. The problem of dependent regressors is introduced using a supply and demand example. This section explains the terminology used for variables in a system of regression equations and introduces the SYSLIN procedure statements for declaring the roles the variables play. The syntax used for the different estimation methods and the output produced is shown.

# **An Example Model**

In simultaneous systems of equations, endogenous variables are determined jointly rather than sequentially. Consider the following supply and demand functions for some product:

$$Q_D = a_1 + b_1 P + c_1 Y + d_1 S + \epsilon_1 \text{(demand)}$$

$$Q_S = a_2 + b_2 P + c_2 U + \epsilon_2 \text{(supply)}$$

$$Q = Q_D = Q_S \text{(market equilibrium)}$$

The variables in this system are as follows:

$Q_D$	quantity demanded
$Q_S$	quantity supplied
Q	the observed quantity sold, which equates quantity supplied and quantity demanded in equilibrium
P	price per unit
Y	income
S	price of substitutes
U	unit cost
$\epsilon_1$	the random error term for the demand equation
$\epsilon_2$	the random error term for the supply equation

In this system, quantity demanded depends on price, income, and the price of substitutes. Consumers normally purchase more of a product when prices are lower and when income and the price of substitute goods are higher. Quantity supplied depends on price and the unit cost of production. Producers supply more when price is high and when unit cost is low. The actual price and quantity sold are determined jointly by the values that equate demand and supply.

Since price and quantity are jointly endogenous variables, both structural equations are necessary to adequately describe the observed values. A critical assumption of OLS is that the regressors are uncorrelated with the residual. When current endogenous variables appear as regressors in other equations (endogenous variables depend on each other), this assumption is violated and the OLS parameter estimates are biased and inconsistent. The bias caused by the violated assumptions is called simultaneous equation bias. Neither the demand nor supply equation can be estimated consistently by OLS.

# Variables in a System of Equations

Before explaining how to use the SYSLIN procedure, it is useful to define some terms. The variables in a system of equations can be classified as follows:

- Endogenous variables, which are also called jointly dependent or response variables, are the variables determined by the system. Endogenous variables can also appear on the right-hand side of equations.
- Exogenous variables are independent variables that do not depend on any of the endogenous variables in the system.
- Predetermined variables include both the exogenous variables and lagged endogenous variables, which are past values of endogenous variables determined at previous time periods. PROC SYSLIN does not compute lagged values; any lagged endogenous variables must be computed in a preceding DATA step.
- Instrumental variables are predetermined variables used in obtaining predicted values for the current period endogenous variables by a first-stage regression. The use of instrumental variables characterizes estimation methods such as two-stage least squares and three-stage least squares. Instrumental variables estimation methods substitute these first-stage predicted values for endogenous variables when they appear as regressors in model equations.

# **Using PROC SYSLIN**

First specify the input data set and estimation method in the PROC SYSLIN statement. If any model uses dependent regressors, and you are using an instrumental variables regression method, declare the dependent regressors with an ENDOGENOUS statement and declare the instruments with an INSTRUMENTS statement. Next, use MODEL statements to specify the structural equations of the system.

The use of different estimation methods is shown by the following examples. These examples use the simulated data set WORK.IN, which follows:

```
data in;
   label q = "Quantity"
         p = "Price"
         s = "Price of Substitutes"
         v = "Income"
         u = "Unit Cost";
   drop i e1 e2;
   p = 0; q = 0;
   do i = 1 to 60;
      y = 1 + .05*i + .15*rannor(123);
                     + .05*rannor(123) + .05*rannor(123);
      s = 4 - .001*(i-10)*(i-110) + .5*rannor(123);
      e1 = .15 * rannor(123);
      e2 = .15 * rannor(123);
      demandx = 1 + .3 * y + .35 * s + e1;
      supplyx = -1 - 1 * u + e2 - .4*e1;
      q = 1.4/2.15 * demandx + .75/2.15 * supplyx;
      p = ( -q + supplyx ) / -1.4;
      output;
   end;
run;
```

# **OLS Estimation**

PROC SYSLIN performs OLS regression if you do not specify a method of estimation in the PROC SYSLIN statement. OLS does not use instruments, so the ENDOGENOUS and INSTRUMENTS statements can be omitted.

The following statements estimate the supply and demand model shown previously:

```
proc syslin data=in;
  demand: model q = p y s;
  supply: model q = p u;
run;
```

The PROC SYSLIN output for the demand equation is shown in Figure 35.1, and the output for the supply equation is shown in Figure 35.2.

Figure 35.1 OLS Results for Demand Equation

# The SYSLIN Procedure Ordinary Least Squares Estimation

Model	DEMAND
Dependent Variable	q
Label	Quantity

Figure 35.1 continued

Analysis of Variance						
Sour	ce	DF	• • • • • • • • • • • • • • • • • • • •	Mean Square	F Value	Pr > F
Mode	el	3	9.587901	3.195967	398.31	<.0001
Error		56	0.449338	0.008024		
Corre	ected Total	59	10.03724			
	Root MSE		0.0895	8 <b>R-Squa</b> i	re 0.9552	23
	Dependent	Me	an 1.3009	5 <b>Adj R-S</b>	<b>q</b> 0.9528	33
	Coeff Var		6.8854	2		

Parameter Estimates							
Parameter Standard Variable Variable DF Estimate Error t Value Pr >  t  Label							
Intercept	1	-0.47677	0.210239	-2.27	0.0272	Intercept	
р	1	0.123326	0.105177	1.17	0.2459	Price	
у	1	0.201282	0.032403	6.21	<.0001	Income	
S	1	0.167258	0.024091	6.94	<.0001	Price of Substitutes	

Figure 35.2 OLS Results for Supply Equation

# The SYSLIN Procedure Ordinary Least Squares Estimation

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Analysis of Variance						
_						
Source	DF	Squares	Square	F Value	Pr > F	
Model	2	9.033902	4.516951	256.61	<.0001	
Error	57	1.003337	0.017602			
Corrected Total	59	10.03724				

Root MSE	0.13267	R-Square	0.90004
Dependent Mean	1.30095	Adj R-Sq	0.89653
Coeff Var	10.19821		

Parameter Estimates							
Variable	Variable Label						
Intercept	1	-0.30389	0.471397	-0.64	0.5217	Intercept	
р	1	1.218743	0.053914	22.61	<.0001	Price	
u	1	-1.07757	0.234150	-4.60	<.0001	Unit Cost	

For each MODEL statement, the output first shows the model label and dependent variable name and label. This is followed by an analysis-of-variance table for the model, which shows the model, error, and total mean squares, and an F test for the no-regression hypothesis. Next, the procedure prints the root mean squared error, dependent variable mean and coefficient of variation, and the  $R^2$  and adjusted  $R^2$  statistics.

Finally, the table of parameter estimates shows the estimated regression coefficients, standard errors, and t tests. You would expect the price coefficient in a demand equation to be negative. However, note that the OLS estimate of the price coefficient P in the demand equation (0.1233) has a positive sign. This could be caused by simultaneous equation bias.

# **Two-Stage Least Squares Estimation**

In the supply and demand model, P is an endogenous variable, and consequently the OLS estimates are biased. The following example estimates this model using two-stage least squares:

```
proc syslin data=in 2sls;
   endogenous p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The 2SLS option in the PROC SYSLIN statement specifies the two-stage least squares method. The ENDOGENOUS statement specifies that P is an endogenous regressor for which first-stage predicted values are substituted. You need to declare an endogenous variable in the ENDOGENOUS statement only if it is used as a regressor; thus although Q is endogenous in this model, it is not necessary to list it in the ENDOGENOUS statement.

Usually, all predetermined variables that appear in the system are used as instruments. The INSTRUMENTS statement specifies that the exogenous variables Y, U, and S are used as instruments for the first-stage regression to predict P.

The 2SLS results are shown in Figure 35.3 and Figure 35.4. The first-stage regressions are not shown. To see the first-stage regression results, use the FIRST option in the PROC SYSLIN statement.

Figure 35.3 2SLS Results for Demand Equation

# The SYSLIN Procedure Two-Stage Least Squares Estimation

Model	DEMAND
Dependent Variable	q
Label	Quantity

Analysis of Variance										
	Sum of Mean									
Source	DF	Squares	Square	F Value	Pr > F					
Model	3	9.670892	3.223631	115.58	<.0001					
Error	56	1.561956	0.027892							
Corrected Total	59	10.03724								

Root MSE	0.16701	R-Square	0.86095
Dependent Mean	1.30095	Adj R-Sq	0.85350
Coeff Var	12.83744		
Coeii vai	12.03744		

Figure 35.3 continued

	Parameter Estimates											
Parameter Standard Variable Variable DF Estimate Error t Value Pr >  t  Label												
Intercept	1	1.901048	1.171231	1.62	0.1102	Intercept						
p	1	-1.11519	0.607395	-1.84	0.0717	Price						
у	1	0.419546	0.117955	3.56	0.0008	Income						
s	1	0.331475	0.088472	3.75	0.0004	Price of Substitutes						

Figure 35.4 2SLS Results for Supply Equation

# The SYSLIN Procedure Two-Stage Least Squares Estimation

Mo	odel		SUPPLY							
De	pen	dent Varia	ble	q						
La	bel		Quar	ntity						
	Ar	alysis of \	/ariance							
Source	Sum of Mean									
Model	2	9.646109	4.823054	253.96	<.0001					
Error	57	1.082503	0.018991							
Corrected Total	59	10.03724								
Root MSE		0.1378	1 R-Squa	are 0.899	10					
Dependent	Mea	n 1.3009	5 <b>Adj R-S</b>	<b>5q</b> 0.895	56					
Coeff Var		10.5929	1		_					
	Da	rameter E	ctimates							

	Parameter Estimates												
Variable	Variable Label												
Intercept	1	-0.51878	0.490999	-1.06	0.2952	Intercept							
р	1	1.333080	0.059271	22.49	<.0001	Price							
u	1	-1.14623	0.243491	-4.71	<.0001	Unit Cost							

The 2SLS output is similar in form to the OLS output. However, the 2SLS results are based on predicted values for the endogenous regressors from the first stage instrumental regressions. This makes the analysis-of-variance table and the  $R^2$  statistics difficult to interpret. For more information, see the sections "ANOVA Table for Instrumental Variables Methods" on page 2633 and "The R-Square Statistics" on page 2633.

Note that, unlike the OLS results, the 2SLS estimate for the P coefficient in the demand equation (-1.115) is negative.

# LIML, K-Class, and MELO Estimation

To obtain limited information maximum likelihood, general K-class, or minimum expected loss estimates, use the ENDOGENOUS, INSTRUMENTS, and MODEL statements as in the 2SLS case but specify the

LIML, K=, or MELO option instead of 2SLS in the PROC SYSLIN statement. The following statements show this for K-class estimation:

```
proc syslin data=in k=.5;
  endogenous p;
  instruments y u s;
  demand: model q = p y s;
  supply: model q = p u;
run:
```

For more information about these estimation methods, see the section "Estimation Methods" on page 2630 and consult econometrics textbooks.

# SUR, 3SLS, and FIML Estimation

In a multivariate regression model, the errors in different equations might be correlated. In this case, the efficiency of the estimation might be improved by taking these cross-equation correlations into account.

# **Seemingly Unrelated Regression**

Seemingly unrelated regression (SUR), also called joint generalized least squares (JGLS) or Zellner estimation, is a generalization of OLS for multi-equation systems. Like OLS, the SUR method assumes that all the regressors are independent variables, but SUR uses the correlations among the errors in different equations to improve the regression estimates. The SUR method requires an initial OLS regression to compute residuals. The OLS residuals are used to estimate the cross-equation covariance matrix.

The SUR option in the PROC SYSLIN statement specifies seemingly unrelated regression, as shown in the following statements:

```
proc syslin data=in sur;
  demand: model q = p y s;
  supply: model q = p u;
run;
```

INSTRUMENTS and ENDOGENOUS statements are not needed for SUR, because the SUR method assumes there are no endogenous regressors. For SUR to be effective, the models must use different regressors. SUR produces the same results as OLS unless the model contains at least one regressor not used in the other equations.

# **Three-Stage Least Squares**

The three-stage least squares method generalizes the two-stage least squares method to take into account the correlations between equations in the same way that SUR generalizes OLS. Three-stage least squares requires three steps: first-stage regressions to get predicted values for the endogenous regressors; a two-stage least squares step to get residuals to estimate the cross-equation correlation matrix; and the final 3SLS estimation step.

The 3SLS option in the PROC SYSLIN statement specifies the three-stage least squares method, as shown in the following statements:

```
proc syslin data=in 3sls;
   endogenous p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The 3SLS output begins with a two-stage least squares regression to estimate the cross-model correlation matrix. This output is the same as the 2SLS results shown in Figure 35.3 and Figure 35.4, and is not repeated here. The next part of the 3SLS output prints the cross-model correlation matrix computed from the 2SLS residuals. This output is shown in Figure 35.5 and includes the cross-model covariances, correlations, the inverse of the correlation matrix, and the inverse covariance matrix.

Figure 35.5 Estimated Cross-Model Covariances Used for 3SLS Estimates

# The SYSLIN Procedure Three-Stage Least Squares Estimation

Cross N	Model Cova	ariance
	DEMAND	SUPPLY
DEMAND	0.027892	011283
SUPPLY	011283	0.018991
Cross M	lodel Corr	elation
	DEMAND	SUPPLY
DEMAND	1.00000	-0.49022
SUPPLY	-0.49022	1.00000
0.000	Model Inv	erse
C	Correlation	
	DEMAND	SUPPLY
DEMAND	1.31634	0.64530
SUPPLY	0.64530	1.31634
0.000	Model Inv	erse
C	Covariance	
	DEMAND	SUPPLY
DEMAND	47.1941	28.0379
SUPPLY	28.0379	69.3130

The final 3SLS estimates are shown in Figure 35.6.

Figure 35.6 Three-Stage Least Squares Results

System Weighted MSE	0.57	11			
Degrees of freedom	1	13			
System Weighted R-Sq	<b>uare</b> 0.96	0.9627			
		_			
Model	DEMAND				
<b>Dependent Variable</b> q					
Label	Quantity				

Figure 35.6 continued

	Parameter Estimates									
Variable	DF	Parameter Estimate			t Valu	e Pr>	t	Vari Lab		
Intercept	1	1.980269	1.16	9176	1.6	9 0.09	59	Inter	cept	
р	1	-1.17654	0.60	)5015	-1.94	4 0.05	68	Price	е	
у	1	0.404117	0.11	7179	3.4	5 0.00	11	Inco	me	
S	1	0.359204	0.08	35077	4.2	2 <.00	01	Price	e of Substitu	utes
Model SUPPLY										
		IVIO	iei		3	OPPL	. Y			
		Dep	ende	nt Vai	iable		q			
		Lab	el			Quanti	ty			
			Para	neter	Estima	ites				
		Parar	neter	Stan	dard				Variable	
Vari	able	DF Est	imate	E	rror t	Value	Pr	>  t	Label	
Inter	cept	: 1 -0.5	1878	0.49	0999	-1.06	0.2	2952	Intercept	
р		1 1.33	3080	0.05	9271	22.49	<.(	0001	Price	
u		1 -1.1	4623	0.24	3491	-4.71	<.(	0001	Unit Cost	

This output first prints the system weighted mean squared error and system weighted  $R^2$  statistics. The system weighted MSE and system weighted  $R^2$  measure the fit of the joint model obtained by stacking all the models together and performing a single regression with the stacked observations weighted by the inverse of the model error variances. For more information, see the section "The R-Square Statistics" on page 2633.

Next, the table of 3SLS parameter estimates for each model is printed. This output has the same form as for the other estimation methods.

Note that, in some cases, the 3SLS and 2SLS results can be the same. Such a case could arise because of the same principle that causes OLS and SUR results to be identical, unless an equation includes a regressor not used in the other equations of the system. However, the application of this principle is more complex when instrumental variables are used. When all the exogenous variables are used as instruments, linear combinations of all the exogenous variables appear in the third-stage regressions through substitution of first-stage predicted values.

In this example, 3SLS produces different (and, it is hoped, more efficient) estimates for the demand equation. However, the 3SLS and 2SLS results for the supply equation are the same. This is because the supply equation has one endogenous regressor and one exogenous regressor not used in other equations. In contrast, the demand equation has fewer endogenous regressors than exogenous regressors not used in other equations in the system.

# **Full Information Maximum Likelihood**

The FIML option in the PROC SYSLIN statement specifies the full information maximum likelihood method, as shown in the following statements:

```
proc syslin data=in fiml;
  endogenous p q;
  instruments y u s;
  demand: model q = p y s;
  supply: model q = p u;
run;
```

The FIML results are shown in Figure 35.7.

Figure 35.7 FIML Results

# The SYSLIN Procedure Full-Information Maximum Likelihood Estimation

Model DEMAND

Dependent Variable q

Label Quantity

	Parameter Estimates											
Parameter Standard Variable Variable DF Estimate Error t Value Pr >  t  Label												
Intercept	1	1.988538	1.233632	1.61	0.1126	Intercept						
p	1	-1.18148	0.652278	-1.81	0.0755	Price						
у	1	0.402312	0.107270	3.75	0.0004	Income						
s	1	0.361345	0.103817	3.48	0.0010	Price of Substitutes						

Model	SUPPLY
<b>Dependent Variable</b>	q
Label	Quantity

	Parameter Estimates													
Parameter Standard Varial Variable DF Estimate Error t Value Pr >  t  Label														
		-0.52443												
р	1	1.336083	0.057939	23.06	<.0001	Price								
u	1	-1.14804	0.237793	-4.83	<.0001	Unit Cost								

# **Computing Reduced Form Estimates**

A system of structural equations with endogenous regressors can be represented as functions of only the predetermined variables. For this to be possible, there must be as many equations as endogenous variables. If there are more endogenous variables than regression models, you can use IDENTITY statements to complete the system. For more information, see the section "Reduced Form Estimates" on page 2635.

The REDUCED option in the PROC SYSLIN statement prints reduced form estimates. The following statements show this by using the 3SLS estimates of the structural parameters:

```
proc syslin data=in 3sls reduced;
  endogenous p;
  instruments y u s;
  demand: model q = p y s;
  supply: model q = p u;
run;
```

The first four pages of this output were as shown previously and are not repeated here. (See Figure 35.3, Figure 35.4, Figure 35.5, and Figure 35.6.) The final page of the output from this example contains the reduced form coefficients from the 3SLS structural estimates, as shown in Figure 35.8.

Figure 35.8 Reduced Form 3SLS Results

# The SYSLIN Procedure Three-Stage Least Squares Estimation

			En	doge	nous	Va	riables	5			
							р	q			
			DEI	MANE	1.1	7654	43	1			
			SUI	PPLY	-1.:	3330	08	1			
			Ex	ogen	ous	Vari	ables			_	_
			Interc	ept		У		s			u
DE	MA	ND	1.980	269 C	.404	117	0.359	204			0
SU	PP	LY	-0.51	878		0		0	-1.	146	523
			In	verse V	End ariab		nous				
				DEN	/AN	D S	UPPL	Υ			
			р	0.39	9846	6 -C	).3984	7			
			q	0.53	3118	7 0.	46881	3			
				Red	uced	For	rm				
		Inte	rcept		у		s			u	
	p	0.99	95788	0.161	1027	0.1	43131	0.4	567	35	
	q	0.80	08682	0.214	1662	0.19	90804	-0.5	537	37	

# **Restricting Parameter Estimates**

You can impose restrictions on the parameter estimates with RESTRICT and SRESTRICT statements. The RESTRICT statement imposes linear restrictions on parameters in the equation specified by the preceding MODEL statement. The SRESTRICT statement imposes linear restrictions that relate parameters in different models.

To impose restrictions involving parameters in different equations, use the SRESTRICT statement. Specify the parameters in the linear hypothesis as model-label.regressor-name. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.)

Tests for the significance of the restrictions are printed when RESTRICT or SRESTRICT statements are used. You can label RESTRICT and SRESTRICT statements to identify the restrictions in the output.

The RESTRICT statement in the following example restricts the price coefficient in the demand equation to equal 0.015. The SRESTRICT statement restricts the estimate of the income coefficient in the demand equation to be 0.01 times the estimate of the unit cost coefficient in the supply equation.

```
proc syslin data=in 3sls;
  endogenous p;
  instruments y u s;
  demand: model q = p y s;
  peq015: restrict p = .015;
  supply: model q = p u;
  yeq01u: srestrict demand.y = .01 * supply.u;
run;
```

The restricted estimation results are shown in Figure 35.9.

Figure 35.9 Restricted Estimates

# The SYSLIN Procedure Three-Stage Least Squares Estimation

Model	DEMAND
<b>Dependent Variable</b>	q
Label	Quantity

Parameter Estimates							
Variable		Parameter Estimate		t Value	Pr >  t	Variable Label	
Intercept	1	-0.46584	0.053307	-8.74	<.0001	Intercept	
р	1	0.015000	0			Price	
у	1	-0.00679	0.002357	-2.88	0.0056	Income	
s	1	0.325589	0.009872	32.98	<.0001	Price of Substitutes	
RESTRICT	-1	50.59353	7.464988	6.78	<.0001	PEQ015	

Model	SUPPLY
<b>Dependent Variable</b>	q
Label	Quantity

Parameter Estimates									
Variable	Parameter Standard able DF Estimate Error t Value Pr >  t								
Intercept	1	-1.31894	0.477633	-2.76	0.0077	Intercept			
р	1	1.291718	0.059101	21.86	<.0001	Price			
u	1	-0.67887	0.235679	-2.88	0.0056	Unit Cost			

		Parameter Estimates							
Variable D		Parameter Estimate		t Value	Pr >  t	Variable Label			
RESTRICT -	-1	342.3605	38.12094	8.98	<.0001	YEQ01U			

The standard error for P in the demand equation is 0, since the value of the P coefficient was specified by the RESTRICT statement and not estimated from the data. The "Parameter Estimates" table for the demand equation contains an additional row for the restriction specified by the RESTRICT statement. The parameter estimate for the restriction is the value of the Lagrange multiplier used to impose the restriction. The restriction is highly significant (t = 6.777), which means that the data are not consistent with the restriction, and the model does not fit as well with the restriction imposed. For more information, see the section "RESTRICT Statement" on page 2625.

Following the "Parameter Estimates" table for the supply equation, the results for the cross model restrictions are printed. This shows that the restriction specified by the SRESTRICT statement is not consistent with the data (t = 8.98). For more information, see the section "SRESTRICT Statement" on page 2626.

# **Testing Parameters**

You can test linear hypotheses about the model parameters with TEST and STEST statements. The TEST statement tests hypotheses about parameters in the equation specified by the preceding MODEL statement. The STEST statement tests hypotheses that relate parameters in different models.

For example, the following statements test the hypothesis that the price coefficient in the demand equation is equal to 0.015:

```
proc syslin data=in 3sls;
  endogenous p;
  instruments y u s;
  demand: model q = p y s;
  test_1: test p = .015;
  supply: model q = p u;
run;
```

The TEST statement results are shown in Figure 35.10. This reports an F test for the hypothesis specified by the TEST statement. In this case, the F statistic is 6.79 (3.879/.571) with 1 and 113 degrees of freedom. The p-value for this F statistic is 0.0104, which indicates that the hypothesis tested is almost but not quite rejected at the 0.01 level. For more information, see the section "TEST Statement" on page 2628.

Figure 35.10 TEST Statement Results

# The SYSLIN Procedure Three-Stage Least Squares Estimation

System Weighted MSE		0.5711
Degrees of freedom	113	
System Weighted R-So	0.9627	
Model	DEM	AND
Dependent Variable		q

Parameter Estimates								
Parameter Standard Variable Variable DF Estimate Error t Value Pr >  t  Label								
Intercept	1	1.980269	1.169176	1.69	0.0959	Intercept		
р	1	-1.17654	0.605015	-1.94	0.0568	Price		
у	1	0.404117	0.117179	3.45	0.0011	Income		
s	1	0.359204	0.085077	4.22	<.0001	Price of Substitutes		

Test Results						
Num DF Den DF F Value Pr > F Label						
1	113	6.79	0.0104	TEST_1		

To test hypotheses that involve parameters in different equations, use the STEST statement. Specify the parameters in the linear hypothesis as *model-label.regressor-name*. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.)

For example, the following statements test the hypothesis that the income coefficient in the demand equation is 0.01 times the unit cost coefficient in the supply equation:

```
proc syslin data=in 3sls;
  endogenous p;
  instruments y u s;
  demand: model q = p y s;
  supply: model q = p u;
  stest1: stest demand.y = .01 * supply.u;
run;
```

The STEST statement results are shown in Figure 35.11. The form and interpretation of the STEST statement results are like the TEST statement results. In this case, the *F* test produces a *p*-value less than 0.0001 and strongly rejects the hypothesis tested. For more information, see the section "STEST Statement" on page 2627.

Figure 35.11 STEST Statement Results

# The SYSLIN Procedure Three-Stage Least Squares Estimation

System Weighted MSE		0.5711			
Degrees of freedom	113				
System Weighted R-Square 0.962					
Model	DEM	AND			
Model Dependent Variable	DEM.	AND q			

Parameter Estimates							
Parameter Standard					Variable		
Variable	DF	Estimate	Error	t Value	Pr >  t	Label	
Intercept	1	1.980269	1.169176	1.69	0.0959	Intercept	
p	1	-1.17654	0.605015	-1.94	0.0568	Price	
у	1	0.404117	0.117179	3.45	0.0011	Income	
s	1	0.359204	0.085077	4.22	<.0001	Price of Substitutes	

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Parameter Estimates								
Variable	Parameter Standard ariable DF Estimate Error t Value Pr >  t							
Intercept	1	-0.51878	0.490999	-1.06	0.2952	Intercept		
р	1	1.333080	0.059271	22.49	<.0001	Price		
u	1	-1.14623	0.243491	-4.71	<.0001	Unit Cost		

Figure 35.11 continued

Test Results					
Num DF	Den DF	F Value	Pr > F	Label	
1	113	22.46	0.0001	STEST1	

You can combine TEST and STEST statements with RESTRICT and SRESTRICT statements to perform hypothesis tests for restricted models. Of course, the validity of the TEST and STEST statement results depends on the correctness of any restrictions you impose on the estimates.

# Saving Residuals and Predicted Values

You can store predicted values and residuals from the estimated models in a SAS data set. Specify the OUT= option in the PROC SYSLIN statement and use the OUTPUT statement to specify names for new variables to contain the predicted and residual values.

For example, the following statements store the predicted quantity from the supply and demand equations in the data set PRED:

```
proc syslin data=in out=pred 3sls;
   endogenous p;
   instruments y u s;
   demand: model q = p y s;
   output predicted=q_demand;
   supply: model q = p u;
   output predicted=q_supply;
run;
```

# **Plotting Residuals**

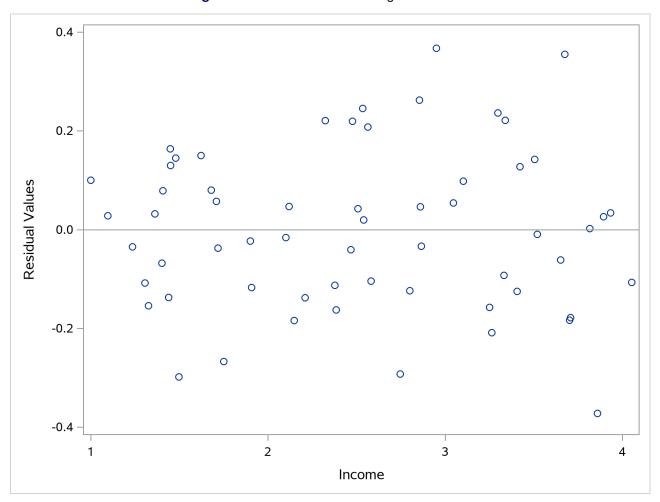
You can plot the residuals against the regressors by using the PROC SGPLOT. For example, the following statements plot the 2SLS residuals for the demand model against price, income, and price of substitutes:

```
proc syslin data=in 2sls out=out;
   endogenous p;
   instruments y u s;
   demand: model q = p y s;
   output residual=residual_q;
run;
proc sgplot data=out;
   scatter x=p y=residual_q;
   refline 0 / axis=y;
run;
proc sgplot data=out;
   scatter x=y y=residual_q;
   refline 0 / axis=y;
run;
```

```
proc sgplot data=out;
    scatter x=s y=residual_q;
    refline 0 / axis=y;
run;
```

The plot for income is shown in Figure 35.12. The other plots are not shown.

Figure 35.12 Plot of Residuals against Income



# **Syntax: SYSLIN Procedure**

The SYSLIN procedure uses the following statements:

```
PROC SYSLIN options;
BY variables;
ENDOGENOUS variables;
IDENTITY identities;
INSTRUMENTS variables;
MODEL response = regressors / options;
OUTPUT PREDICTED=variable RESIDUAL=variable;
RESTRICT restrictions;
SRESTRICT restrictions;
STEST equations;
TEST equations;
VAR variables;
WEIGHT variable;
```

# **Functional Summary**

The SYSLIN procedure statements and options are summarized in Table 35.1.

**Table 35.1** Functional Summary

Description	Statement	Option
Data Set Options		
Specify the input data set	PROC SYSLIN	DATA=
Specify the output data set	PROC SYSLIN	OUT=
Write parameter estimates to an output data set	PROC SYSLIN	OUTEST=
Write covariances to the OUTEST= data set	PROC SYSLIN	OUTCOV
		OUTCOV3
Write the SSCP matrix to an output data set	PROC SYSLIN	OUTSSCP=
<b>Estimation Method Options</b>		
Specify full information maximum likelihood estimation	PROC SYSLIN	FIML
Specify iterative SUR estimation	PROC SYSLIN	ITSUR
Specify iterative 3SLS estimation	PROC SYSLIN	IT3SLS
Specify K-class estimation	PROC SYSLIN	K=
Specify limited information maximum	PROC SYSLIN	LIML
likelihood estimation		
Specify minimum expected loss estimation	PROC SYSLIN	MELO
Specify ordinary least squares estimation	PROC SYSLIN	OLS
Specify seemingly unrelated estimation	PROC SYSLIN	SUR
Specify two-stage least squares estimation	PROC SYSLIN	2SLS
Specify three-stage least squares estimation	PROC SYSLIN	3SLS

Table 35.1 continued

Table 00.1 Continued				
Description	Statement	Option		
Specify Fuller's modification to LIML	PROC SYSLIN	ALPHA=		
Specify convergence criterion	PROC SYSLIN	CONVERGE=		
Specify maximum number of iterations	PROC SYSLIN	MAXIT=		
Use diagonal of S instead of S	PROC SYSLIN	SDIAG		
Exclude RESTRICT statements in final stage	PROC SYSLIN	NOINCLUDE		
Specify criterion for testing for singularity	PROC SYSLIN	SINGULAR=		
Specify denominator for variance estimates	PROC SYSLIN	VARDEF=		
<b>Printing Control Options</b>				
Print all results	PROC SYSLIN	ALL		
Print first-stage regression statistics	PROC SYSLIN	FIRST		
Print estimates and SSE at each iteration	PROC SYSLIN	ITPRINT		
Print the reduced form estimates	PROC SYSLIN	REDUCED		
Print descriptive statistics	PROC SYSLIN	SIMPLE		
Print uncorrected SSCP matrix	PROC SYSLIN	USSCP		
Print correlations of the parameter estimates	MODEL	CORRB		
Print covariances of the parameter estimates	MODEL	COVB		
print Durbin-Watson statistics	MODEL	DW		
Print Basmann's test	MODEL	OVERID		
Plot residual values against regressors	MODEL	PLOT		
Print standardized parameter estimates	MODEL	STB		
Print unrestricted parameter estimates	MODEL	UNREST		
Print the model crossproducts matrix	MODEL	XPX		
Print the inverse of the crossproducts matrix	MODEL	I		
Suppress printed output	MODEL	NOPRINT		
Suppress all printed output	PROC SYSLIN	NOPRINT		
<b>Model Specification</b>				
Specify structural equations	MODEL			
Suppress the intercept parameter	MODEL	NOINT		
Specify linear relationship among variables	IDENTITY			
Perform weighted regression	WEIGHT			
<b>Tests and Restrictions on Parameters</b>				
Place restrictions on parameter estimates	RESTRICT			
Place restrictions on parameter estimates	SRESTRICT			
Test linear hypothesis	STEST			
Test linear hypothesis	TEST			
Other Statements				
Specify BY-group processing	BY			
Specify the endogenous variables	ENDOGENOUS			
Specify instrumental variables	S INSTRUMENTS			

Table 35.1 continued

Description	Statement	Option
Write predicted and residual values to a data	OUTPUT	
set Name variable for predicted values	OUTPUT	PREDICTED=
Name variable for residual values	OUTPUT	RESIDUAL=
Include additional variables in $X'X$ matrix	VAR	

# **PROC SYSLIN Statement**

# PROC SYSLIN options;

The following options can be used with the PROC SYSLIN statement.

# **Data Set Options**

#### DATA=SAS-data-set

specifies the input data set. If the DATA= option is omitted, the most recently created SAS data set is used. In addition to ordinary SAS data sets, PROC SYSLIN can analyze data sets of TYPE=CORR, TYPE=COV, TYPE=UCORR, TYPE=UCOV, and TYPE=SSCP. For more information, see the section "Special TYPE= Input Data Sets" on page 2630.

# OUT=SAS-data-set

specifies an output SAS data set for residuals and predicted values. The OUT= option is used in conjunction with the OUTPUT statement. For more information, see the section "OUT= Data Set" on page 2637.

## **OUTEST=**SAS-data-set

writes the parameter estimates to an output data set. For more information, see the section "OUTEST= Data Set" on page 2637.

#### **OUTCOV**

#### COVOUT

writes the covariance matrix of the parameter estimates to the OUTEST= data set in addition to the parameter estimates.

#### **OUTCOV3**

#### **COV3OUT**

writes covariance matrices for each model in a system to the OUTEST= data set when the 3SLS, SUR, or FIML option is used.

## **OUTSSCP=**SAS-data-set

writes the sum-of-squares-and-crossproducts matrix to an output data set. For more information, see the section "OUTSSCP= Data Set" on page 2638.

# **Estimation Method Options**

## 2SLS

specifies the two-stage least squares estimation method.

## 3SLS

specifies the three-stage least squares estimation method.

#### ALPHA=value

specifies Fuller's modification to the LIML estimation method. For more information, see the section "Fuller's Modification to LIML" on page 2636.

#### CONVERGE=value

specifies the convergence criterion for the iterative estimation methods IT3SLS, ITSUR, and FIML. The default is CONVERGE=0.0001.

#### **FIML**

specifies the full information maximum likelihood estimation method.

#### **ITSUR**

specifies the iterative seemingly unrelated estimation method.

#### **IT3SLS**

specifies the iterative three-stage least squares estimation method.

#### K=value

specifies the K-class estimation method.

# LIML

specifies the limited information maximum likelihood estimation method.

#### MAXITER=n

specifies the maximum number of iterations allowed for the IT3SLS, ITSUR, and FIML estimation methods. The MAXITER= option can be abbreviated as MAXIT=. The default is MAXITER=30.

# **MELO**

specifies the minimum expected loss estimation method.

## **NOINCLUDE**

excludes the RESTRICT statements from the final stage for the 3SLS, IT3SLS, SUR, and ITSUR estimation methods.

#### **OLS**

specifies the ordinary least squares estimation method. This is the default.

#### **SDIAG**

uses the diagonal of **S** instead of **S** to do the estimation, where **S** is the covariance matrix of equation errors. For more information, see the section "Uncorrelated Errors across Equations" on page 2636.

## SINGULAR=value

specifies a criterion for testing singularity of the crossproducts matrix. This is a tuning parameter used to make PROC SYSLIN more or less sensitive to singularities. The value must be between 0 and 1. The default is SINGULAR=1E-8.

#### **SUR**

specifies the seemingly unrelated estimation method.

# **Printing Control Options**

#### ALL

specifies the CORRB, COVB, DW, I, OVERID, PLOT, STB, and XPX options for every MODEL statement.

#### **FIRST**

prints first-stage regression statistics for the endogenous variables regressed on the instruments. This output includes sums of squares, estimates, variances, and standard deviations.

#### **ITPRINT**

prints parameter estimates, system-weighted residual sum of squares, and  $R^2$  at each iteration for the IT3SLS and ITSUR estimation methods. For the FIML method, the ITPRINT option prints parameter estimates, negative of log-likelihood function, and norm of gradient vector at each iteration.

## **NOPRINT**

suppresses all printed output. Specifying NOPRINT in the PROC SYSLIN statement is equivalent to specifying NOPRINT in every MODEL statement.

#### **REDUCED**

prints the reduced form estimates. If the REDUCED option is specified, you should specify any IDENTITY statements needed to make the system square. For more information, see the section "Reduced Form Estimates" on page 2635.

# **SIMPLE**

prints descriptive statistics for the dependent variables. The statistics printed include the sum, mean, uncorrected sum of squares, variance, and standard deviation.

# **USSCP**

prints the uncorrected sum-of-squares-and-crossproducts matrix.

# **USSCP2**

prints the uncorrected sum-of-squares-and-crossproducts matrix for all variables used in the analysis, including predicted values of variables generated by the procedure.

# VARDEF=DF | N | WEIGHT | WGT

specifies the denominator to use in calculating cross-equation error covariances and parameter standard errors and covariances. The default is VARDEF=DF, which corrects for model degrees of freedom. VARDEF=N specifies no degrees-of-freedom correction. VARDEF=WEIGHT specifies the sum of the observation weights. VARDEF=WGT specifies the sum of the observation weights minus the model degrees of freedom. For more information, see the section "Computation of Standard Errors" on page 2635.

# **BY Statement**

BY variables:

A BY statement can be used with PROC SYSLIN to obtain separate analyses on observations in groups defined by the BY variables.

# **ENDOGENOUS Statement**

# **ENDOGENOUS** variables;

The ENDOGENOUS statement declares the jointly dependent variables that are projected in the first-stage regression through the instrument variables. The ENDOGENOUS statement is not needed for the SUR, ITSUR, or OLS estimation methods. The default ENDOGENOUS list consists of all the dependent variables in the MODEL and IDENTITY statements that do not appear in the INSTRUMENTS statement.

# **IDENTITY Statement**

# **IDENTITY** equation;

The IDENTITY statement specifies linear relationships among variables to write to the OUTEST= data set. It provides extra information in the OUTEST= data set but does not create or compute variables. The OUTEST= data set can be processed by the SIMLIN procedure in a later step.

The IDENTITY statement is also used to compute reduced form coefficients when the REDUCED option in the PROC SYSLIN statement is specified. For more information, see the section "Reduced Form Estimates" on page 2635.

The *equation* given by the IDENTITY statement has the same form as equations in the MODEL statement. A label can be specified for an IDENTITY statement as follows:

label: IDENTITY ...;

# **INSTRUMENTS Statement**

## **INSTRUMENTS** variables;

The INSTRUMENTS statement declares the variables used in obtaining first-stage predicted values. All the instruments specified are used in each first-stage regression. The INSTRUMENTS statement is required for the 2SLS, 3SLS, IT3SLS, LIML, MELO, and K-class estimation methods. The INSTRUMENTS statement is not needed for the SUR, ITSUR, OLS, or FIML estimation methods.

# **MODEL Statement**

**MODEL** response = regressors / options;

The MODEL statement regresses the response variable on the left side of the equal sign against the regressors listed on the right side.

Models can be given labels. Model labels are used in the printed output to identify the results for different models. Model labels are also used in SRESTRICT and STEST statements to refer to parameters in different

models. If no label is specified, the response variable name is used as the label for the model. The model label is specified as follows:

```
label: MODEL ...;
```

The following options can be used in the MODEL statement after a slash (/):

#### ALL

specifies the CORRB, COVB, DW, I, OVERID, PLOT, STB, and XPX options.

#### ALPHA=value

specifies the  $\alpha$  parameter for Fuller's modification to the LIML estimation method. For more information, see the section "Fuller's Modification to LIML" on page 2636.

#### **CORRB**

prints the matrix of estimated correlations between the parameter estimates.

## **COVB**

prints the matrix of estimated covariances between the parameter estimates.

#### DW

prints Durbin-Watson statistics and autocorrelation coefficients for the residuals. If there are missing values, d' is calculated according to Savin and White (1978). Use the DW option only if the data set to be analyzed is an ordinary SAS data set with time series observations sorted in time order. The Durbin-Watson test is not valid for models with lagged dependent regressors.

ı

prints the inverse of the crossproducts matrix for the model,  $(\mathbf{X}'\mathbf{X})^{-1}$ . If restrictions are specified, the crossproducts matrix printed is adjusted for the restrictions. For more information, see the section "Computational Details" on page 2634.

# K=value

specifies K-class estimation.

#### **NOINT**

suppresses the intercept parameter from the model.

#### **NOPRINT**

suppresses the normal printed output.

## **OVERID**

prints Basmann's (1960) test for over identifying restrictions. For more information, see the section "Overidentification Restrictions" on page 2636.

## **PLOT**

plots residual values against regressors. A plot of the residuals for each regressor is printed.

## STB

prints standardized parameter estimates. Sometimes known as a standard partial regression coefficient, a standardized parameter estimate is a parameter estimate multiplied by the standard deviation of the associated regressor and divided by the standard deviation of the response variable.

#### UNREST

prints parameter estimates computed before restrictions are applied. The UNREST option is valid only if a RESTRICT statement is specified.

#### **XPX**

prints the model crossproducts matrix, X'X. For more information, see the section "Computational Details" on page 2634.

# **OUTPUT Statement**

```
OUTPUT < PREDICTED=variable > < RESIDUAL=variable > ;
```

The OUTPUT statement writes predicted values and residuals from the preceding model to the data set specified by the OUT= option in the PROC SYSLIN statement. An OUTPUT statement must come after the MODEL statement to which it applies. The OUT= option must be specified in the PROC SYSLIN statement.

The following options can be specified in the OUTPUT statement:

#### PREDICTED=variable

names a new variable to contain the predicted values for the response variable. The PREDICTED= option can be abbreviated as PREDICT=, PRED=, or P=.

# RESIDUAL=variable

names a new variable to contain the residual values for the response variable. The RESIDUAL= option can be abbreviated as RESID= or R=.

For example, the following statements create an output data set named B. In addition to the variables in the input data set, the data set B contains the variable YHAT, with values that are predicted values of the response variable Y, and the YRESID, with values that are the residual values of Y.

```
proc syslin data=a out=b;
  model y = x1 x2;
  output p=yhat r=yresid;
run;
```

For example, the following statements create an output data set named PRED. In addition to the variables in the input data set, the data set PRED contains the variables Q\_DEMAND and Q\_SUPPLY, with values that are predicted values of the response variable Q for the demand and supply equations, respectively, and the variables R\_DEMAND and R\_SUPPLY, with values that are the residual values of the demand and supply equations, respectively.

```
proc syslin data=in out=pred;
  demand: model q = p y s;
  output p=q_demand r=r_demand;
  supply: model q = p u;
  output p=q_supply r=r_supply;
run;
```

For more information, see the section "OUT= Data Set" on page 2637.

# **RESTRICT Statement**

```
RESTRICT equation , . . . , equation ;
```

The RESTRICT statement places restrictions on the parameter estimates for the preceding MODEL statement. Any number of RESTRICT statements can follow a MODEL statement. Each restriction is written as a linear equation. If more than one restriction is specified in a single RESTRICT statement, the restrictions are separated by commas.

Parameters are referred to by the name of the corresponding regressor variable. Each name used in the equation must be a regressor in the preceding MODEL statement. The keyword INTERCEPT is used to refer to the intercept parameter in the model.

RESTRICT statements can be given labels. The labels are used in the printed output to distinguish results for different restrictions. Labels are specified as follows:

```
label: RESTRICT ...;
```

The following is an example of the use of the RESTRICT statement, in which the coefficients of the regressors X1 and X2 are required to sum to 1:

```
proc syslin data=a;
  model y = x1 x2;
  restrict x1 + x2 = 1;
run;
```

Variable names can be multiplied by constants. When no equal sign appears, the linear combination is set equal to 0. Note that the parameters associated with the variables are restricted, not the variables themselves. Here are some examples of valid RESTRICT statements:

```
restrict x1 + x2 = 1;
restrict x1 + x2 - 1;
restrict 2 * x1 = x2 + x3 , intercept + x4 = 0;
restrict x1 = x2 = x3 = 1;
restrict 2 * x1 - x2;
```

Restricted parameter estimates are computed by introducing a Lagrangian parameter  $\lambda$  for each restriction (Pringle and Rayner 1971). The estimates of these Lagrangian parameters are printed in the "Parameter Estimates" table. If a restriction cannot be applied, its parameter value and degrees of freedom are listed as 0.

The Lagrangian parameter  $\lambda$  measures the sensitivity of the sum of squared errors (SSE) to the restriction. If the restriction is changed by a small amount  $\epsilon$ , the SSE is changed by  $2\lambda\epsilon$ .

The t ratio tests the significance of the restrictions. If  $\lambda$  is zero, the restricted estimates are the same as the unrestricted.

Any number of restrictions can be specified in a RESTRICT statement, and any number of RESTRICT statements can be used. The estimates are computed subject to all restrictions specified. However, restrictions should be consistent and not redundant.

**NOTE:** The RESTRICT statement is not supported for the FIML estimation method.

```
SRESTRICT equation , . . . , equation ;
```

The SRESTRICT statement imposes linear restrictions that involve parameters in two or more MODEL statements. The SRESTRICT statement is like the RESTRICT statement but is used to impose restrictions across equations, whereas the RESTRICT statement applies only to parameters in the immediately preceding MODEL statement.

Each restriction is written as a linear equation. Parameters are referred to as *label.variable*, where *label* is the model label and *variable* is the name of the regressor to which the parameter is attached. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.) Each variable name used must be a regressor in the indicated MODEL statement. The keyword INTERCEPT is used to refer to intercept parameters.

SRESTRICT statements can be given labels. The labels are used in the printed output to distinguish results for different restrictions. Labels are specified as follows:

```
label: SRESTRICT ...;
```

The following is an example of the use of the SRESTRICT statement, in which the coefficient for the regressor X2 is constrained to be the same in both models:

```
proc syslin data=a 3sls;
  endogenous y1 y2;
  instruments x1 x2;
  model y1 = y2 x1 x2;
  model y2 = y1 x2;
  srestrict y1.x2 = y2.x2;
run;
```

When no equal sign is used, the linear combination is set equal to 0. Thus, the restriction in the preceding example can also be specified as

```
srestrict y1.x2 - y2.x2;
```

Any number of restrictions can be specified in an SRESTRICT statement, and any number of SRESTRICT statements can be used. The estimates are computed subject to all restrictions specified. However, restrictions should be consistent and not redundant.

When a system restriction is requested for a single equation estimation method (such as OLS or 2SLS), PROC SYSLIN produces the restricted estimates by actually using a corresponding system method. For example, when an SRESTRICT statement is specified along with OLS, PROC SYSLIN produces the restricted OLS estimates via a two-step process equivalent to using SUR estimation with the SDIAG option. First, the unrestricted OLS results are produced. Then, the GLS (SUR) estimation with the system restriction is performed, using the diagonal of the covariance matrix of the residuals. When an SRESTRICT statement is specified along with 2SLS, PROC SYSLIN produces the restricted 2SLS estimates via a multistep process equivalent to using 3SLS estimation with the SDIAG option. First, the unrestricted 2SLS results are produced. Then, the GLS (3SLS) estimation with the system restriction is performed, using the diagonal of the covariance matrix of the residuals.

The results of the SRESTRICT statements are printed after the parameter estimates for all the models in the system. The format of the SRESTRICT statement output is the same as the "Parameter Estimates" table. In this output the parameter estimate is the Lagrangian parameter  $\lambda$  used to impose the restriction.

The Lagrangian parameter  $\lambda$  measures the sensitivity of the system sum of square errors to the restriction. The system SSE is the system MSE shown in the printed output multiplied by the degrees of freedom. If the restriction is changed by a small amount  $\epsilon$ , the system SSE is changed by  $2\lambda\epsilon$ .

The t ratio tests the significance of the restriction. If  $\lambda$  is zero, the restricted estimates are the same as the unrestricted estimates.

The model degrees of freedom are not adjusted for the cross-model restrictions imposed by SRESTRICT statements.

**NOTE:** The SRESTRICT statement is only supported for 2SLS, 3SLS, IT3SLS, OLS, SUR and ITSUR estimation methods.

# **STEST Statement**

```
STEST equation , ..., equation / options;
```

The STEST statement performs an F test for the joint hypotheses specified in the statement.

The hypothesis is represented in matrix notation as

$$L\beta = c$$

and the F test is computed as

$$\frac{(\mathbf{L}b - \mathbf{c})'(\mathbf{L}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{L}')^{-1}(\mathbf{L}b - \mathbf{c})}{m\hat{\sigma}^2}$$

where b is the estimate of  $\beta$ , m is the number of restrictions, and  $\hat{\sigma}^2$  is the system weighted mean squared error. For information about the matrix X'X, see the section "Computational Details" on page 2634.

Each hypothesis to be tested is written as a linear equation. Parameters are referred to as *label.variable*, where *label* is the model label and *variable* is the name of the regressor to which the parameter is attached. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.) Each variable name used must be a regressor in the indicated MODEL statement. The keyword INTERCEPT is used to refer to intercept parameters.

STEST statements can be given labels. The label is used in the printed output to distinguish different tests. Any number of STEST statements can be specified. Labels are specified as follows:

```
label: STEST ...;
```

The following is an example of the STEST statement:

```
proc syslin data=a 3sls;
  endogenous y1 y2;
  instruments x1 x2;
  model y1 = y2 x1 x2;
  model y2 = y1 x2;
  stest y1.x2 = y2.x2;
run;
```

The test performed is exact only for ordinary least squares, given the OLS assumptions of the linear model. For other estimation methods, the *F* test is based on large sample theory and is only approximate in finite samples.

If RESTRICT or SRESTRICT statements are used, the tests computed by the STEST statement are conditional on the restrictions specified. The validity of the tests can be compromised if incorrect restrictions are imposed on the estimates.

The following are examples of STEST statements:

The PRINT option can be specified in the STEST statement after a slash (/):

#### **PRINT**

prints intermediate calculations for the hypothesis tests.

**NOTE:** The STEST statement is only supported for 2SLS, 3SLS, IT3SLS, OLS, SUR and ITSUR estimation methods.

# **TEST Statement**

```
TEST equation , . . . , equation / options ;
```

The TEST statement performs *F* tests of linear hypotheses about the parameters in the preceding MODEL statement. Each equation specifies a linear hypothesis to be tested. If more than one equation is specified, the equations are separated by commas.

Variable names must correspond to regressors in the preceding MODEL statement, and each name represents the coefficient of the corresponding regressor. The keyword INTERCEPT is used to refer to the model intercept.

TEST statements can be given labels. The label is used in the printed output to distinguish different tests. Any number of TEST statements can be specified. Labels are specified as follows:

```
label: TEST ...;
```

The following is an example of the use of TEST statement, which tests the hypothesis that the coefficients of X1 and X2 are the same:

```
proc syslin data=a;
  model y = x1 x2;
  test x1 = x2;
run;
```

The following statements perform F tests for the hypothesis that the coefficients of X1 and X2 are equal, for the hypothesis that the sum of the X1 and X2 coefficients is twice the intercept, and for the joint hypothesis:

```
proc syslin data=a;
   model y = x1 x2;
   xleqx2: test x1 = x2;
   sumeq2i: test x1 + x2 = 2 * intercept;
   joint: test x1 = x2, x1 + x2 = 2 * intercept;
run;
```

The following are additional examples of TEST statements:

```
test x1 + x2 = 1;
test x1 = x2 = x3 = 1;
test 2 * x1 = x2 + x3, intercept + x4 = 0;
test 2 * x1 - x2;
```

The TEST statement performs an *F* test for the joint hypotheses specified. The hypothesis is represented in matrix notation as follows:

$$L\beta = c$$

The F test is computed as

$$\frac{(\mathbf{L}b - \mathbf{c})'(\mathbf{L}(\mathbf{X}'\mathbf{X})^{-}\mathbf{L}')^{-1}(\mathbf{L}b - \mathbf{c})}{m\hat{\sigma}^{2}}$$

where b is the estimate of  $\beta$ , m is the number of restrictions, and  $\hat{\sigma}^2$  is the model mean squared error. For information about the matrix X'X, see the section "Computational Details" on page 2634.

The test performed is exact only for ordinary least squares, given the OLS assumptions of the linear model. For other estimation methods, the F test is based on large sample theory and is only approximate in finite samples.

If RESTRICT or SRESTRICT statements are used, the tests computed by the TEST statement are conditional on the restrictions specified. The validity of the tests can be compromised if incorrect restrictions are imposed on the estimates.

The PRINT option can be specified in the TEST statement after a slash (/):

## **PRINT**

prints intermediate calculations for the hypothesis tests.

**NOTE:** The TEST statement is not supported for the FIML estimation method.

# **VAR Statement**

```
VAR variables;
```

The VAR statement is used to include variables in the crossproducts matrix that are not specified in any MODEL statement. This statement is rarely used with PROC SYSLIN and is used only with the OUTSSCP= option in the PROC SYSLIN statement.

# **WEIGHT Statement**

## WEIGHT variable;

The WEIGHT statement is used to perform weighted regression. The WEIGHT statement names a variable in the input data set whose values are relative weights for a weighted least squares fit. If the weight value is proportional to the reciprocal of the variance for each observation, the weighted estimates are the best linear unbiased estimates (BLUE).

# **Details: SYSLIN Procedure**

# Input Data Set

PROC SYSLIN does not compute new values for regressors. For example, if you need a lagged variable, you must create it with a DATA step. No values are computed by IDENTITY statements; all values must be in the input data set.

# Special TYPE= Input Data Sets

The input data set for most applications of the SYSLIN procedure contains standard rectangular data. However, PROC SYSLIN can also process input data in the form of a crossproducts, covariance, or correlation matrix. Data sets that contain such matrices are identified by values of the TYPE= data set option.

These special kinds of input data sets can be used to save computer time. It takes  $nk^2$  operations, where n is the number of observations and k is the number of variables, to calculate cross products; the regressions are of the order  $k^3$ . When n is in the thousands and k is much smaller, you can save most of the computer time in later runs of PROC SYSLIN by reusing the SSCP matrix rather than recomputing it.

The SYSLIN procedure can process TYPE=CORR, COV, UCORR, UCOV, or SSCP data sets. TYPE=CORR and TYPE=COV data sets, usually created by the CORR procedure, contain means and standard deviations, and correlations or covariances. TYPE=SSCP data sets, usually created in previous runs of PROC SYSLIN, contain sums of squares and cross products. For more information about special SAS data sets, see SAS/STAT User's Guide.

When special SAS data sets are read, you must specify the TYPE= data set option. PROC CORR and PROC SYSLIN automatically set the type for output data sets; however, if you create the data set by some other means, you must specify its type with the TYPE= data set option.

When the special data sets are used, the DW (Durbin-Watson test) and PLOT options in the MODEL statement cannot be performed, and the OUTPUT statements are not valid.

# **Estimation Methods**

A brief description of the methods used by the SYSLIN procedure follows. For more information about these methods, see the references at the end of this chapter.

There are two fundamental methods of estimation for simultaneous equations: least squares and maximum likelihood. There are two approaches within each of these categories: single equation methods (also referred to as limited information methods) and system methods (also referred to as full information methods). System methods take into account cross-equation correlations of the disturbances in estimating parameters, while single equation methods do not.

OLS, 2SLS, MELO, K-class, SUR, ITSUR, 3SLS, and IT3SLS use the least squares method; LIML and FIML use the maximum likelihood method.

OLS, 2SLS, MELO, K-class, and LIML are single equation methods. The system methods are SUR, ITSUR, 3SLS, IT3SLS, and FIML.

# **Single Equation Estimation Methods**

Single equation methods do not take into account correlations of errors across equations. As a result, these estimators are not asymptotically efficient compared to full information methods; however, there are instances in which they may be preferred. (For more information, see the section "Choosing a Method for Simultaneous Equations" on page 2632.)

Let  $y_i$  be the dependent endogenous variable in equation i, and  $X_i$  and  $Y_i$  be the matrices of exogenous and endogenous variables appearing as regressors in the same equation.

The 2SLS method owes its name to the fact that, in a first stage, the instrumental variables are used as regressors to obtain a projected value  $\hat{Y}_i$  that is uncorrelated with the residual in equation i. In a second stage,  $\hat{Y}_i$  replaces  $Y_i$  on the right-hand side to obtain consistent least squares estimators.

Normally, the predetermined variables of the system are used as the instruments. It is possible to use variables other than predetermined variables from your system as instruments; however, the estimation might not be as efficient. For consistent estimates, the instruments must be uncorrelated with the residual and correlated with the endogenous variables.

The LIML method results in consistent estimates that are equal to the 2SLS estimates when an equation is exactly identified. LIML can be viewed as a least-variance ratio estimation or as a maximum likelihood estimation. LIML involves minimizing the ratio  $\lambda = (rvar\_eq)/(rvar\_sys)$ , where  $rvar\_eq$  is the residual variance associated with regressing the weighted endogenous variables on all predetermined variables that appear in that equation, and  $rvar\_sys$  is the residual variance associated with regressing weighted endogenous variables on all predetermined variables in the system.

The MELO method computes the minimum expected loss estimator. MELO estimators "minimize the posterior expectation of generalized quadratic loss functions for structural coefficients of linear structural models" (Judge et al. 1985, p. 635).

K-class estimators are a class of estimators that depends on a user-specified parameter k. A k value less than 1 is recommended but not required. The parameter k can be deterministic or stochastic, but its probability limit must equal 1 for consistent parameter estimates. When all the predetermined variables are listed as instruments, they include all the other single equation estimators supported by PROC SYSLIN. The instance when some of the predetermined variables are not listed among the instruments is not supported by PROC SYSLIN for the general K-class estimation. However, it is supported for the other methods.

For k = 1, the K-class estimator is the 2SLS estimator, while for k = 0, the K-class estimator is the OLS estimator. The K-class interpretation of LIML is that  $k = \lambda$ . Note that k is stochastic in the LIML method, unlike for OLS and 2SLS.

MELO is a Bayesian K-class estimator. It yields estimates that can be expressed as a matrix-weighted average of the OLS and 2SLS estimates. MELO estimators have finite second moments and hence finite risk. Other frequently used K-class estimators might not have finite moments under some commonly encountered circumstances, and hence there can be infinite risk relative to quadratic and other loss functions.

One way of comparing K-class estimators is to note that when k = 1, the correlation between regressor and the residual is completely corrected for. In all other cases, it is only partially corrected for.

For more information about K-class estimators, see the section "Computational Details" on page 2634.

# **SUR and 3SLS Estimation Methods**

SUR might improve the efficiency of parameter estimates when there is contemporaneous correlation of errors across equations. In practice, the contemporaneous correlation matrix is estimated using OLS residuals. Under two sets of circumstances, SUR parameter estimates are the same as those produced by OLS: when there is no contemporaneous correlation of errors across equations (the estimate of the contemporaneous correlation matrix is diagonal) and when the independent variables are the same across equations.

Theoretically, SUR parameter estimates are always at least as efficient as OLS in large samples, provided that your equations are correctly specified. However, in small samples the need to estimate the covariance matrix from the OLS residuals increases the sampling variability of the SUR estimates. This effect can cause SUR to be less efficient than OLS. If the sample size is small and the cross-equation correlations are small, then OLS is preferred to SUR. The consequences of specification error are also more serious with SUR than with OLS.

The 3SLS method combines the ideas of the 2SLS and SUR methods. Like 2SLS, the 3SLS method uses  $\hat{Y}$  instead of Y for endogenous regressors, which results in consistent estimates. Like SUR, the 3SLS method takes the cross-equation error correlations into account to improve large sample efficiency. For 3SLS, the 2SLS residuals are used to estimate the cross-equation error covariance matrix.

The SUR and 3SLS methods can be iterated by recomputing the estimate of the cross-equation covariance matrix from the SUR or 3SLS residuals and then computing new SUR or 3SLS estimates based on this updated covariance matrix estimate. Continuing this iteration until convergence produces ITSUR or IT3SLS estimates.

#### FIML Estimation Method

The FIML estimator is a system generalization of the LIML estimator. The FIML method involves minimizing the determinant of the covariance matrix associated with residuals of the reduced form of the equation system. From a maximum likelihood standpoint, the LIML method involves assuming that the errors are normally distributed and then maximizing the likelihood function subject to restrictions on a particular equation. FIML is similar, except that the likelihood function is maximized subject to restrictions on all of the parameters in the model, not just those in the equation being estimated.

**NOTE:** The RESTRICT, SRESTRICT, TEST, and STEST statements are not supported when the FIML method is used.

# **Choosing a Method for Simultaneous Equations**

A number of factors should be taken into account in choosing an estimation method. Although system methods are asymptotically most efficient in the absence of specification error, system methods are more sensitive to specification error than single equation methods.

In practice, models are never perfectly specified. It is a matter of judgment whether the misspecification is serious enough to warrant avoidance of system methods.

Another factor to consider is sample size. With small samples, 2SLS might be preferred to 3SLS. In general, it is difficult to say much about the small sample properties of K-class estimators because the results depend on the regressors used.

LIML and FIML are invariant to the normalization rule imposed but are computationally more expensive than 2SLS or 3SLS.

If the reason for contemporaneous correlation among errors across equations is a common, omitted variable, it is not necessarily best to apply SUR. SUR parameter estimates are more sensitive to specification error than OLS. OLS might produce better parameter estimates under these circumstances. SUR estimates are also affected by the sampling variation of the error covariance matrix. There is some evidence from Monte Carlo studies that SUR is less efficient than OLS in small samples.

### **ANOVA Table for Instrumental Variables Methods**

In the instrumental variables methods (2SLS, LIML, K-class, MELO), first-stage predicted values are substituted for the endogenous regressors. As a result, the regression sum of squares (RSS) and the error sum of squares (ESS) do not sum to the total corrected sum of squares for the dependent variable (TSS). The analysis-of-variance table included in the second-stage results gives these sums of squares and the mean squares that are used for the F test, but this table is not a variance decomposition in the usual sense.

The F test shown in the instrumental variables case is a valid test of the no-regression hypothesis that the true coefficients of all regressors are 0. However, because of the first-stage projection of the regression mean square, this is a Wald-type test statistic, which is asymptotically F but not exactly F-distributed in finite samples. Thus, for small samples the F test is only approximate when instrumental variables are used.

# The R-Square Statistics

As explained in the section "ANOVA Table for Instrumental Variables Methods" on page 2633, when instrumental variables are used, the regression sum of squares (RSS) and the error sum of squares (ESS) do not sum to the total corrected sum of squares. In this case, there are several ways that the  $R^2$  statistic can be defined.

The definition of  $R^2$  used by the SYSLIN procedure is

$$R^2 = \frac{RSS}{RSS + ESS}$$

This definition is consistent with the F test of the null hypothesis that the true coefficients of all regressors are zero. However, this  $R^2$  might not be a good measure of the goodness of fit of the model.

### System Weighted R-Square and System Weighted Mean Squared Error

The system weighted  $R^2$ , printed for the 3SLS, IT3SLS, SUR, ITSUR, and FIML methods, is computed as follows.

$$R^2 = Y'WR(X'X)^{-1}R'WY/Y'WY$$

In this equation, the matrix X'X is R'WR and W is the projection matrix of the instruments:

$$W = S^{-1} \otimes Z(Z'Z)^{-1}Z'$$

The matrix Z is the instrument set, R is the regressor set, and S is the estimated cross-model covariance matrix.

The system weighted MSE, printed for the 3SLS, IT3SLS, SUR, ITSUR, and FIML methods, is computed as follows:

$$MSE = \frac{1}{tdf} (\mathbf{Y}'\mathbf{W}\mathbf{Y} - \mathbf{Y}'\mathbf{W}\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'\mathbf{W}\mathbf{Y})$$

In this equation, tdf is the sum of the error degrees of freedom for the equations in the system.

### **Computational Details**

This section discusses various computational details.

### **Computation of Least Squares-Based Estimators**

Let the system be composed of G equations, and let the ith equation be expressed in the form

$$y_i = Y_i \boldsymbol{\beta}_i + X_i \boldsymbol{\gamma}_i + \mathbf{u}$$

where

 $y_i$  is the vector of observations on the dependent variable

 $Y_i$  is the matrix of observations on the endogenous variables included in the equation

 $\beta_i$  is the vector of parameters associated with  $Y_i$ 

 $X_i$  is the matrix of observations on the predetermined variables included in the equation

 $\gamma_i$  is the vector of parameters associated with  $X_i$ 

u is a vector of errors

Let  $\hat{V}_i = Y_i - \hat{Y}_i$ , where  $\hat{Y}_i$  is the projection of  $Y_i$  onto the space spanned by the instruments matrix **Z**.

Let

$$\delta_i = \begin{bmatrix} \boldsymbol{\beta}_i \\ \boldsymbol{\gamma}_i \end{bmatrix}$$

be the vector of parameters associated with both the endogenous and exogenous variables.

The K-class of estimators (Theil 1971) is defined by

$$\hat{\delta}_{i,k} = \begin{bmatrix} Y_i'Y_i - k\hat{V}_i'\hat{V}_i & Y_i'X_i \\ X_i'Y_i & X_i'X_i \end{bmatrix}^{-1} \begin{bmatrix} (Y_i - kV_i)'y_i \\ X_i'y_i \end{bmatrix}$$

where k is a user-defined value.

Let

$$\mathbf{R} = [Y_i X_i]$$

and

$$\hat{\mathbf{R}} = [\hat{Y}_i \ X_i]$$

The 2SLS estimator is defined as

$$\hat{\boldsymbol{\delta}}_{i,2\text{SLS}} = [\hat{R}'_i \ \hat{R}_i]^{-1} \hat{R}'_i y_i$$

Let y and  $\delta$  be the vectors obtained by stacking the vectors of dependent variables and parameters for all G equations, and let **R** and  $\hat{\mathbf{R}}$  be the block diagonal matrices formed by  $R_i$  and  $\hat{R}_i$ , respectively.

The SUR and ITSUR estimators are defined as

$$\hat{\boldsymbol{\delta}}_{(\mathrm{IT})\mathrm{SUR}} = \left[\mathbf{R}'\left(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}\right)\mathbf{R}\right]^{-1}\mathbf{R}'\left(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}\right)\mathbf{y}$$

while the 3SLS and IT3SLS estimators are defined as

$$\hat{\boldsymbol{\delta}}_{(\mathrm{IT})3\mathrm{SLS}} = \left[\hat{\mathbf{R}}'\left(\hat{\Sigma}^{-1} \otimes \mathbf{I}\right)\hat{\mathbf{R}}\right]^{-1}\hat{\mathbf{R}}'\left(\hat{\Sigma}^{-1} \otimes \mathbf{I}\right)\mathbf{y}$$

where **I** is the identity matrix and  $\hat{\Sigma}$  is an estimator of the cross-equation correlation matrix. For 3SLS,  $\hat{\Sigma}$  is obtained from the 2SLS estimation, while for SUR it is derived from the OLS estimation. For IT3SLS and ITSUR, it is obtained iteratively from the previous estimation step, until convergence.

### **Computation of Standard Errors**

The VARDEF= option in the PROC SYSLIN statement controls the denominator used in calculating the cross-equation covariance estimates and the parameter standard errors and covariances. The values of the VARDEF= option and the resulting denominator are as follows:

N uses the number of nonmissing observations.

DF uses the number of nonmissing observations less the degrees of freedom in the model.

WEIGHT uses the sum of the observation weights given by the WEIGHTS statement.

WDF uses the sum of the observation weights given by the WEIGHTS statement less the degrees

of freedom in the model.

The VARDEF= option does not affect the model mean squared error, root mean squared error, or  $R^2$  statistics. These statistics are always based on the error degrees of freedom, regardless of the VARDEF= option. The VARDEF= option also does not affect the dependent variable coefficient of variation (CV).

#### **Reduced Form Estimates**

The REDUCED option in the PROC SYSLIN statement computes estimates of the reduced form coefficients. The REDUCED option requires that the equation system be square. If there are fewer models than endogenous variables, IDENTITY statements can be used to complete the equation system.

The reduced form coefficients are computed as follows. Represent the equation system, with all endogenous variables moved to the left-hand side of the equations and identities, as

$$BY = \Gamma X$$

Here **B** is the estimated coefficient matrix for the endogenous variables **Y**, and  $\Gamma$  is the estimated coefficient matrix for the exogenous (or predetermined) variables **X**.

The system can be solved for Y as follows, provided B is square and nonsingular:

$$Y = B^{-1}\Gamma X$$

The reduced form coefficients are the matrix  $B^{-1}\Gamma$ .

### **Uncorrelated Errors across Equations**

The SDIAG option in the PROC SYSLIN statement computes estimates by assuming uncorrelated errors across equations. As a result, when the SDIAG option is used, the 3SLS estimates are identical to 2SLS estimates, and the SUR estimates are the same as the OLS estimates.

#### **Overidentification Restrictions**

The OVERID option in the MODEL statement can be used to test for overidentifying restrictions on parameters of each equation. The null hypothesis is that the predetermined variables that do not appear in any equation have zero coefficients. The alternative hypothesis is that at least one of the assumed zero coefficients is nonzero. The test is approximate and rejects the null hypothesis too frequently for small sample sizes.

The formula for the test is given as follows. Let  $y_i = \beta_i \mathbf{Y}_i + \gamma_i \mathbf{Z}_i + e_i$  be the *i*th equation.  $\mathbf{Y}_i$  are the endogenous variables that appear as regressors in the *i*th equation, and  $\mathbf{Z}_i$  are the instrumental variables that appear as regressors in the *i*th equation. Let  $N_i$  be the number of variables in  $\mathbf{Y}_i$  and  $\mathbf{Z}_i$ .

Let  $v_i = y_i - \mathbf{Y}_i \hat{\beta}_i$ . Let **Z** represent all instrumental variables, T be the total number of observations, and K be the total number of instrumental variables. Define  $\hat{l}$  as follows:

$$\hat{l} = \frac{v'_i (\mathbf{I} - \mathbf{Z}_i (\mathbf{Z}'_i \mathbf{Z}_i)^{-1} \mathbf{Z}'_i) v_i}{v'_i (\mathbf{I} - \mathbf{Z} (\mathbf{Z}'\mathbf{Z})^{-1} \mathbf{Z}') v_i}$$

Then the test statistic

$$\frac{T-K}{K-N_i}(\hat{l}-1)$$

is distributed approximately as an F with  $K - N_i$  and T - K degrees of freedom. For more information, see Basmann (1960).

#### **Fuller's Modification to LIML**

The ALPHA= option in the PROC SYSLIN and MODEL statements parameterizes Fuller's modification to LIML. This modification is  $k = \gamma - (\alpha/(n-g))$ , where  $\alpha$  is the value of the ALPHA= option,  $\gamma$  is the LIML k value,n is the number of observations, and g is the number of predetermined variables. Fuller's modification is not used unless the ALPHA= option is specified. For more information, see Fuller (1977).

# **Missing Values**

Observations that have a missing value for any variable in the analysis are excluded from the computations.

### **OUT= Data Set**

The output SAS data set produced by the OUT= option in the PROC SYSLIN statement contains all the variables in the input data set and the variables that contain predicted values and residuals specified by OUTPUT statements.

The residuals are computed as actual values minus predicted values. Predicted values never use lags of other predicted values, as would be desirable for dynamic simulation. For these applications, PROC SIMLIN is available to predict or simulate values from the estimated equations.

### OUTEST= Data Set

The OUTEST= option produces a TYPE=EST output SAS data set that contains estimates from the regressions. The variables in the OUTEST= data set are as follows:

BY variables	identifies the BY statement variables that are included in the OUTEST= data set.
_TYPE_	identifies the estimation type for the observations. The _TYPE_ value INST indicates first-stage regression estimates. Other values indicate the estimation method used: 2SLS indicates two-stage least squares results, 3SLS indicates three-stage least squares results, LIML indicates limited information maximum likelihood results, and so forth. Observations added by IDENTITY statements have the _TYPE_ value IDENTITY.
_STATUS_	identifies the convergence status of the estimation. The value of _STATUS_ is 0 when convergence criteria are met. Otherwise, the value of _STATUS_ is 1 when the estimation converges with a note, 2 when it converges with a warning, or 3 when it fails to converge.
_MODEL_	identifies the model label. The model label is the label specified in the MODEL statement or the dependent variable name if no label is specified. For first-stage regression estimates, _MODEL_ has the value FIRST.
_DEPVAR_	identifies the name of the dependent variable for the model.
_NAME_	identifies the names of the regressors for the rows of the covariance matrix, if the COVOUT option is specifiedNAME_ has a blank value for the parameter estimates observations. The _NAME_ variable is not included in the OUTEST= data set unless the COVOUT option is used to output the covariance of parameter estimates matrix.
_SIGMA_	contains the root mean squared error for the model, which is an estimate of the standard deviation of the error term. The _SIGMA_ variable contains the same values reported as Root MSE in the printed output.
INTERCEPT	identifies the intercept parameter estimates.
regressors	identifies the regressor variables from all the MODEL statements that are included in the

OUTEST= data set.

OUTEST= data set. Variables used in IDENTIFY statements are also included in the

The parameter estimates are stored under the names of the regressor variables. The intercept parameters are stored in the variable INTERCEPT. The dependent variable of the model is given a coefficient of -1. Variables that are not in a model have missing values for the OUTEST= observations for that model.

Some estimation methods require computation of preliminary estimates. All estimates computed are output to the OUTEST= data set. For each BY group and each estimation, the OUTEST= data set contains one observation for each MODEL or IDENTITY statement. Results for different estimations are identified by the TYPE variable.

For example, consider the following statements:

```
proc syslin data=a outest=est 3sls;
  by b;
  endogenous y1 y2;
  instruments x1-x4;
  model y1 = y2 x1 x2;
  model y2 = y1 x3 x4;
  identity x1 = x3 + x4;
run;
```

The 3SLS method requires both a preliminary 2SLS stage and preliminary first-stage regressions for the endogenous variable. The OUTEST= data set thus contains three different kinds of estimates. The observations for the first-stage regression estimates have the \_TYPE\_ value INST. The observations for the 2SLS estimates have the \_TYPE\_ value 2SLS. The observations for the final 3SLS estimates have the \_TYPE\_ value 3SLS.

Since there are two endogenous variables in this example, there are two first-stage regressions and two \_TYPE\_=INST observations in the OUTEST= data set. Since there are two model statements, there are two OUTEST= observations with \_TYPE\_=2SLS and two observations with \_TYPE\_=3SLS. In addition, the OUTEST= data set contains an observation with the \_TYPE\_ value IDENTITY that contains the coefficients specified by the IDENTITY statement. All these observations are repeated for each BY group in the input data set defined by the values of the BY variable B.

When the COVOUT option is specified, the estimated covariance matrix for the parameter estimates is included in the OUTEST= data set. Each observation for parameter estimates is followed by observations that contain the rows of the parameter covariance matrix for that model. The row of the covariance matrix is identified by the variable \_NAME\_. For observations that contain parameter estimates, \_NAME\_ is blank. For covariance observations, \_NAME\_ contains the regressor name for the row of the covariance matrix and the regressor variables contain the covariances.

For an example of the OUTEST= data set, see Example 35.1.

### OUTSSCP= Data Set

The OUTSSCP= option produces a TYPE=SSCP output SAS data set that contains sums of squares and cross products. The data set contains all variables used in the MODEL, IDENTITY, and VAR statements. Observations are identified by the variable \_NAME\_.

The OUTSSCP= data set can be useful when a large number of observations are to be explored in many different PROC SYSLIN runs. The sum-of-squares-and-crossproducts matrix can be saved with the OUTSSCP= option and used as the DATA= data set on subsequent PROC SYSLIN runs. This is much less expensive computationally because PROC SYSLIN never reads the original data again. In the step that creates the OUTSSCP= data set, include in the VAR statement all the variables you expect to use.

### **Printed Output**

The printed output produced by the SYSLIN procedure is as follows:

- 1. If the SIMPLE option is used, a table of descriptive statistics is printed that shows the sum, mean, sum of squares, variance, and standard deviation for all the variables used in the models.
- 2. If the FIRST option is specified and an instrumental variables method is used, first-stage regression results are printed. The results show the regression of each endogenous variable on the variables in the INSTRUMENTS list.
- 3. The results of the second-stage regression are printed for each model. (For more information, see the section "Printed Output for Each Model" on page 2639.)
- 4. If a systems method like 3SLS, SUR, or FIML is used, the cross-equation error covariance matrix is printed. This matrix is shown four ways: the covariance matrix itself, the correlation matrix form, the inverse of the correlation matrix, and the inverse of the covariance matrix.
- 5. If a systems method like 3SLS, SUR, or FIML is used, the system weighted mean squared error and system weighted  $R^2$  statistics are printed. The system weighted MSE and  $R^2$  measure the fit of the joint model obtained by stacking all the models together and performing a single regression with the stacked observations weighted by the inverse of the model error variances.
- 6. If a systems method like 3SLS, SUR, or FIML is used, the final results are printed for each model.
- 7. If the REDUCED option is used, the reduced form coefficients are printed. These consist of the structural coefficient matrix for the endogenous variables, the structural coefficient matrix for the exogenous variables, the inverse of the endogenous coefficient matrix, and the reduced form coefficient matrix. The reduced form coefficient matrix is the product of the inverse of the endogenous coefficient matrix and the exogenous structural coefficient matrix.

### **Printed Output for Each Model**

The results printed for each model include the analysis-of-variance table, the "Parameter Estimates" table, and optional items requested by TEST statements or by options in the MODEL statement.

The printed output produced for each model is described in the following.

The analysis-of-variance table includes the following:

- the model degrees of freedom, sum of squares, and mean square
- the error degrees of freedom, sum of squares, and mean square. The error mean square is computed by dividing the error sum of squares by the error degrees of freedom and is not affected by the VARDEF= option.
- the corrected total degrees of freedom and total sum of squares. Note that for instrumental variables methods, the model and error sums of squares do not add to the total sum of squares.

- the F ratio, labeled "F Value," and its significance, labeled "PROB>F," for the test of the hypothesis that all the nonintercept parameters are 0
- the root mean squared error. This is the square root of the error mean square.
- the dependent variable mean
- the coefficient of variation (CV) of the dependent variable
- the  $R^2$  statistic. This  $R^2$  is computed consistently with the calculation of the F statistic. It is valid for hypothesis tests but might not be a good measure of fit for models estimated by instrumental variables methods.
- the  $R^2$  statistic adjusted for model degrees of freedom, labeled "Adj R-SQ"

The "Parameter Estimates" table includes the following:

- estimates of parameters for regressors in the model and the Lagrangian parameter for each restriction specified
- a degrees of freedom column labeled DF. Estimated model parameters have 1 degree of freedom. Restrictions have a DF of -1. Regressors or restrictions dropped from the model due to collinearity have a DF of 0.
- the standard errors of the parameter estimates
- the t statistics, which are the parameter estimates divided by the standard errors
- the significance of the *t* tests for the hypothesis that the true parameter is 0, labeled "Pr > ltl." As previously noted, the significance tests are strictly valid in finite samples only for OLS estimates but are asymptotically valid for the other methods.
- the standardized regression coefficients, if the STB option is specified. This is the parameter estimate multiplied by the ratio of the standard deviation of the regressor to the standard deviation of the dependent variable.
- the labels of the regressor variables or restriction labels

In addition to the analysis-of-variance table and the "Parameter Estimates" table, the results printed for each model can include the following:

- If TEST statements are specified, the test results are printed.
- If the DW option is specified, the Durbin-Watson statistic and first-order autocorrelation coefficient are printed.
- If the OVERID option is specified, the results of Basmann's test for overidentifying restrictions are printed.
- If the PLOT option is used, plots of residual against each regressor are printed.

If the COVB or CORRB options are specified, the results for each model also include the covariance
or correlation matrix of the parameter estimates. For systems methods like 3SLS and FIML, the
COVB and CORB output is printed for the whole system after the output for the last model, instead of
separately for each model.

The third-stage output for 3SLS, SUR, IT3SLS, ITSUR, and FIML does not include the analysis-of-variance table. When a systems method is used, the second-stage output does not include the optional output, except for the COVB and CORRB matrices.

### **ODS Table Names**

PROC SYSLIN assigns a name to each table it creates. You can use these names to reference the table when you use the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in Table 35.2. If the estimation method used is 3SLS, IT3SLS, ITSUR or SUR, you can obtain tables by specifying ODS OUTPUT CorrResiduals, InvCorrResiduals, InvCovResiduals.

Table 35.2 ODS Tables Produced in PROC SYSLIN

ODS Table Name	Description	Option
ANOVA	Summary of the SSE, MSE for the equations	Default
AugXPXMat	Model crossproducts	XPX or USSCP
AutoCorrStat	Autocorrelation statistics	DW
ConvergenceStatus	Convergence status	Default
CorrB	Correlations of parameters	CORRB
CorrResiduals	Correlations of residuals	
CovB	Covariance of parameters	COVB
CovResiduals	Covariance of residuals	
EndoMat	Endogenous variables	REDUCED
ExogMat	Exogenous variables	REDUCED
FitStatistics	Statistics of fit	Default
InvCorrResiduals	Inverse correlations of residuals	
InvCovResiduals	Inverse covariance of residuals	
InvEndoMat	Inverse endogenous variables	REDUCED
InvXPX	X'X inverse for system	I
IterHistory	Iteration printing	ITPRINT
MissingValues	Missing values generated by the program	Default
ModelVars	Name and label for the model	Default
ParameterEstimates	Parameter estimates	Default
RedMat	Reduced form	REDUCED
SimpleStatistics	Descriptive statistics	SIMPLE
SSCP	Model crossproducts	XPX or USSCP
TestResults	Test for overidentifying restrictions	
Weight	Weighted model statistics	

### **ODS Graphics**

This section describes the use of ODS for creating graphics with the SYSLIN procedure.

### **ODS Graph Names**

PROC SYSLIN assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when you use ODS. The names are listed in Table 35.3.

To request these graphs, you must specify the ODS GRAPHICS statement.

Table 35.3 ODS Graphics Produced by PROC SYSLIN

ODS Graph Name	Plot Description
DiagnosticsPanel	All applicable plots listed below
ActualByPredicted	Predicted versus actual plot
QQPlot	Q-Q plot of residuals
ResidualHistogram	Histogram of the residuals
ResidualPlot	Residual plot

# **Examples: SYSLIN Procedure**

# **Example 35.1: Klein's Model I Estimated with LIML and 3SLS**

This example uses PROC SYSLIN to estimate the classic Klein Model I. For a discussion of this model, see Theil (1971). The following statements read the data:

```
*-----*
| By L.R. Klein, Economic Fluctuations in the United States, 1921-1941 |
| (1950), NY: John Wiley. A macro-economic model of the U.S. with
| three behavioral equations, and several identities. See Theil, p.456.|
*-----;
data klein;
input year c p w i x wp g t k wsum;
  date=mdy(1,1,year);
  format date monyy.;
  y = c+i+g-t;
  yr =year-1931;
  klag=lag(k);
  plag=lag(p);
  xlag=lag(x);
  label year='Year'
      date='Date'
      c = 'Consumption'
      p ='Profits'
        ='Private Wage Bill'
```

```
='Investment'
        i
        k = 'Capital Stock'
           ='National Income'
        У
           ='Private Production'
        x
        wsum='Total Wage Bill'
        wp = 'Govt Wage Bill'
           ='Govt Demand'
           ='Taxes'
        i
        klag='Capital Stock Lagged'
        plag='Profits Lagged'
        xlag='Private Product Lagged'
        yr = 'YEAR-1931';
datalines;
1920
          12.7
                          44.9
                                               182.8
                                     3.9
1921 41.9 12.4 25.5 -0.2
                         45.6 2.7
                                          7.7 182.6 28.2
1922 45.0 16.9 29.3 1.9 50.1 2.9
                                     3.2
                                         3.9 184.5
                                                     32.2
1923 49.2 18.4 34.1 5.2 57.2 2.9
                                     2.8 4.7 189.7 37.0
1924 50.6 19.4 33.9 3.0 57.1 3.1
                                     3.5
                                          3.8 192.7
                                                     37.0
1925 52.6 20.1 35.4 5.1 61.0 3.2
                                     3.3
                                          5.5 197.8 38.6
1926 55.1 19.6 37.4 5.6 64.0 3.3
                                     3.3
                                         7.0 203.4 40.7
1927 56.2 19.8 37.9 4.2 64.4 3.6
                                     4.0
                                           6.7 207.6 41.5
1928 57.3 21.1 39.2 3.0 64.5 3.7
                                     4.2
                                           4.2 210.6 42.9
                                          4.0 215.7 45.3
1929 57.8 21.7 41.3 5.1 67.0 4.0
                                     4.1
1930 55.0 15.6 37.9 1.0 61.2 4.2 5.2 7.7 216.7 42.1
   ... more lines ...
```

The following statements estimate the Klein model using the limited information maximum likelihood method. In addition, the parameter estimates are written to a SAS data set with the OUTEST= option.

```
proc syslin data=klein outest=b liml;
  endogenous c p w i x wsum k y;
  instruments klag plag xlag wp g t yr;
  consume: model c = p plag wsum;
  invest: model i = p plag klag;
  labor: model w = x xlag yr;
run;

proc print data=b;
run;
```

The PROC SYSLIN estimates are shown in Output 35.1.1 through Output 35.1.3.

Output 35.1.1 LIML Estimates for Consumption

# The SYSLIN Procedure Limited-Information Maximum Likelihood Estimation

Model	CONSUME
Dependent Variable	С
Label	Consumption

Output 35.1.1 continued

Analysis of Variance								
Sum of Mean								
Source	DF	Squares	Square	F Value	Pr > F			
Model	3	854.3541	284.7847	118.42	<.0001			
Error	17	40.88419	2.404952					
<b>Corrected Total</b>	20	941.4295						

Root MSE	1.55079	R-Square	0.95433
Dependent Mean	53.99524	Adj R-Sq	0.94627
Coeff Var	2.87209		

Parameter Estimates								
Parameter Standard Variab Variable DF Estimate Error t Value Pr > It  Label								
Variable	DF	Estimate	Error	t value	Pr >  t	Label		
Intercept	1	17.14765	2.045374	8.38	<.0001	Intercept		
р	1	-0.22251	0.224230	-0.99	0.3349	Profits		
plag	1	0.396027	0.192943	2.05	0.0558	Profits Lagged		
wsum	1	0.822559	0.061549	13.36	<.0001	Total Wage Bill		

# Output 35.1.2 LIML Estimates for Investments

### The SYSLIN Procedure Limited-Information Maximum Likelihood Estimation

Model	INVEST
<b>Dependent Variable</b>	i
Label	Taxes

Analysis of Variance								
Sum of Mean Source DF Squares Square F Value Pr > 1								
Model		210 3790						
Error	·	34.99649	, 0200 .	54.00	\.UUU1			
Corrected Total	20	252.3267						

Root MSE	1.43479	R-Square	0.85738
Dependent Mean	1.26667	Adj R-Sq	0.83221
Coeff Var	113.27274		

Parameter Estimates								
Variable	DF	Parameter Estimate		t Value	Pr >  t	Variable Label		
Intercept	1	22.59083	9.498146	2.38	0.0294	Intercept		
p	1	0.075185	0.224712	0.33	0.7420	Profits		
plag	1	0.680386	0.209145	3.25	0.0047	Profits Lagged		
klag	1	-0.16826	0.045345	-3.71	0.0017	Capital Stock Lagged		

Output 35.1.3 LIML Estimates for Labor

# The SYSLIN Procedure Limited-Information Maximum Likelihood Estimation

Model			LABOR			
Depen	Dependent Variable					
Label	Label					
	An	alysis of \	/ariance			
		Sum of	Mean			
Source	DF	Squares	Square	F Value	Pr > F	
Model	3	696.1485	232.0495	393.62	<.0001	
Error	17	10.02192	0.589525			
<b>Corrected Total</b>	20	794.9095				
Root MSE		0.7678	31 <b>R-Squ</b> a	re 0.985	<u> </u>	
Dependent	0 <b>Adj R-</b> S	<b>5q</b> 0.983	30			
Coeff Var		2.1115	6			

Parameter Estimates										
		Parameter	Standard			Variable				
Variable	DF	Estimate	Error	t Value	Pr >  t	Label				
Intercept	1	1.526187	1.320838	1.16	0.2639	Intercept				
x	1	0.433941	0.075507	5.75	<.0001	Private Production				
xlag	1	0.151321	0.074527	2.03	0.0583	Private Product Lagged				
yr	1	0.131593	0.035995	3.66	0.0020	YEAR-1931				

The OUTEST= data set is shown in part in Output 35.1.4. Note that the data set contains the parameter estimates and root mean squared errors,  $\_SIGMA\_$ , for the first-stage instrumental regressions as well as the parameter estimates and  $\sigma$  for the LIML estimates for the three structural equations.

Output 35.1.4 The OUTEST= Data Set

Obs _TYPE_	_STATUS_	_MODEL_	_DEPVAR_	_SIGMA_	Intercept	klag	plag	xlag	wp
1 LIML	0 Converged	CONSUME	С	1.55079	17.1477		0.39603		
2 LIML	0 Converged	INVEST	i	1.43479	22.5908	-0.16826	0.68039	•	
3 LIML	0 Converged	LABOR	W	0.76781	1.5262			0.15132	

Obs g t	yr	С	р	W	i	x	wsum	k	у
1		-1	-0.22251				0.82256		_
2	•		0.07518		-1				
3	0.13159			-1		0.43394			

The following statements estimate the model using the 3SLS method. The reduced form estimates are produced by the REDUCED option; IDENTITY statements are used to make the model complete.

The preliminary 2SLS results and estimated cross-model covariance matrix are not shown. The 3SLS estimates are shown in Output 35.1.5 through Output 35.1.7. The reduced form estimates are shown in Output 35.1.8 through Output 35.1.11.

Output 35.1.5 3SLS Estimates for Consumption

# The SYSLIN Procedure Three-Stage Least Squares Estimation

System Weighted MS	<b>SE</b> 5.9342			
Degrees of freedom	51			
System Weighted R-	<b>Square</b> 0.9550			
Model	CONSUME			
Dependent Variable	С			
Label	Consumption			

Parameter Estimates										
Parameter Standard Variable Variable DF Estimate Error t Value Pr >  t  Label										
Intercept	1	16.44079	1.449925	11.34	<.0001	Intercept				
р	1	0.124890	0.120179	1.04	0.3133	Profits				
plag	1	0.163144	0.111631	1.46	0.1621	Profits Lagged				
wsum	1	0.790081	0.042166	18.74	<.0001	Total Wage Bill				

Output 35.1.6 3SLS Estimates for Investments

Model	INVEST
<b>Dependent Variable</b>	i
Label	Taxes

Parameter Estimates										
Variable	DF	Parameter Estimate		t Value	Pr >  t	Variable Label				
Intercept	1	28.17785	7.550853	3.73	0.0017	Intercept				
р	1	-0.01308	0.179938	-0.07	0.9429	Profits				
plag	1	0.755724	0.169976	4.45	0.0004	Profits Lagged				
klag	1	-0.19485	0.036156	-5.39	<.0001	Capital Stock Lagged				

Model	LABOR
<b>Dependent Variable</b>	W
Label	Private Wage Bill

	Parameter Estimates										
Variable	DF	Parameter Estimate		t Value	Pr >  t	Variable Label					
Intercept	1	1.797218	1.240203	1.45	0.1655	Intercept					
x	1	0.400492	0.035359	11.33	<.0001	Private Production					
xlag	1	0.181291	0.037965	4.78	0.0002	Private Product Lagged					
yr	1	0.149674	0.031048	4.82	0.0002	YEAR-1931					

Output 35.1.8 Reduced Form Estimates

	Endogenous Variables										
	С	р	w	i	x	wsum	k	у			
CONSUME	1	-0.12489	0	0	0	-0.79008	0	0			
INVEST	0	0.013079	0	1	0	0	0	0			
LABOR	0	0	1	0	-0.40049	0	0	0			
PRODUCT	-1	0	0	-1	1	0	0	0			
INCOME	-1	0	0	-1	0	0	0	1			
PROFIT	0	1	1	0	0	0	0	-1			
STOCK	0	0	0	-1	0	0	1	0			
WAGE	0	0	-1	0	0	1	0	0			

Output 35.1.9 Reduced Form Estimates

	Exogenous Variables										
	Intercept	plag	klag	xlag	yr	g	t	wp			
CONSUME	16.44079	0.163144	0	0	0	0	0	0			
INVEST	28.17785	0.755724	-0.19485	0	0	0	0	0			
LABOR	1.797218	0	0	0.181291	0.149674	0	0	0			
PRODUCT	0	0	0	0	0	1	0	0			
INCOME	0	0	0	0	0	1	-1	0			
PROFIT	0	0	0	0	0	0	0	0			
STOCK	0	0	1	0	0	0	0	0			
WAGE	0	0	0	0	0	0	0	_1			

			Inverse E	ndogenous	Variables			
	CONSUME	INVEST	LABOR	PRODUCT	INCOME	PROFIT	STOCK	WAGE
С	1.634654	0.634654	1.095657	0.438802	0.195852	0.195852	0	1.291509
р	0.972364	0.972364	-0.34048	-0.13636	1.108721	1.108721	0	0.768246
w	0.649572	0.649572	1.440585	0.576943	0.072629	0.072629	0	0.513215
i	-0.01272	0.987282	0.004453	0.001783	-0.0145	-0.0145	0	-0.01005
x	1.621936	1.621936	1.10011	1.440585	0.181351	0.181351	0	1.281461
wsum	0.649572	0.649572	1.440585	0.576943	0.072629	0.072629	0	1.513215
k	-0.01272	0.987282	0.004453	0.001783	-0.0145	-0.0145	1	-0.01005
у	1.621936	1.621936	1.10011	0.440585	1.181351	0.181351	0	1.281461

Output 35.1.10 Reduced Form Estimates

Output 35.1.11 Reduced Form Estimates

	Reduced Form							
	Intercept	plag	klag	xlag	yr	g	t	wp
С	46.7273	0.746307	-0.12366	0.198633	0.163991	0.634654	-0.19585	1.291509
р	42.77363	0.893474	-0.18946	-0.06173	-0.05096	0.972364	-1.10872	0.768246
w	31.57207	0.596871	-0.12657	0.261165	0.215618	0.649572	-0.07263	0.513215
i	27.6184	0.744038	-0.19237	0.000807	0.000667	-0.01272	0.014501	-0.01005
x	74.3457	1.490345	-0.31603	0.19944	0.164658	1.621936	-0.18135	1.281461
wsum	31.57207	0.596871	-0.12657	0.261165	0.215618	0.649572	-0.07263	1.513215
k	27.6184	0.744038	0.80763	0.000807	0.000667	-0.01272	0.014501	-0.01005
у	74.3457	1.490345	-0.31603	0.19944	0.164658	1.621936	-1.18135	1.281461

# **Example 35.2: Grunfeld's Model Estimated with SUR**

The following example was used by Zellner in his classic 1962 paper on seemingly unrelated regressions. Different stock prices often move in the same direction at a given point in time. The SUR technique might provide more efficient estimates than OLS in this situation.

The following statements read the data. (The prefix GE stands for General Electric and WH stands for Westinghouse.)

```
*-----* Seemingly Unrelated Technique-----*
| A. Zellner, "An Efficient Method of Estimating Seemingly
| Unrelated Regressions and Tests for Aggregation Bias,"
| JASA 57(1962) pp.348-364
| J.C.G. Boot, "Investment Demand: an Empirical Contribution |
| to the Aggregation Problem, " IER 1(1960) pp.3-30.
| Y. Grunfeld, "The Determinants of Corporate Investment,"
| Unpublished thesis, Chicago, 1958
data grunfeld;
  input year ge_i ge_f ge_c wh_i wh_f wh_c;
  label ge_i = 'Gross Investment, GE'
```

The following statements compute the SUR estimates for the Grunfeld model:

```
proc syslin data=grunfeld sur;
  ge:    model ge_i = ge_f ge_c;
  westing: model wh_i = wh_f wh_c;
run;
```

The PROC SYSLIN output is shown in Output 35.2.1 through Output 35.2.5.

### Output 35.2.1 PROC SYSLIN Output for SUR

# The SYSLIN Procedure Ordinary Least Squares Estimation

Model	GE
<b>Dependent Variable</b>	ge_i
Label	Gross Investment, GE

	Analysis of Variance				
Source	DF	Sum of Squares		F Value	Pr > F
Model	2	31632.03	15816.02	20.34	<.0001
Error	17	13216.59	777.4463		
Corrected Total	19	44848.62			

Root MSE	27.88272	R-Square	0.70531
Dependent Mean	102.29000	Adj R-Sq	0.67064
Coeff Var	27.25850		

Parameter Estimates							
	Parameter Standard Variable						
Variable	DF	Estimate	Error	t Value	Pr >  t	Label	
Intercept	1	-9.95631	31.37425	-0.32	0.7548	Intercept	
ge_f	1	0.026551	0.015566	1.71	0.1063	Value of Outstanding Shares Lagged, GE	
ge_c	1	0.151694	0.025704	5.90	<.0001	Capital Stock Lagged, GE	

### Output 35.2.2 PROC SYSLIN Output for SUR

### The SYSLIN Procedure **Ordinary Least Squares Estimation**

Model	WESTING
Dependent Variable	wh_i
Label	Gross Investment, WH

Analysis of Variance						
Sum of Mean						
Source	DF	Squares	Square	F Value	Pr > F	
Model	2	5165.553	2582.776	24.76	<.0001	
Error	17	1773.234	104.3079			
<b>Corrected Total</b>	19	6938.787				

10.21312 **R-Square** 0.74445 Root MSE **Dependent Mean** 42.89150 **Adj R-Sq** 0.71438 Coeff Var 23.81153

Parameter Estimates							
		Parameter	Standard			Variable	
Variable	DF	Estimate	Error	t Value	Pr >  t	Label	
Intercept	1	-0.50939	8.015289	-0.06	0.9501	Intercept	
wh_f	1	0.052894	0.015707	3.37	0.0037	Value of Outstanding Shares Lagged, WH	
wh_c	1	0.092406	0.056099	1.65	0.1179	Capital Stock Lagged, WH	

# Output 35.2.3 PROC SYSLIN Output for SUR

### The SYSLIN Procedure **Seemingly Unrelated Regression Estimation**

Cross Model Covariance				
	GE	WESTING		
GE	777.446	207.587		
WESTING	207.587	104.308		
Cross M	lodel Cor	relation		
	GE	WESTING		
GE	1.00000	0.72896		
WESTING	0.72896	1.00000		
0.000	Model In			
C	orrelatio	n		
	GE	WESTING		
GE	2.13397	-1.55559		
WESTING	-1.55559	2.13397		

Output 35.2.3 continued

Cross Model Inverse Covariance					
	GE	WESTING			
GE	0.002745	005463			
WESTING	005463	0.020458			

### Output 35.2.4 PROC SYSLIN Output for SUR

System Weighted MSE	0.9719
Degrees of freedom	34
System Weighted R-Square	0.6284

Model	GE
Dependent Variable	ge_i
Label	Gross Investment, GE

Parameter Estimates						
Parameter Standard Variable						
Variable	DF	Estimate	Error	t Value	Pr >  t	Label
Intercept	1	-27.7193	29.32122	-0.95	0.3577	Intercept
ge_f	1	0.038310	0.014415	2.66	0.0166	Value of Outstanding Shares Lagged, GE
ge_c	1	0.139036	0.024986	5.56	<.0001	Capital Stock Lagged, GE

### Output 35.2.5 PROC SYSLIN Output for SUR

Model	WESTING
<b>Dependent Variable</b>	wh_i
Label	Gross Investment, WH

Parameter Estimates						
		Parameter	Standard			Variable
Variable	DF	Estimate	Error	t Value	Pr >  t	Label
Intercept	1	-1.25199	7.545217	-0.17	0.8702	Intercept
wh_f	1	0.057630	0.014546	3.96	0.0010	Value of Outstanding Shares Lagged, WH
wh_c	1	0.063978	0.053041	1.21	0.2443	Capital Stock Lagged, WH

# **Example 35.3: Illustration of ODS Graphics**

This example illustrates the use of ODS graphics. This is a continuation of the section "Example 35.1: Klein's Model I Estimated with LIML and 3SLS" on page 2642. These graphical displays are requested by specifying the ODS GRAPHICS statement before running PROC SYSLIN. For information about the graphics available in the SYSLIN procedure, see the section "ODS Graphics" on page 2642.

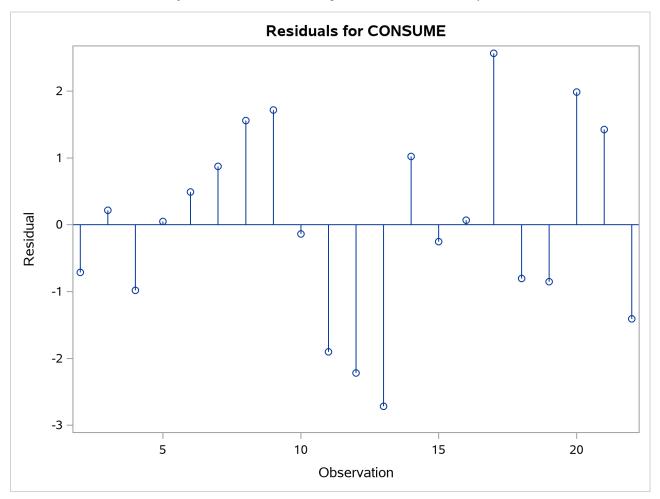
The following statements show how to generate ODS graphics plots with the SYSLIN procedure. The plots of residuals for each one of the equations in the model are displayed in Figure 35.3.1 through Figure 35.3.3.

... more lines ...

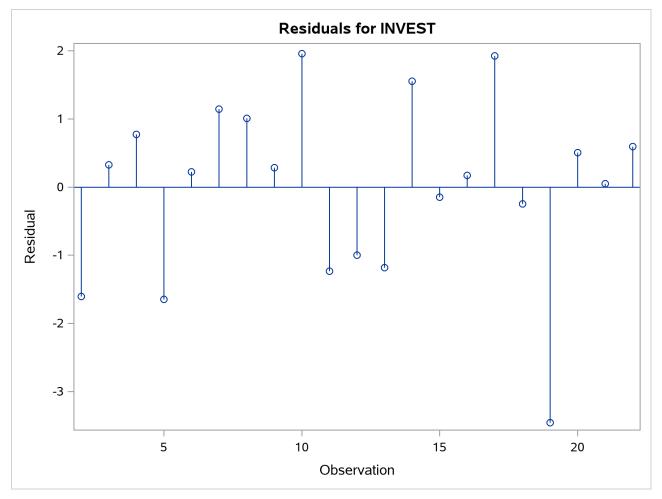
```
*-----*
| By L.R. Klein, Economic Fluctuations in the United States, 1921-1941 |
| (1950), NY: John Wiley. A macro-economic model of the U.S. with
| three behavioral equations, and several identities. See Theil, p.456.|
*-----*:
data klein;
input year c p w i x wp g t k wsum;
  date=mdy(1,1,year);
  format date monyy.;
  y = c+i+q-t;
  yr =year-1931;
  klag=lag(k);
  plag=lag(p);
  xlag=lag(x);
  label year='Year'
       date='Date'
       c = 'Consumption'
       р
         ='Profits'
         ='Private Wage Bill'
       w
       i ='Investment'
       k = 'Capital Stock'
          ='National Income'
       У
       x
         ='Private Production'
       wsum='Total Wage Bill'
       wp = 'Govt Wage Bill'
          ='Govt Demand'
       g
          ='Taxes'
       i
       klag='Capital Stock Lagged'
       plag='Profits Lagged'
       xlag='Private Product Lagged'
       yr = 'YEAR-1931';
datalines;
      . 12.7
                . . 44.9
                                        . 182.8
1920
1921 41.9 12.4 25.5 -0.2 45.6 2.7 3.9 7.7 182.6 28.2
1922 45.0 16.9 29.3 1.9 50.1 2.9 3.2 3.9 184.5 32.2
1923 49.2 18.4 34.1 5.2 57.2 2.9 2.8 4.7 189.7 37.0
1924 50.6 19.4 33.9 3.0 57.1 3.1 3.5 3.8 192.7 37.0
1925 52.6 20.1 35.4 5.1 61.0 3.2 3.3 5.5 197.8 38.6
1926 55.1 19.6 37.4 5.6 64.0 3.3 3.3 7.0 203.4 40.7
1927 56.2 19.8 37.9 4.2 64.4 3.6 4.0 6.7 207.6 41.5
1928 57.3 21.1 39.2 3.0 64.5 3.7 4.2 4.2 210.6 42.9
1929 57.8 21.7 41.3 5.1 67.0 4.0 4.1 4.0 215.7 45.3
1930 55.0 15.6 37.9 1.0 61.2 4.2 5.2 7.7 216.7 42.1
```

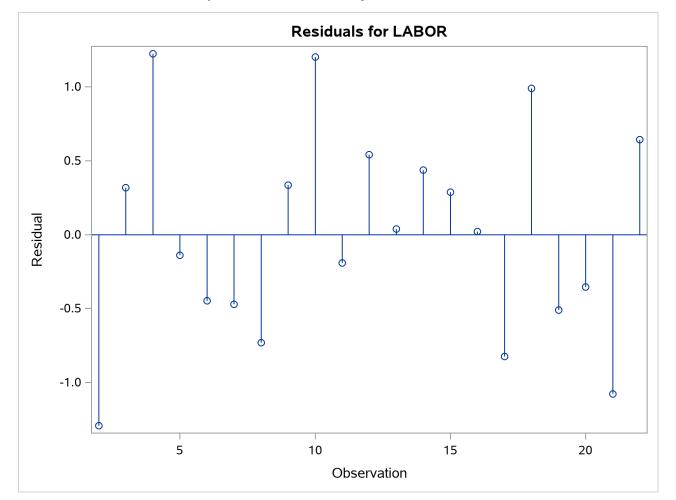
```
ods graphics on;
proc syslin data=klein outest=b liml plots(unpack only)=residual;
  endogenous c p w i x wsum k y;
  instruments klag plag xlag wp g t yr;
  consume: model c = p plag wsum;
  invest: model i = p plag klag;
  labor: model w = x xlag yr;
run;
```

Output 35.3.1 Residuals Diagnostic Plots for Consumption



Output 35.3.2 Residuals Diagnostic Plots for Investments





Output 35.3.3 Residuals Diagnostic Plots for Labor

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