SAS/ETS® 14.1 User's Guide

The SYSLIN Procedure

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Chapter 36
The SYSLIN Procedure

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Overview: SYSLIN Procedure

The SYSLIN procedure estimates parameters in an interdependent system of linear regression equations.

Ordinary least squares (OLS) estimates are biased and inconsistent when current period endogenous variables appear as regressors in other equations in the system. The errors of a set of related regression equations are often correlated, and the efficiency of the estimates can be improved by taking these correlations into account. The SYSLIN procedure provides several techniques that produce consistent and asymptotically efficient estimates for systems of regression equations.

The SYSLIN procedure provides the following estimation methods:

- ordinary least squares (OLS)
- two-stage least squares (2SLS)
- limited information maximum likelihood (LIML)
- K-class
- seemingly unrelated regressions (SUR)
- iterated seemingly unrelated regressions (ITSUR)
- three-stage least squares (3SLS)
- iterated three-stage least squares (IT3SLS)
- full information maximum likelihood (FIML)
- minimum expected loss (MELO)

Other features of the SYSLIN procedure enable you to:

- impose linear restrictions on the parameter estimates

- test linear hypotheses about the parameters
- write predicted and residual values to an output SAS data set
- write parameter estimates to an output SAS data set
- write the crossproducts matrix (SSCP) to an output SAS data set
- use raw data, correlations, covariances, or cross products as input

Getting Started: SYSLIN Procedure

This section introduces the use of the SYSLIN procedure. The problem of dependent regressors is introduced using a supply and demand example. This section explains the terminology used for variables in a system of regression equations and introduces the SYSLIN procedure statements for declaring the roles the variables play. The syntax used for the different estimation methods and the output produced is shown.

An Example Model

In simultaneous systems of equations, endogenous variables are determined jointly rather than sequentially. Consider the following supply and demand functions for some product:

$$Q_D = a_1 + b_1 P + c_1 Y + d_1 S + \epsilon_1 (\text{demand})$$

$$Q_S = a_2 + b_2 P + c_2 U + \epsilon_2 (\text{supply})$$

$$Q = Q_D = Q_S (\text{market equilibrium})$$

The variables in this system are as follows:

Q_D	quantity demanded
Q_S	quantity supplied
Q	the observed quantity sold, which equates quantity supplied and quantity demanded in equilibrium
P	price per unit
Y	income
S	price of substitutes
U	unit cost
ϵ_1	the random error term for the demand equation
ϵ_2	the random error term for the supply equation

In this system, quantity demanded depends on price, income, and the price of substitutes. Consumers normally purchase more of a product when prices are lower and when income and the price of substitute goods are higher. Quantity supplied depends on price and the unit cost of production. Producers supply more when price is high and when unit cost is low. The actual price and quantity sold are determined jointly by the values that equate demand and supply.

Since price and quantity are jointly endogenous variables, both structural equations are necessary to adequately describe the observed values. A critical assumption of OLS is that the regressors are uncorrelated with the residual. When current endogenous variables appear as regressors in other equations (endogenous variables depend on each other), this assumption is violated and the OLS parameter estimates are biased and inconsistent. The bias caused by the violated assumptions is called *simultaneous equation bias.* Neither the demand nor supply equation can be estimated consistently by OLS.

Variables in a System of Equations

Before explaining how to use the SYSLIN procedure, it is useful to define some terms. The variables in a system of equations can be classified as follows:

- *Endogenous variables,* which are also called *jointly dependent* or *response variables,* are the variables determined by the system. Endogenous variables can also appear on the right-hand side of equations.
- *Exogenous variables* are independent variables that do not depend on any of the endogenous variables in the system.
- *Predetermined variables* include both the exogenous variables and *lagged endogenous variables,* which are past values of endogenous variables determined at previous time periods. PROC SYSLIN does not compute lagged values; any lagged endogenous variables must be computed in a preceding DATA step.
- *Instrumental variables* are predetermined variables used in obtaining predicted values for the current period endogenous variables by a first-stage regression. The use of instrumental variables characterizes estimation methods such as two-stage least squares and three-stage least squares. Instrumental variables estimation methods substitute these first-stage predicted values for endogenous variables when they appear as regressors in model equations.

Using PROC SYSLIN

First specify the input data set and estimation method in the PROC SYSLIN statement. If any model uses dependent regressors, and you are using an instrumental variables regression method, declare the dependent regressors with an ENDOGENOUS statement and declare the instruments with an INSTRUMENTS statement. Next, use MODEL statements to specify the structural equations of the system.

The use of different estimation methods is shown by the following examples. These examples use the simulated dataset WORK.IN given below.

```
data in;
   label q = "Quantity"
         p = "Price"
         s = "Price of Substitutes"
         y = "Income"
         u = "Unit Cost";
   drop i e1 e2;
   p = 0; q = 0;
   do i = 1 to 60;
      y = 1 + .05*i  + .15*rannor(123);
      u = 2          + .05*rannor(123) + .05*rannor(123);
      s = 4 - .001*(i-10)*(i-110) + .5*rannor(123);
      e1 = .15 * rannor(123);
      e2 = .15 * rannor(123);
      demandx = 1 + .3 * y + .35 * s + e1;
      supplyx = -1 - 1 * u + e2 - .4*e1;
      q = 1.4/2.15 * demandx + .75/2.15 * supplyx;
      p = ( - q + supplyx ) / -1.4;
      output;
   end;
run;
```

OLS Estimation

PROC SYSLIN performs OLS regression if you do not specify a method of estimation in the PROC SYSLIN statement. OLS does not use instruments, so the ENDOGENOUS and INSTRUMENTS statements can be omitted.

The following statements estimate the supply and demand model shown previously:

```
proc syslin data=in;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The PROC SYSLIN output for the demand equation is shown in Figure 36.1, and the output for the supply equation is shown in Figure 36.2.

Figure 36.1 OLS Results for Demand Equation

The SYSLIN Procedure
Ordinary Least Squares Estimation

Model	DEMAND
Dependent Variable	q
Label	Quantity

Figure 36.1 *continued*

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	9.587901	3.195967	398.31	<.0001
Error	56	0.449338	0.008024		
Corrected Total	59	10.03724			

Root MSE	0.08958	R-Square	0.95523
Dependent Mean	1.30095	Adj R-Sq	0.95283
Coeff Var	6.88542		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.47677	0.210239	-2.27	0.0272	Intercept
p	1	0.123326	0.105177	1.17	0.2459	Price
y	1	0.201282	0.032403	6.21	<.0001	Income
s	1	0.167258	0.024091	6.94	<.0001	Price of Substitutes

Figure 36.2 OLS Results for Supply Equation

The SYSLIN Procedure
Ordinary Least Squares Estimation

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	9.033902	4.516951	256.61	<.0001
Error	57	1.003337	0.017602		
Corrected Total	59	10.03724			

Root MSE	0.13267	R-Square	0.90004
Dependent Mean	1.30095	Adj R-Sq	0.89653
Coeff Var	10.19821		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.30389	0.471397	-0.64	0.5217	Intercept
p	1	1.218743	0.053914	22.61	<.0001	Price
u	1	-1.07757	0.234150	-4.60	<.0001	Unit Cost

For each MODEL statement, the output first shows the model label and dependent variable name and label. This is followed by an analysis-of-variance table for the model, which shows the model, error, and total mean squares, and an F test for the no-regression hypothesis. Next, the procedure prints the root mean squared error, dependent variable mean and coefficient of variation, and the R^2 and adjusted R^2 statistics.

Finally, the table of parameter estimates shows the estimated regression coefficients, standard errors, and *t* tests. You would expect the price coefficient in a demand equation to be negative. However, note that the OLS estimate of the price coefficient P in the demand equation (0.1233) has a positive sign. This could be caused by simultaneous equation bias.

Two-Stage Least Squares Estimation

In the supply and demand model, P is an endogenous variable, and consequently the OLS estimates are biased. The following example estimates this model using two-stage least squares.

```
proc syslin data=in 2sls;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The 2SLS option in the PROC SYSLIN statement specifies the two-stage least squares method. The ENDOGENOUS statement specifies that P is an endogenous regressor for which first-stage predicted values are substituted. You need to declare an endogenous variable in the ENDOGENOUS statement only if it is used as a regressor; thus although Q is endogenous in this model, it is not necessary to list it in the ENDOGENOUS statement.

Usually, all predetermined variables that appear in the system are used as instruments. The INSTRUMENTS statement specifies that the exogenous variables Y, U, and S are used as instruments for the first-stage regression to predict P.

The 2SLS results are shown in Figure 36.3 and Figure 36.4. The first-stage regressions are not shown. To see the first-stage regression results, use the FIRST option in the PROC SYSLIN statement.

Figure 36.3 2SLS Results for Demand Equation

The SYSLIN Procedure
Two-Stage Least Squares Estimation

Model	DEMAND
Dependent Variable	q
Label	Quantity

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	9.670892	3.223631	115.58	<.0001
Error	56	1.561956	0.027892		
Corrected Total	59	10.03724			

Root MSE	0.16701	**R-Square**	0.86095
Dependent Mean	1.30095	**Adj R-Sq**	0.85350
Coeff Var	12.83744		

Figure 36.3 *continued*

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.901048	1.171231	1.62	0.1102	Intercept
p	1	-1.11519	0.607395	-1.84	0.0717	Price
y	1	0.419546	0.117955	3.56	0.0008	Income
s	1	0.331475	0.088472	3.75	0.0004	Price of Substitutes

Figure 36.4 2SLS Results for Supply Equation

The SYSLIN Procedure
Two-Stage Least Squares Estimation

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	9.646109	4.823054	253.96	<.0001
Error	57	1.082503	0.018991		
Corrected Total	59	10.03724			

Root MSE	0.13781	R-Square	0.89910
Dependent Mean	1.30095	Adj R-Sq	0.89556
Coeff Var	10.59291		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.51878	0.490999	-1.06	0.2952	Intercept
p	1	1.333080	0.059271	22.49	<.0001	Price
u	1	-1.14623	0.243491	-4.71	<.0001	Unit Cost

The 2SLS output is similar in form to the OLS output. However, the 2SLS results are based on predicted values for the endogenous regressors from the first stage instrumental regressions. This makes the analysis-of-variance table and the R^2 statistics difficult to interpret. See the sections "ANOVA Table for Instrumental Variables Methods" on page 2581 and "The R-Square Statistics" on page 2581 for details.

Note that, unlike the OLS results, the 2SLS estimate for the P coefficient in the demand equation (–1.115) is negative.

LIML, K-Class, and MELO Estimation

To obtain limited information maximum likelihood, general K-class, or minimum expected loss estimates, use the ENDOGENOUS, INSTRUMENTS, and MODEL statements as in the 2SLS case but specify the

LIML, K=, or MELO option instead of 2SLS in the PROC SYSLIN statement. The following statements show this for K-class estimation.

```
proc syslin data=in k=.5;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

For more information about these estimation methods, see the section "Estimation Methods" on page 2578 and consult econometrics textbooks.

SUR, 3SLS, and FIML Estimation

In a multivariate regression model, the errors in different equations might be correlated. In this case, the efficiency of the estimation might be improved by taking these cross-equation correlations into account.

Seemingly Unrelated Regression

Seemingly unrelated regression (SUR), also called joint generalized least squares (JGLS) or Zellner estimation, is a generalization of OLS for multi-equation systems. Like OLS, the SUR method assumes that all the regressors are independent variables, but SUR uses the correlations among the errors in different equations to improve the regression estimates. The SUR method requires an initial OLS regression to compute residuals. The OLS residuals are used to estimate the cross-equation covariance matrix.

The SUR option in the PROC SYSLIN statement specifies seemingly unrelated regression, as shown in the following statements:

```
proc syslin data=in sur;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

INSTRUMENTS and ENDOGENOUS statements are not needed for SUR, because the SUR method assumes there are no endogenous regressors. For SUR to be effective, the models must use different regressors. SUR produces the same results as OLS unless the model contains at least one regressor not used in the other equations.

Three-Stage Least Squares

The three-stage least squares method generalizes the two-stage least squares method to take into account the correlations between equations in the same way that SUR generalizes OLS. Three-stage least squares requires three steps: first-stage regressions to get predicted values for the endogenous regressors; a two-stage least squares step to get residuals to estimate the cross-equation correlation matrix; and the final 3SLS estimation step.

The 3SLS option in the PROC SYSLIN statement specifies the three-stage least squares method, as shown in the following statements.

```
proc syslin data=in 3sls;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The 3SLS output begins with a two-stage least squares regression to estimate the cross-model correlation matrix. This output is the same as the 2SLS results shown in Figure 36.3 and Figure 36.4, and is not repeated here. The next part of the 3SLS output prints the cross-model correlation matrix computed from the 2SLS residuals. This output is shown in Figure 36.5 and includes the cross-model covariances, correlations, the inverse of the correlation matrix, and the inverse covariance matrix.

Figure 36.5 Estimated Cross-Model Covariances Used for 3SLS Estimates

The SYSLIN Procedure
Three-Stage Least Squares Estimation

Cross Model Covariance	DEMAND	SUPPLY
DEMAND	0.027892	-.011283
SUPPLY	-.011283	0.018991

Cross Model Correlation	DEMAND	SUPPLY
DEMAND	1.00000	-0.49022
SUPPLY	-0.49022	1.00000

Cross Model Inverse Correlation	DEMAND	SUPPLY
DEMAND	1.31634	0.64530
SUPPLY	0.64530	1.31634

Cross Model Inverse Covariance	DEMAND	SUPPLY
DEMAND	47.1941	28.0379
SUPPLY	28.0379	69.3130

The final 3SLS estimates are shown in Figure 36.6.

Figure 36.6 Three-Stage Least Squares Results

System Weighted MSE	0.5711
Degrees of freedom	113
System Weighted R-Square	0.9627

Model	DEMAND
Dependent Variable	q
Label	Quantity

Figure 36.6 *continued*

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.980269	1.169176	1.69	0.0959	Intercept
p	1	-1.17654	0.605015	-1.94	0.0568	Price
y	1	0.404117	0.117179	3.45	0.0011	Income
s	1	0.359204	0.085077	4.22	<.0001	Price of Substitutes

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.51878	0.490999	-1.06	0.2952	Intercept
p	1	1.333080	0.059271	22.49	<.0001	Price
u	1	-1.14623	0.243491	-4.71	<.0001	Unit Cost

This output first prints the system weighted mean squared error and system weighted R^2 statistics. The system weighted MSE and system weighted R^2 measure the fit of the joint model obtained by stacking all the models together and performing a single regression with the stacked observations weighted by the inverse of the model error variances. See the section "The R-Square Statistics" on page 2581 for details.

Next, the table of 3SLS parameter estimates for each model is printed. This output has the same form as for the other estimation methods.

Note that, in some cases, the 3SLS and 2SLS results can be the same. Such a case could arise because of the same principle that causes OLS and SUR results to be identical, unless an equation includes a regressor not used in the other equations of the system. However, the application of this principle is more complex when instrumental variables are used. When all the exogenous variables are used as instruments, linear combinations of all the exogenous variables appear in the third-stage regressions through substitution of first-stage predicted values.

In this example, 3SLS produces different (and, it is hoped, more efficient) estimates for the demand equation. However, the 3SLS and 2SLS results for the supply equation are the same. This is because the supply equation has one endogenous regressor and one exogenous regressor not used in other equations. In contrast, the demand equation has fewer endogenous regressors than exogenous regressors not used in other equations in the system.

Full Information Maximum Likelihood

The FIML option in the PROC SYSLIN statement specifies the full information maximum likelihood method, as shown in the following statements.

```
proc syslin data=in fiml;
   endogenous  p q;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The FIML results are shown in Figure 36.7.

Figure 36.7 FIML Results

The SYSLIN Procedure
Full-Information Maximum Likelihood Estimation

NOTE: Convergence criterion met at iteration 3.

Model	DEMAND
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.988538	1.233632	1.61	0.1126	Intercept
p	1	-1.18148	0.652278	-1.81	0.0755	Price
y	1	0.402312	0.107270	3.75	0.0004	Income
s	1	0.361345	0.103817	3.48	0.0010	Price of Substitutes

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.52443	0.479522	-1.09	0.2787	Intercept
p	1	1.336083	0.057939	23.06	<.0001	Price
u	1	-1.14804	0.237793	-4.83	<.0001	Unit Cost

Computing Reduced Form Estimates

A system of structural equations with endogenous regressors can be represented as functions of only the predetermined variables. For this to be possible, there must be as many equations as endogenous variables. If there are more endogenous variables than regression models, you can use IDENTITY statements to complete the system. See the section "Reduced Form Estimates" on page 2583 for details.

The REDUCED option in the PROC SYSLIN statement prints reduced form estimates. The following statements show this by using the 3SLS estimates of the structural parameters.

```
proc syslin data=in 3sls reduced;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
run;
```

The first four pages of this output were as shown previously and are not repeated here. (See Figure 36.3, Figure 36.4, Figure 36.5, and Figure 36.6.) The final page of the output from this example contains the reduced form coefficients from the 3SLS structural estimates, as shown in Figure 36.8.

Figure 36.8 Reduced Form 3SLS Results

The SYSLIN Procedure
Three-Stage Least Squares Estimation

Endogenous Variables	p	q
DEMAND	1.176543	1
SUPPLY	-1.33308	1

Exogenous Variables	Intercept	y	s	u
DEMAND	1.980269	0.404117	0.359204	0
SUPPLY	-0.51878	0	0	-1.14623

Inverse Endogenous Variables	DEMAND	SUPPLY
p	0.398466	-0.39847
q	0.531187	0.468813

Reduced Form	Intercept	y	s	u
p	0.995788	0.161027	0.143131	0.456735
q	0.808682	0.214662	0.190804	-0.53737

Restricting Parameter Estimates

You can impose restrictions on the parameter estimates with RESTRICT and SRESTRICT statements. The RESTRICT statement imposes linear restrictions on parameters in the equation specified by the preceding MODEL statement. The SRESTRICT statement imposes linear restrictions that relate parameters in different models.

To impose restrictions involving parameters in different equations, use the SRESTRICT statement. Specify the parameters in the linear hypothesis as *model-label.regressor-name*. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.)

Tests for the significance of the restrictions are printed when RESTRICT or SRESTRICT statements are used. You can label RESTRICT and SRESTRICT statements to identify the restrictions in the output.

The RESTRICT statement in the following example restricts the price coefficient in the demand equation to equal 0.015. The SRESTRICT statement restricts the estimate of the income coefficient in the demand equation to be 0.01 times the estimate of the unit cost coefficient in the supply equation.

```
proc syslin data=in 3sls;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   peq015: restrict p = .015;
   supply: model q = p u;
   yeq01u: srestrict demand.y = .01 * supply.u;
run;
```

The restricted estimation results are shown in Figure 36.9.

Figure 36.9 Restricted Estimates

The SYSLIN Procedure
Three-Stage Least Squares Estimation

Model	DEMAND
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.46584	0.053307	-8.74	<.0001	Intercept
p	1	0.015000	0	.	.	Price
y	1	-0.00679	0.002357	-2.88	0.0056	Income
s	1	0.325589	0.009872	32.98	<.0001	Price of Substitutes
RESTRICT	-1	50.59353	7.464988	6.78	<.0001	PEQ015

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-1.31894	0.477633	-2.76	0.0077	Intercept
p	1	1.291718	0.059101	21.86	<.0001	Price
u	1	-0.67887	0.235679	-2.88	0.0056	Unit Cost

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
RESTRICT	-1	342.3605	38.12094	8.98	<.0001	YEQ01U

The standard error for P in the demand equation is 0, since the value of the P coefficient was specified by the RESTRICT statement and not estimated from the data. The "Parameter Estimates" table for the demand equation contains an additional row for the restriction specified by the RESTRICT statement. The parameter estimate for the restriction is the value of the Lagrange multiplier used to impose the restriction. The restriction is highly significant ($t = 6.777$), which means that the data are not consistent with the restriction, and the model does not fit as well with the restriction imposed. See the section "RESTRICT Statement" on page 2573 for details.

Following the "Parameter Estimates" table for the supply equation, the results for the cross model restrictions are printed. This shows that the restriction specified by the SRESTRICT statement is not consistent with the data ($t = 8.98$). See the section "SRESTRICT Statement" on page 2574 for details.

Testing Parameters

You can test linear hypotheses about the model parameters with TEST and STEST statements. The TEST statement tests hypotheses about parameters in the equation specified by the preceding MODEL statement. The STEST statement tests hypotheses that relate parameters in different models.

For example, the following statements test the hypothesis that the price coefficient in the demand equation is equal to 0.015.

```
proc syslin data=in 3sls;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   test_1: test p = .015;
   supply: model q = p u;
run;
```

The TEST statement results are shown in Figure 36.10. This reports an F test for the hypothesis specified by the TEST statement. In this case, the F statistic is 6.79 (3.879/.571) with 1 and 113 degrees of freedom. The p value for this F statistic is 0.0104, which indicates that the hypothesis tested is almost but not quite rejected at the 0.01 level. See the section "TEST Statement" on page 2576 for details.

Figure 36.10 TEST Statement Results

The SYSLIN Procedure
Three-Stage Least Squares Estimation

System Weighted MSE	0.5711
Degrees of freedom	113
System Weighted R-Square	0.9627

Model	DEMAND
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.980269	1.169176	1.69	0.0959	Intercept
p	1	-1.17654	0.605015	-1.94	0.0568	Price
y	1	0.404117	0.117179	3.45	0.0011	Income
s	1	0.359204	0.085077	4.22	<.0001	Price of Substitutes

Test Results

Num DF	Den DF	F Value	Pr > F	Label
1	113	6.79	0.0104	TEST_1

To test hypotheses that involve parameters in different equations, use the STEST statement. Specify the parameters in the linear hypothesis as *model-label.regressor-name.* (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.)

For example, the following statements test the hypothesis that the income coefficient in the demand equation is 0.01 times the unit cost coefficient in the supply equation:

```
proc syslin data=in 3sls;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   supply: model q = p u;
   stest1: stest demand.y = .01 * supply.u;
run;
```

The STEST statement results are shown in Figure 36.11. The form and interpretation of the STEST statement results are like the TEST statement results. In this case, the F test produces a p value less than 0.0001, and strongly rejects the hypothesis tested. See the section "STEST Statement" on page 2575 for details.

Figure 36.11 STEST Statement Results

The SYSLIN Procedure
Three-Stage Least Squares Estimation

System Weighted MSE	0.5711
Degrees of freedom	113
System Weighted R-Square	0.9627

Model	DEMAND
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.980269	1.169176	1.69	0.0959	Intercept
p	1	-1.17654	0.605015	-1.94	0.0568	Price
y	1	0.404117	0.117179	3.45	0.0011	Income
s	1	0.359204	0.085077	4.22	<.0001	Price of Substitutes

Model	SUPPLY
Dependent Variable	q
Label	Quantity

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.51878	0.490999	-1.06	0.2952	Intercept
p	1	1.333080	0.059271	22.49	<.0001	Price
u	1	-1.14623	0.243491	-4.71	<.0001	Unit Cost

Figure 36.11 *continued*

Test Results				
Num DF	Den DF	F Value	Pr > F	Label
1	113	22.46	0.0001	STEST1

You can combine TEST and STEST statements with RESTRICT and SRESTRICT statements to perform hypothesis tests for restricted models. Of course, the validity of the TEST and STEST statement results depends on the correctness of any restrictions you impose on the estimates.

Saving Residuals and Predicted Values

You can store predicted values and residuals from the estimated models in a SAS data set. Specify the OUT= option in the PROC SYSLIN statement and use the OUTPUT statement to specify names for new variables to contain the predicted and residual values.

For example, the following statements store the predicted quantity from the supply and demand equations in a data set PRED:

```
proc syslin data=in out=pred 3sls;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   output predicted=q_demand;
   supply: model q = p u;
   output predicted=q_supply;
run;
```

Plotting Residuals

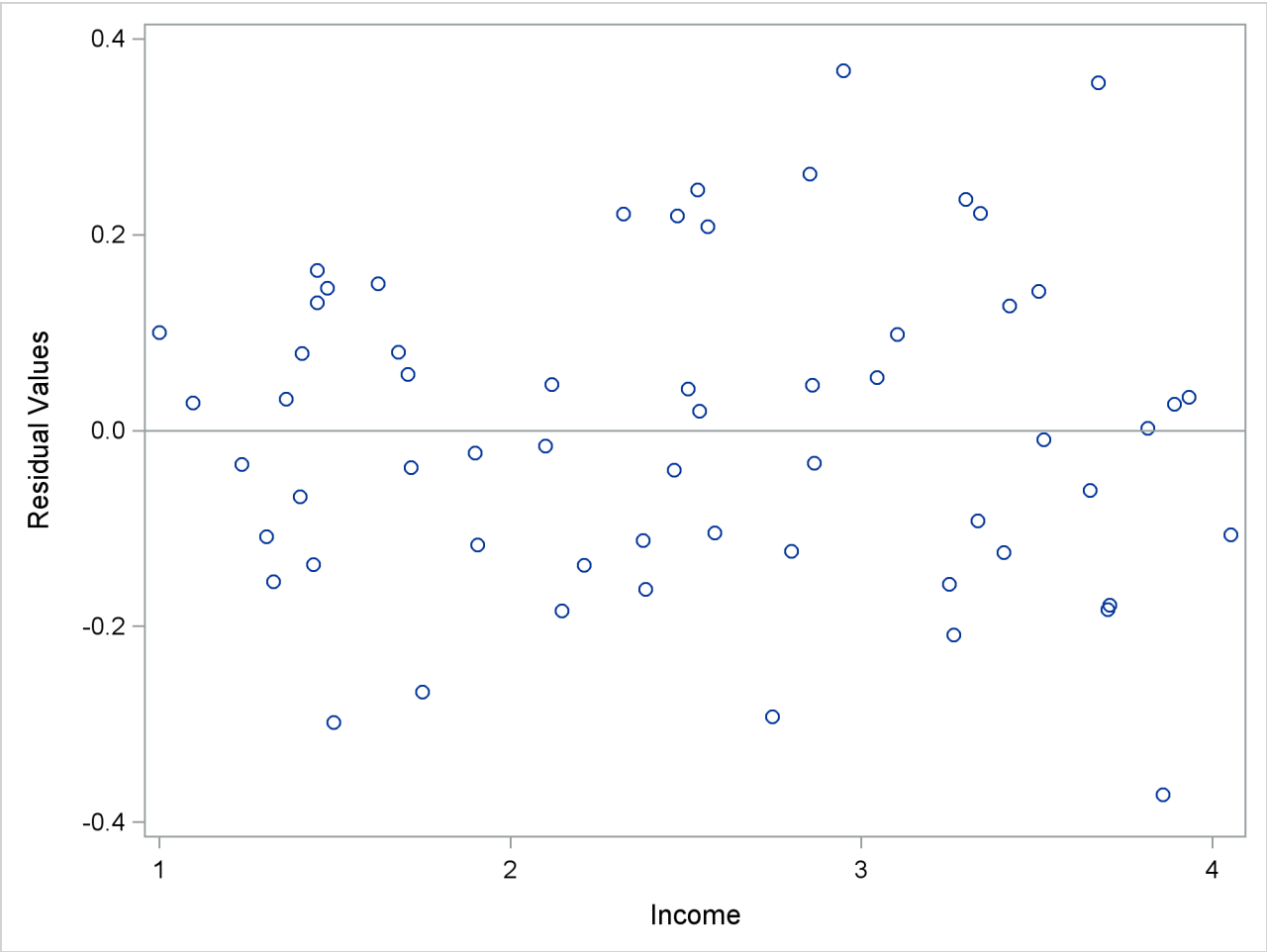
You can plot the residuals against the regressors by using the PROC SGPLOT. For example, the following statements plot the 2SLS residuals for the demand model against price, income, and price of substitutes.

```
proc syslin data=in 2sls out=out;
   endogenous  p;
   instruments y u s;
   demand: model q = p y s;
   output residual=residual_q;
run;

proc sgplot data=out;
   scatter x=p y=residual_q;
   refline 0 / axis=y;
run;

proc sgplot data=out;
   scatter x=y y=residual_q;
   refline 0 / axis=y;
run;
```

```
proc sgplot data=out;
   scatter x=s y=residual_q;
   refline 0 / axis=y;
run;
```

The plot for income is shown in Figure 36.12. The other plots are not shown.

Figure 36.12 Plot of Residuals against Income



Syntax: SYSLIN Procedure

The SYSLIN procedure uses the following statements:

```
PROC SYSLIN options ;
    BY variables ;
    ENDOGENOUS variables ;
    IDENTITY identities ;
    INSTRUMENTS variables ;
    MODEL response = regressors / options ;
    OUTPUT PREDICTED= variable RESIDUAL= variable ;
    RESTRICT restrictions ;
    SRESTRICT restrictions ;
    STEST equations ;
    TEST equations ;
    VAR variables ;
    WEIGHT variable ;
```

Functional Summary

The SYSLIN procedure statements and options are summarized in the following table.

Description	Statement	Option
Data Set Options		
specify the input data set	PROC SYSLIN	DATA=
specify the output data set	PROC SYSLIN	OUT=
write parameter estimates to an output data set	PROC SYSLIN	OUTEST=
write covariances to the OUTEST= data set	PROC SYSLIN	OUTCOV OUTCOV3
write the SSCP matrix to an output data set	PROC SYSLIN	OUTSSCP=
Estimation Method Options		
specify full information maximum likelihood estimation	PROC SYSLIN	FIML
specify iterative SUR estimation	PROC SYSLIN	ITSUR
specify iterative 3SLS estimation	PROC SYSLIN	IT3SLS
specify K-class estimation	PROC SYSLIN	K=
specify limited information maximum likelihood estimation	PROC SYSLIN	LIML
specify minimum expected loss estimation	PROC SYSLIN	MELO
specify ordinary least squares estimation	PROC SYSLIN	OLS
specify seemingly unrelated estimation	PROC SYSLIN	SUR
specify two-stage least squares estimation	PROC SYSLIN	2SLS
specify three-stage least squares estimation	PROC SYSLIN	3SLS

Description	Statement	Option
specify Fuller's modification to LIML	PROC SYSLIN	ALPHA=
specify convergence criterion	PROC SYSLIN	CONVERGE=
specify maximum number of iterations	PROC SYSLIN	MAXIT=
use diagonal of **S** instead of **S**	PROC SYSLIN	SDIAG
exclude RESTRICT statements in final stage	PROC SYSLIN	NOINCLUDE
specify criterion for testing for singularity	PROC SYSLIN	SINGULAR=
specify denominator for variance estimates	PROC SYSLIN	VARDEF=
Printing Control Options		
print all results	PROC SYSLIN	ALL
print first-stage regression statistics	PROC SYSLIN	FIRST
print estimates and SSE at each iteration	PROC SYSLIN	ITPRINT
print the reduced form estimates	PROC SYSLIN	REDUCED
print descriptive statistics	PROC SYSLIN	SIMPLE
print uncorrected SSCP matrix	PROC SYSLIN	USSCP
print correlations of the parameter estimates	MODEL	CORRB
print covariances of the parameter estimates	MODEL	COVB
print Durbin-Watson statistics	MODEL	DW
print Basmann's test	MODEL	OVERID
plot residual values against regressors	MODEL	PLOT
print standardized parameter estimates	MODEL	STB
print unrestricted parameter estimates	MODEL	UNREST
print the model crossproducts matrix	MODEL	XPX
print the inverse of the crossproducts matrix	MODEL	I
suppress printed output	MODEL	NOPRINT
suppress all printed output	PROC SYSLIN	NOPRINT
Model Specification		
specify structural equations	MODEL	
suppress the intercept parameter	MODEL	NOINT
specify linear relationship among variables	IDENTITY	
perform weighted regression	WEIGHT	
Tests and Restrictions on Parameters		
place restrictions on parameter estimates	RESTRICT	
place restrictions on parameter estimates	SRESTRICT	
test linear hypothesis	STEST	
test linear hypothesis	TEST	
Other Statements		
specify BY-group processing	BY	

Description	Statement	Option
specify the endogenous variables	ENDOGENOUS	
specify instrumental variables	INSTRUMENTS	
write predicted and residual values to a data set	OUTPUT	
name variable for predicted values	OUTPUT	PREDICTED=
name variable for residual values	OUTPUT	RESIDUAL=
include additional variables in $X'X$ matrix	VAR	

PROC SYSLIN Statement

PROC SYSLIN *options* **;**

The following options can be used with the PROC SYSLIN statement.

Data Set Options

DATA=*SAS-data-set*

specifies the input data set. If the DATA= option is omitted, the most recently created SAS data set is used. In addition to ordinary SAS data sets, PROC SYSLIN can analyze data sets of TYPE=CORR, TYPE=COV, TYPE=UCORR, TYPE=UCOV, and TYPE=SSCP. See the section "Special TYPE= Input Data Sets" on page 2578 for details.

OUT=*SAS-data-set*

specifies an output SAS data set for residuals and predicted values. The OUT= option is used in conjunction with the OUTPUT statement. See the section "OUT= Data Set" on page 2585 for details.

OUTEST=*SAS-data-set*

writes the parameter estimates to an output data set. See the section "OUTEST= Data Set" on page 2585 for details.

OUTCOV

COVOUT

writes the covariance matrix of the parameter estimates to the OUTEST= data set in addition to the parameter estimates.

OUTCOV3

COV3OUT

writes covariance matrices for each model in a system to the OUTEST= data set when the 3SLS, SUR, or FIML option is used.

OUTSSCP=*SAS-data-set*

writes the sum-of-squares-and-crossproducts matrix to an output data set. See the section "OUTSSCP= Data Set" on page 2586 for details.

Estimation Method Options

2SLS
specifies the two-stage least squares estimation method.

3SLS
specifies the three-stage least squares estimation method.

ALPHA=*value*
specifies Fuller's modification to the LIML estimation method. See the section "Fuller's Modification to LIML" on page 2584 for details.

CONVERGE=*value*
specifies the convergence criterion for the iterative estimation methods IT3SLS, ITSUR, and FIML. The default is CONVERGE=0.0001.

FIML
specifies the full information maximum likelihood estimation method.

ITSUR
specifies the iterative seemingly unrelated estimation method.

IT3SLS
specifies the iterative three-stage least squares estimation method.

K=*value*
specifies the K-class estimation method.

LIML
specifies the limited information maximum likelihood estimation method.

MAXITER=*n*
specifies the maximum number of iterations allowed for the IT3SLS, ITSUR, and FIML estimation methods. The MAXITER= option can be abbreviated as MAXIT=. The default is MAXITER=30.

MELO
specifies the minimum expected loss estimation method.

NOINCLUDE
excludes the RESTRICT statements from the final stage for the 3SLS, IT3SLS, SUR, and ITSUR estimation methods.

OLS
specifies the ordinary least squares estimation method. This is the default.

SDIAG
uses the diagonal of **S** instead of **S** to do the estimation, where **S** is the covariance matrix of equation errors. See the section "Uncorrelated Errors across Equations" on page 2584 for details.

SINGULAR=*value*
specifies a criterion for testing singularity of the crossproducts matrix. This is a tuning parameter used to make PROC SYSLIN more or less sensitive to singularities. The value must be between 0 and 1. The default is SINGULAR=1E–8.

SUR
specifies the seemingly unrelated estimation method.

Printing Control Options

ALL
specifies the CORRB, COVB, DW, I, OVERID, PLOT, STB, and XPX options for every MODEL statement.

FIRST
prints first-stage regression statistics for the endogenous variables regressed on the instruments. This output includes sums of squares, estimates, variances, and standard deviations.

ITPRINT
prints parameter estimates, system-weighted residual sum of squares, and R^2 at each iteration for the IT3SLS and ITSUR estimation methods. For the FIML method, the ITPRINT option prints parameter estimates, negative of log-likelihood function, and norm of gradient vector at each iteration.

NOPRINT
suppresses all printed output. Specifying NOPRINT in the PROC SYSLIN statement is equivalent to specifying NOPRINT in every MODEL statement.

REDUCED
prints the reduced form estimates. If the REDUCED option is specified, you should specify any IDENTITY statements needed to make the system square. See the section "Reduced Form Estimates" on page 2583 for details.

SIMPLE
prints descriptive statistics for the dependent variables. The statistics printed include the sum, mean, uncorrected sum of squares, variance, and standard deviation.

USSCP
prints the uncorrected sum-of-squares-and-crossproducts matrix.

USSCP2
prints the uncorrected sum-of-squares-and-crossproducts matrix for all variables used in the analysis, including predicted values of variables generated by the procedure.

VARDEF=DF | N | WEIGHT | WGT
specifies the denominator to use in calculating cross-equation error covariances and parameter standard errors and covariances. The default is VARDEF=DF, which corrects for model degrees of freedom. VARDEF=N specifies no degrees-of-freedom correction. VARDEF=WEIGHT specifies the sum of the observation weights. VARDEF=WGT specifies the sum of the observation weights minus the model degrees of freedom. See the section "Computation of Standard Errors" on page 2583 for details.

BY Statement

BY *variables* **;**

A BY statement can be used with PROC SYSLIN to obtain separate analyses on observations in groups defined by the BY variables.

ENDOGENOUS Statement

ENDOGENOUS *variables* **;**

The ENDOGENOUS statement declares the jointly dependent variables that are projected in the first-stage regression through the instrument variables. The ENDOGENOUS statement is not needed for the SUR, ITSUR, or OLS estimation methods. The default ENDOGENOUS list consists of all the dependent variables in the MODEL and IDENTITY statements that do not appear in the INSTRUMENTS statement.

IDENTITY Statement

IDENTITY *equation* **;**

The IDENTITY statement specifies linear relationships among variables to write to the OUTEST= data set. It provides extra information in the OUTEST= data set but does not create or compute variables. The OUTEST= data set can be processed by the SIMLIN procedure in a later step.

The IDENTITY statement is also used to compute reduced form coefficients when the REDUCED option in the PROC SYSLIN statement is specified. See the section "Reduced Form Estimates" on page 2583 for details.

The *equation* given by the IDENTITY statement has the same form as equations in the MODEL statement. A label can be specified for an IDENTITY statement as follows:

label **: IDENTITY . . . ;**

INSTRUMENTS Statement

INSTRUMENTS *variables* **;**

The INSTRUMENTS statement declares the variables used in obtaining first-stage predicted values. All the instruments specified are used in each first-stage regression. The INSTRUMENTS statement is required for the 2SLS, 3SLS, IT3SLS, LIML, MELO, and K-class estimation methods. The INSTRUMENTS statement is not needed for the SUR, ITSUR, OLS, or FIML estimation methods.

MODEL Statement

MODEL *response = regressors / options* **;**

The MODEL statement regresses the response variable on the left side of the equal sign against the regressors listed on the right side.

Models can be given labels. Model labels are used in the printed output to identify the results for different models. Model labels are also used in SRESTRICT and STEST statements to refer to parameters in different models. If no label is specified, the response variable name is used as the label for the model. The model label is specified as follows:

label **: MODEL . . . ;**

The following options can be used in the MODEL statement after a slash (/).

ALL
specifies the CORRB, COVB, DW, I, OVERID, PLOT, STB, and XPX options.

ALPHA=*value*
specifies the α parameter for Fuller's modification to the LIML estimation method. See the section "Fuller's Modification to LIML" on page 2584 for details.

CORRB
prints the matrix of estimated correlations between the parameter estimates.

COVB
prints the matrix of estimated covariances between the parameter estimates.

DW
prints Durbin-Watson statistics and autocorrelation coefficients for the residuals. If there are missing values, d' is calculated according to Savin and White (1978). Use the DW option only if the data set to be analyzed is an ordinary SAS data set with time series observations sorted in time order. The Durbin-Watson test is not valid for models with lagged dependent regressors.

I
prints the inverse of the crossproducts matrix for the model, $(\mathbf{X}'\mathbf{X})^{-1}$. If restrictions are specified, the crossproducts matrix printed is adjusted for the restrictions. See the section "Computational Details" on page 2582 for details.

K=*value*
specifies K-class estimation.

NOINT
suppresses the intercept parameter from the model.

NOPRINT
suppresses the normal printed output.

OVERID
prints Basmann's (1960) test for over identifying restrictions. See the section "Overidentification Restrictions" on page 2584 for details.

PLOT
plots residual values against regressors. A plot of the residuals for each regressor is printed.

STB
prints standardized parameter estimates. Sometimes known as a standard partial regression coefficient, a standardized parameter estimate is a parameter estimate multiplied by the standard deviation of the associated regressor and divided by the standard deviation of the response variable.

UNREST
prints parameter estimates computed before restrictions are applied. The UNREST option is valid only if a RESTRICT statement is specified.

XPX

prints the model crossproducts matrix, $X'X$. See the section "Computational Details" on page 2582 for details.

OUTPUT Statement

OUTPUT < *PREDICTED=variable* > < *RESIDUAL=variable* > **;**

The OUTPUT statement writes predicted values and residuals from the preceding model to the data set specified by the OUT= option in the PROC SYSLIN statement. An OUTPUT statement must come after the MODEL statement to which it applies. The OUT= option must be specified in the PROC SYSLIN statement.

The following options can be specified in the OUTPUT statement:

PREDICTED=*variable*

names a new variable to contain the predicted values for the response variable. The PREDICTED= option can be abbreviated as PREDICT=, PRED=, or P=.

RESIDUAL=*variable*

names a new variable to contain the residual values for the response variable. The RESIDUAL= option can be abbreviated as RESID= or R=.

For example, the following statements create an output data set named B. In addition to the variables in the input data set, the data set B contains the variable YHAT, with values that are predicted values of the response variable Y, and YRESID, with values that are the residual values of Y.

```
proc syslin data=a out=b;
   model y = x1 x2;
   output p=yhat r=yresid;
run;
```

For example, the following statements create an output data set named PRED. In addition to the variables in the input data set, the data set PRED contains the variables Q_DEMAND and Q_SUPPLY, with values that are predicted values of the response variable Q for the demand and supply equations respectively, and R_DEMAND and R_SUPPLY, with values that are the residual values of the demand and supply equations respectively.

```
proc syslin data=in out=pred;
   demand: model q = p y s;
   output p=q_demand r=r_demand;
   supply: model q = p u;
   output p=q_supply r=r_supply;
run;
```

See the section "OUT= Data Set" on page 2585 for details.

RESTRICT Statement

RESTRICT *equation , . . . , equation* **;**

The RESTRICT statement places restrictions on the parameter estimates for the preceding MODEL statement. Any number of RESTRICT statements can follow a MODEL statement. Each restriction is written as a linear equation. If more than one restriction is specified in a single RESTRICT statement, the restrictions are separated by commas.

Parameters are referred to by the name of the corresponding regressor variable. Each name used in the equation must be a regressor in the preceding MODEL statement. The keyword INTERCEPT is used to refer to the intercept parameter in the model.

RESTRICT statements can be given labels. The labels are used in the printed output to distinguish results for different restrictions. Labels are specified as follows:

label **: RESTRICT . . . ;**

The following is an example of the use of the RESTRICT statement, in which the coefficients of the regressors X1 and X2 are required to sum to 1.

```
proc syslin data=a;
   model y = x1 x2;
   restrict x1 + x2 = 1;
run;
```

Variable names can be multiplied by constants. When no equal sign appears, the linear combination is set equal to 0. Note that the parameters associated with the variables are restricted, not the variables themselves. Here are some examples of valid RESTRICT statements:

```
restrict x1 + x2 = 1;
restrict x1 + x2 - 1;
restrict 2 * x1 = x2 + x3 , intercept + x4 = 0;
restrict x1 = x2 = x3 = 1;
restrict 2 * x1 - x2;
```

Restricted parameter estimates are computed by introducing a Lagrangian parameter λ for each restriction (Pringle and Rayner 1971). The estimates of these Lagrangian parameters are printed in the "Parameter Estimates" table. If a restriction cannot be applied, its parameter value and degrees of freedom are listed as 0.

The Lagrangian parameter λ measures the sensitivity of the sum of squared errors (SSE) to the restriction. If the restriction is changed by a small amount ϵ, the SSE is changed by $2\lambda\epsilon$.

The t ratio tests the significance of the restrictions. If λ is zero, the restricted estimates are the same as the unrestricted.

Any number of restrictions can be specified on a RESTRICT statement, and any number of RESTRICT statements can be used. The estimates are computed subject to all restrictions specified. However, restrictions should be consistent and not redundant.

NOTE: The RESTRICT statement is not supported for the FIML estimation method.

SRESTRICT Statement

SRESTRICT *equation , . . . , equation* **;**

The SRESTRICT statement imposes linear restrictions that involve parameters in two or more MODEL statements. The SRESTRICT statement is like the RESTRICT statement but is used to impose restrictions across equations, whereas the RESTRICT statement applies only to parameters in the immediately preceding MODEL statement.

Each restriction is written as a linear equation. Parameters are referred to as *label.variable,* where *label* is the model label and *variable* is the name of the regressor to which the parameter is attached. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.) Each variable name used must be a regressor in the indicated MODEL statement. The keyword INTERCEPT is used to refer to intercept parameters.

SRESTRICT statements can be given labels. The labels are used in the printed output to distinguish results for different restrictions. Labels are specified as follows:

label **: SRESTRICT . . . ;**

The following is an example of the use of the SRESTRICT statement, in which the coefficient for the regressor X2 is constrained to be the same in both models.

```
proc syslin data=a 3sls;
   endogenous y1 y2;
   instruments x1 x2;
   model y1 = y2 x1 x2;
   model y2 = y1 x2;
   srestrict y1.x2 = y2.x2;
run;
```

When no equal sign is used, the linear combination is set equal to 0. Thus, the restriction in the preceding example can also be specified as

```
   srestrict y1.x2 - y2.x2;
```

Any number of restrictions can be specified on an SRESTRICT statement, and any number of SRESTRICT statements can be used. The estimates are computed subject to all restrictions specified. However, restrictions should be consistent and not redundant.

When a system restriction is requested for a single equation estimation method (such as OLS or 2SLS), PROC SYSLIN produces the restricted estimates by actually using a corresponding system method. For example, when SRESTRICT is specified along with OLS, PROC SYSLIN produces the restricted OLS estimates via a two-step process equivalent to using SUR estimation with the SDIAG option. First, the unrestricted OLS results are produced. Then, the GLS (SUR) estimation with the system restriction is performed, using the diagonal of the covariance matrix of the residuals. When SRESTRICT is specified along with 2SLS, PROC SYSLIN produces the restricted 2SLS estimates via a multistep process equivalent to using 3SLS estimation with the SDIAG option. First, the unrestricted 2SLS results are produced. Then, the GLS (3SLS) estimation with the system restriction is performed, using the diagonal of the covariance matrix of the residuals.

The results of the SRESTRICT statements are printed after the parameter estimates for all the models in the system. The format of the SRESTRICT statement output is the same as the "Parameter Estimates" table. In this output the parameter estimate is the Lagrangian parameter λ used to impose the restriction.

The Lagrangian parameter λ measures the sensitivity of the system sum of square errors to the restriction. The system SSE is the system MSE shown in the printed output multiplied by the degrees of freedom. If the restriction is changed by a small amount ϵ, the system SSE is changed by $2\lambda\epsilon$.

The t ratio tests the significance of the restriction. If λ is zero, the restricted estimates are the same as the unrestricted estimates.

The model degrees of freedom are not adjusted for the cross-model restrictions imposed by SRESTRICT statements.

NOTE: The SRESTRICT statement is not supported for the LIML and the FIML estimation methods.

STEST Statement

STEST *equation , . . . , equation / options* **;**

The STEST statement performs an F test for the joint hypotheses specified in the statement.

The hypothesis is represented in matrix notation as

$$\mathbf{L}\beta = \mathbf{c}$$

and the F test is computed as

$$\frac{(\mathbf{L}b - \mathbf{c})'(\mathbf{L}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{L}')^{-1}(\mathbf{L}b - \mathbf{c})}{m\hat{\sigma}^2}$$

where b is the estimate of β, m is the number of restrictions, and $\hat{\sigma}^2$ is the system weighted mean squared error. See the section "Computational Details" on page 2582 for information about the matrix $\mathbf{X}'\mathbf{X}$.

Each hypothesis to be tested is written as a linear equation. Parameters are referred to as *label.variable,* where *label* is the model label and *variable* is the name of the regressor to which the parameter is attached. (If the MODEL statement does not have a label, you can use the dependent variable name as the label for the model, provided the dependent variable uniquely labels the model.) Each variable name used must be a regressor in the indicated MODEL statement. The keyword INTERCEPT is used to refer to intercept parameters.

STEST statements can be given labels. The label is used in the printed output to distinguish different tests. Any number of STEST statements can be specified. Labels are specified as follows:

label **: STEST . . . ;**

The following is an example of the STEST statement:

```
proc syslin data=a 3sls;
   endogenous y1 y2;
   instruments x1 x2;
   model y1 = y2 x1 x2;
   model y2 = y1 x2;
   stest y1.x2 = y2.x2;
run;
```

The test performed is exact only for ordinary least squares, given the OLS assumptions of the linear model. For other estimation methods, the *F* test is based on large sample theory and is only approximate in finite samples.

If RESTRICT or SRESTRICT statements are used, the tests computed by the STEST statement are conditional on the restrictions specified. The validity of the tests can be compromised if incorrect restrictions are imposed on the estimates.

The following are examples of STEST statements:

```
stest a.x1 + b.x2 = 1;
stest 2 * b.x2 = c.x3 + c.x4 ,
      a.intercept + b.x2 = 0;
stest a.x1 = c.x2 = b.x3 = 1;
stest 2 * a.x1 - b.x2 = 0;
```

The PRINT option can be specified in the STEST statement after a slash (/):

PRINT
prints intermediate calculations for the hypothesis tests.

NOTE: The STEST statement is not supported for the FIML estimation method.

TEST Statement

TEST *equation , . . . , equation / options* **;**

The TEST statement performs *F* tests of linear hypotheses about the parameters in the preceding MODEL statement. Each equation specifies a linear hypothesis to be tested. If more than one equation is specified, the equations are separated by commas.

Variable names must correspond to regressors in the preceding MODEL statement, and each name represents the coefficient of the corresponding regressor. The keyword INTERCEPT is used to refer to the model intercept.

TEST statements can be given labels. The label is used in the printed output to distinguish different tests. Any number of TEST statements can be specified. Labels are specified as follows:

label **: TEST . . . ;**

The following is an example of the use of TEST statement, which tests the hypothesis that the coefficients of X1 and X2 are the same:

```
proc syslin data=a;
   model y = x1 x2;
   test x1 = x2;
run;
```

The following statements perform *F* tests for the hypothesis that the coefficients of X1 and X2 are equal, for the hypothesis that the sum of the X1 and X2 coefficients is twice the intercept, and for the joint hypothesis.

```
proc syslin data=a;
   model y = x1 x2;
   x1eqx2:  test x1 = x2;
   sumeq2i: test x1 + x2 = 2 * intercept;
   joint:   test x1 = x2, x1 + x2 = 2 * intercept;
run;
```

The following are additional examples of TEST statements:

```
test x1 + x2 = 1;
test x1 = x2 = x3 = 1;
test 2 * x1 = x2 + x3, intercept + x4 = 0;
test 2 * x1 - x2;
```

The TEST statement performs an F test for the joint hypotheses specified. The hypothesis is represented in matrix notation as follows:

$$\mathbf{L}\beta = \mathbf{c}$$

The F test is computed as

$$\frac{(\mathbf{L}b - \mathbf{c})'(\mathbf{L}(\mathbf{X}'\mathbf{X})^{-}\mathbf{L}')^{-1}(\mathbf{L}b - \mathbf{c})}{m\hat{\sigma}^2}$$

where b is the estimate of β, m is the number of restrictions, and $\hat{\sigma}^2$ is the model mean squared error. See the section "Computational Details" on page 2582 for information about the matrix $\mathbf{X}'\mathbf{X}$.

The test performed is exact only for ordinary least squares, given the OLS assumptions of the linear model. For other estimation methods, the F test is based on large sample theory and is only approximate in finite samples.

If RESTRICT or SRESTRICT statements are used, the tests computed by the TEST statement are conditional on the restrictions specified. The validity of the tests can be compromised if incorrect restrictions are imposed on the estimates.

The PRINT option can be specified in the TEST statement after a slash (/):

PRINT

prints intermediate calculations for the hypothesis tests.

NOTE: The TEST statement is not supported for the FIML estimation method.

VAR Statement

VAR *variables* **;**

The VAR statement is used to include variables in the crossproducts matrix that are not specified in any MODEL statement. This statement is rarely used with PROC SYSLIN and is used only with the OUTSSCP= option in the PROC SYSLIN statement.

WEIGHT Statement

WEIGHT *variable* **;**

The WEIGHT statement is used to perform weighted regression. The WEIGHT statement names a variable in the input data set whose values are relative weights for a weighted least squares fit. If the weight value is proportional to the reciprocal of the variance for each observation, the weighted estimates are the best linear unbiased estimates (BLUE).

Details: SYSLIN Procedure

Input Data Set

PROC SYSLIN does not compute new values for regressors. For example, if you need a lagged variable, you must create it with a DATA step. No values are computed by IDENTITY statements; all values must be in the input data set.

Special TYPE= Input Data Sets

The input data set for most applications of the SYSLIN procedure contains standard rectangular data. However, PROC SYSLIN can also process input data in the form of a crossproducts, covariance, or correlation matrix. Data sets that contain such matrices are identified by values of the TYPE= data set option.

These special kinds of input data sets can be used to save computer time. It takes nk^2 operations, where n is the number of observations and k is the number of variables, to calculate cross products; the regressions are of the order k^3. When n is in the thousands and k is much smaller, you can save most of the computer time in later runs of PROC SYSLIN by reusing the SSCP matrix rather than recomputing it.

The SYSLIN procedure can process TYPE=CORR, COV, UCORR, UCOV, or SSCP data sets. TYPE=CORR and TYPE=COV data sets, usually created by the CORR procedure, contain means and standard deviations, and correlations or covariances. TYPE=SSCP data sets, usually created in previous runs of PROC SYSLIN, contain sums of squares and cross products. For more information about special SAS data sets, see *SAS/STAT User's Guide*.

When special SAS data sets are read, you must specify the TYPE= data set option. PROC CORR and PROC SYSLIN automatically set the type for output data sets; however, if you create the data set by some other means, you must specify its type with the TYPE= data set option.

When the special data sets are used, the DW (Durbin-Watson test) and PLOT options in the MODEL statement cannot be performed, and the OUTPUT statements are not valid.

Estimation Methods

A brief description of the methods used by the SYSLIN procedure follows. For more information about these methods, see the references at the end of this chapter.

There are two fundamental methods of estimation for simultaneous equations: least squares and maximum likelihood. There are two approaches within each of these categories: single equation methods (also referred to as limited information methods) and system methods (also referred to as full information methods). System methods take into account cross-equation correlations of the disturbances in estimating parameters, while single equation methods do not.

OLS, 2SLS, MELO, K-class, SUR, ITSUR, 3SLS, and IT3SLS use the least squares method; LIML and FIML use the maximum likelihood method.

OLS, 2SLS, MELO, K-class, and LIML are single equation methods. The system methods are SUR, ITSUR, 3SLS, IT3SLS, and FIML.

Single Equation Estimation Methods

Single equation methods do not take into account correlations of errors across equations. As a result, these estimators are not asymptotically efficient compared to full information methods; however, there are instances in which they may be preferred. (See the section "Choosing a Method for Simultaneous Equations" on page 2580 for details.)

Let y_i be the dependent endogenous variable in equation i, and X_i and Y_i be the matrices of exogenous and endogenous variables appearing as regressors in the same equation.

The 2SLS method owes its name to the fact that, in a first stage, the instrumental variables are used as regressors to obtain a projected value $\hat{Y}_i$ that is uncorrelated with the residual in equation i. In a second stage, $\hat{Y}_i$ replaces Y_i on the right-hand side to obtain consistent least squares estimators.

Normally, the predetermined variables of the system are used as the instruments. It is possible to use variables other than predetermined variables from your system as instruments; however, the estimation might not be as efficient. For consistent estimates, the instruments must be uncorrelated with the residual and correlated with the endogenous variables.

The LIML method results in consistent estimates that are equal to the 2SLS estimates when an equation is exactly identified. LIML can be viewed as a least-variance ratio estimation or as a maximum likelihood estimation. LIML involves minimizing the ratio $\lambda = (rvar_eq)/(rvar_sys)$, where $rvar_eq$ is the residual variance associated with regressing the weighted endogenous variables on all predetermined variables that appear in that equation, and $rvar_sys$ is the residual variance associated with regressing weighted endogenous variables on all predetermined variables in the system.

The MELO method computes the minimum expected loss estimator. MELO estimators "minimize the posterior expectation of generalized quadratic loss functions for structural coefficients of linear structural models" (Judge et al. 1985, p. 635).

K-class estimators are a class of estimators that depends on a user-specified parameter k. A k value less than 1 is recommended but not required. The parameter k can be deterministic or stochastic, but its probability limit must equal 1 for consistent parameter estimates. When all the predetermined variables are listed as instruments, they include all the other single equation estimators supported by PROC SYSLIN. The instance when some of the predetermined variables are not listed among the instruments is not supported by PROC SYSLIN for the general K-class estimation. However, it is supported for the other methods.

For $k = 1$, the K-class estimator is the 2SLS estimator, while for $k = 0$, the K-class estimator is the OLS estimator. The K-class interpretation of LIML is that $k = \lambda$. Note that k is stochastic in the LIML method, unlike for OLS and 2SLS.

MELO is a Bayesian K-class estimator. It yields estimates that can be expressed as a matrix-weighted average of the OLS and 2SLS estimates. MELO estimators have finite second moments and hence finite risk. Other frequently used K-class estimators might not have finite moments under some commonly encountered circumstances, and hence there can be infinite risk relative to quadratic and other loss functions.

One way of comparing K-class estimators is to note that when k =1, the correlation between regressor and the residual is completely corrected for. In all other cases, it is only partially corrected for.

See "Computational Details" on page 2582 for more details about K-class estimators.

SUR and 3SLS Estimation Methods

SUR might improve the efficiency of parameter estimates when there is contemporaneous correlation of errors across equations. In practice, the contemporaneous correlation matrix is estimated using OLS residuals. Under two sets of circumstances, SUR parameter estimates are the same as those produced by OLS: when there is no contemporaneous correlation of errors across equations (the estimate of the contemporaneous correlation matrix is diagonal) and when the independent variables are the same across equations.

Theoretically, SUR parameter estimates are always at least as efficient as OLS in large samples, provided that your equations are correctly specified. However, in small samples the need to estimate the covariance matrix from the OLS residuals increases the sampling variability of the SUR estimates. This effect can cause SUR to be less efficient than OLS. If the sample size is small and the cross-equation correlations are small, then OLS is preferred to SUR. The consequences of specification error are also more serious with SUR than with OLS.

The 3SLS method combines the ideas of the 2SLS and SUR methods. Like 2SLS, the 3SLS method uses $\hat{Y}$ instead of Y for endogenous regressors, which results in consistent estimates. Like SUR, the 3SLS method takes the cross-equation error correlations into account to improve large sample efficiency. For 3SLS, the 2SLS residuals are used to estimate the cross-equation error covariance matrix.

The SUR and 3SLS methods can be iterated by recomputing the estimate of the cross-equation covariance matrix from the SUR or 3SLS residuals and then computing new SUR or 3SLS estimates based on this updated covariance matrix estimate. Continuing this iteration until convergence produces ITSUR or IT3SLS estimates.

FIML Estimation Method

The FIML estimator is a system generalization of the LIML estimator. The FIML method involves minimizing the determinant of the covariance matrix associated with residuals of the reduced form of the equation system. From a maximum likelihood standpoint, the LIML method involves assuming that the errors are normally distributed and then maximizing the likelihood function subject to restrictions on a particular equation. FIML is similar, except that the likelihood function is maximized subject to restrictions on all of the parameters in the model, not just those in the equation being estimated.

NOTE: The RESTRICT, SRESTRICT, TEST, and STEST statements are not supported when the FIML method is used.

Choosing a Method for Simultaneous Equations

A number of factors should be taken into account in choosing an estimation method. Although system methods are asymptotically most efficient in the absence of specification error, system methods are more sensitive to specification error than single equation methods.

In practice, models are never perfectly specified. It is a matter of judgment whether the misspecification is serious enough to warrant avoidance of system methods.

Another factor to consider is sample size. With small samples, 2SLS might be preferred to 3SLS. In general, it is difficult to say much about the small sample properties of K-class estimators because the results depend on the regressors used.

LIML and FIML are invariant to the normalization rule imposed but are computationally more expensive than 2SLS or 3SLS.

If the reason for contemporaneous correlation among errors across equations is a common, omitted variable, it is not necessarily best to apply SUR. SUR parameter estimates are more sensitive to specification error than OLS. OLS might produce better parameter estimates under these circumstances. SUR estimates are also affected by the sampling variation of the error covariance matrix. There is some evidence from Monte Carlo studies that SUR is less efficient than OLS in small samples.

ANOVA Table for Instrumental Variables Methods

In the instrumental variables methods (2SLS, LIML, K-class, MELO), first-stage predicted values are substituted for the endogenous regressors. As a result, the regression sum of squares (RSS) and the error sum of squares (ESS) do not sum to the total corrected sum of squares for the dependent variable (TSS). The analysis-of-variance table included in the second-stage results gives these sums of squares and the mean squares that are used for the F test, but this table is not a variance decomposition in the usual sense.

The F test shown in the instrumental variables case is a valid test of the no-regression hypothesis that the true coefficients of all regressors are 0. However, because of the first-stage projection of the regression mean square, this is a Wald-type test statistic, which is asymptotically F but not exactly F-distributed in finite samples. Thus, for small samples the F test is only approximate when instrumental variables are used.

The R-Square Statistics

As explained in the section "ANOVA Table for Instrumental Variables Methods" on page 2581, when instrumental variables are used, the regression sum of squares (RSS) and the error sum of squares (ESS) do not sum to the total corrected sum of squares. In this case, there are several ways that the R^2 statistic can be defined.

The definition of R^2 used by the SYSLIN procedure is

$$\mathrm{R}^2 = \frac{RSS}{RSS + ESS}$$

This definition is consistent with the F test of the null hypothesis that the true coefficients of all regressors are zero. However, this R^2 might not be a good measure of the goodness of fit of the model.

System Weighted R-Square and System Weighted Mean Squared Error

The system weighted R^2, printed for the 3SLS, IT3SLS, SUR, ITSUR, and FIML methods, is computed as follows.

$$\mathrm{R}^2 = \mathbf{Y}'\mathbf{W}\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'\mathbf{W}\mathbf{Y}/\mathbf{Y}'\mathbf{W}\mathbf{Y}$$

In this equation, the matrix $\mathbf{X}'\mathbf{X}$ is $\mathbf{R}'\mathbf{W}\mathbf{R}$ and $\mathbf{W}$ is the projection matrix of the instruments:

$$\mathbf{W} = \mathbf{S}^{-1} \otimes \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$$

The matrix $\mathbf{Z}$ is the instrument set, $\mathbf{R}$ is the regressor set, and $\mathbf{S}$ is the estimated cross-model covariance matrix.

The system weighted MSE, printed for the 3SLS, IT3SLS, SUR, ITSUR, and FIML methods, is computed as follows:

$$MSE = \frac{1}{tdf}(\mathbf{Y}'\mathbf{W}\mathbf{Y} - \mathbf{Y}'\mathbf{W}\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'\mathbf{W}\mathbf{Y})$$

In this equation, *tdf* is the sum of the error degrees of freedom for the equations in the system.

Computational Details

This section discusses various computational details.

Computation of Least Squares-Based Estimators

Let the system be composed of G equations and let the ith equation be expressed in this form:

$$y_i = Y_i \boldsymbol{\beta}_i + X_i \boldsymbol{\gamma}_i + \mathbf{u}$$

where

y_i	is the vector of observations on the dependent variable
Y_i	is the matrix of observations on the endogenous variables included in the equation
$\boldsymbol{\beta}_i$	is the vector of parameters associated with Y_i
X_i	is the matrix of observations on the predetermined variables included in the equation
$\boldsymbol{\gamma}_i$	is the vector of parameters associated with X_i
$\mathbf{u}$	is a vector of errors

Let $\hat{V}_i = Y_i - \hat{Y}_i$, where $\hat{Y}_i$ is the projection of Y_i onto the space spanned by the instruments matrix $\mathbf{Z}$.

Let

$$\boldsymbol{\delta}_i = \begin{bmatrix} \boldsymbol{\beta}_i \\ \boldsymbol{\gamma}_i \end{bmatrix}$$

be the vector of parameters associated with both the endogenous and exogenous variables.

The K-class of estimators (Theil 1971) is defined by

$$\hat{\boldsymbol{\delta}}_{i,k} = \begin{bmatrix} Y_i'Y_i - k\hat{V}_i'\hat{V}_i & Y_i'X_i \\ X_i'Y_i & X_i'X_i \end{bmatrix}^{-1} \begin{bmatrix} (Y_i - kV_i)'y_i \\ X_i'y_i \end{bmatrix}$$

where k is a user-defined value.

Let

$$\mathbf{R} = [Y_i X_i]$$

and

$$\hat{\mathbf{R}} = [\hat{Y}_i \; X_i]$$

The 2SLS estimator is defined as

$$\hat{\boldsymbol{\delta}}_{i,2SLS} = [\hat{R}_i' \; \hat{R}_i]^{-1} \hat{R}_i' y_i$$

Let $\mathbf{y}$ and $\boldsymbol{\delta}$ be the vectors obtained by stacking the vectors of dependent variables and parameters for all G equations, and let $\mathbf{R}$ and $\hat{\mathbf{R}}$ be the block diagonal matrices formed by R_i and $\hat{R}_i$, respectively.

The SUR and ITSUR estimators are defined as

$$\hat{\boldsymbol{\delta}}_{(IT)SUR} = \left[\mathbf{R}' \left(\hat{\Sigma}^{-1} \otimes \mathbf{I}\right) \mathbf{R}\right]^{-1} \mathbf{R}' \left(\hat{\Sigma}^{-1} \otimes \mathbf{I}\right) \mathbf{y}$$

while the 3SLS and IT3SLS estimators are defined as

$$\hat{\boldsymbol{\delta}}_{(IT)3SLS} = \left[\hat{\mathbf{R}}' \left(\hat{\Sigma}^{-1} \otimes \mathbf{I}\right) \hat{\mathbf{R}}\right]^{-1} \hat{\mathbf{R}}' \left(\hat{\Sigma}^{-1} \otimes \mathbf{I}\right) \mathbf{y}$$

where $\mathbf{I}$ is the identity matrix, and $\hat{\Sigma}$ is an estimator of the cross-equation correlation matrix. For 3SLS, $\hat{\Sigma}$ is obtained from the 2SLS estimation, while for SUR it is derived from the OLS estimation. For IT3SLS and ITSUR, it is obtained iteratively from the previous estimation step, until convergence.

Computation of Standard Errors

The VARDEF= option in the PROC SYSLIN statement controls the denominator used in calculating the cross-equation covariance estimates and the parameter standard errors and covariances. The values of the VARDEF= option and the resulting denominator are as follows:

N	uses the number of nonmissing observations.
DF	uses the number of nonmissing observations less the degrees of freedom in the model.
WEIGHT	uses the sum of the observation weights given by the WEIGHTS statement.
WDF	uses the sum of the observation weights given by the WEIGHTS statement less the degrees of freedom in the model.

The VARDEF= option does not affect the model mean squared error, root mean squared error, or R^2 statistics. These statistics are always based on the error degrees of freedom, regardless of the VARDEF= option. The VARDEF= option also does not affect the dependent variable coefficient of variation (CV).

Reduced Form Estimates

The REDUCED option in the PROC SYSLIN statement computes estimates of the reduced form coefficients. The REDUCED option requires that the equation system be square. If there are fewer models than endogenous variables, IDENTITY statements can be used to complete the equation system.

The reduced form coefficients are computed as follows. Represent the equation system, with all endogenous variables moved to the left-hand side of the equations and identities, as

$$\mathbf{BY} = \mathbf{\Gamma X}$$

Here $\mathbf{B}$ is the estimated coefficient matrix for the endogenous variables $\mathbf{Y}$, and $\mathbf{\Gamma}$ is the estimated coefficient matrix for the exogenous (or predetermined) variables $\mathbf{X}$.

The system can be solved for $\mathbf{Y}$ as follows, provided $\mathbf{B}$ is square and nonsingular:

$$\mathbf{Y} = \mathbf{B}^{-1}\mathbf{\Gamma X}$$

The reduced form coefficients are the matrix $\mathbf{B}^{-1}\mathbf{\Gamma}$.

Uncorrelated Errors across Equations

The SDIAG option in the PROC SYSLIN statement computes estimates by assuming uncorrelated errors across equations. As a result, when the SDIAG option is used, the 3SLS estimates are identical to 2SLS estimates, and the SUR estimates are the same as the OLS estimates.

Overidentification Restrictions

The OVERID option in the MODEL statement can be used to test for overidentifying restrictions on parameters of each equation. The null hypothesis is that the predetermined variables that do not appear in any equation have zero coefficients. The alternative hypothesis is that at least one of the assumed zero coefficients is nonzero. The test is approximate and rejects the null hypothesis too frequently for small sample sizes.

The formula for the test is given as follows. Let $y_i = \beta_i \mathbf{Y}_i + \gamma_i \mathbf{Z}_i + e_i$ be the ith equation. $\mathbf{Y}_i$ are the endogenous variables that appear as regressors in the ith equation, and $\mathbf{Z}_i$ are the instrumental variables that appear as regressors in ith equation. Let N_i be the number of variables in $\mathbf{Y}_i$ and $\mathbf{Z}_i$.

Let $v_i = y_i - \mathbf{Y}_i \hat{\beta}_i$. Let $\mathbf{Z}$ represent all instrumental variables, T be the total number of observations, and K be the total number of instrumental variables. Define $\hat{l}$ as follows:

$$\hat{l} = \frac{v'_i(\mathbf{I} - \mathbf{Z}_i(\mathbf{Z}'_i\mathbf{Z}_i)^{-1}\mathbf{Z}'_i)v_i}{v'_i(\mathbf{I} - \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}')v_i}$$

Then the test statistic

$$\frac{T-K}{K-N_i}(\hat{l} - 1)$$

is distributed approximately as an F with $K - N_i$ and $T - K$ degrees of freedom. See Basmann (1960) for more information.

Fuller's Modification to LIML

The ALPHA= option in the PROC SYSLIN and MODEL statements parameterizes Fuller's modification to LIML. This modification is $k = \gamma - (\alpha/(n-g))$, where α is the value of the ALPHA= option, γ is the LIML k value,n is the number of observations, and g is the number of predetermined variables. Fuller's modification is not used unless the ALPHA= option is specified. See Fuller (1977) for more information.

Missing Values

Observations that have a missing value for any variable in the analysis are excluded from the computations.

OUT= Data Set

The output SAS data set produced by the OUT= option in the PROC SYSLIN statement contains all the variables in the input data set and the variables that contain predicted values and residuals specified by OUTPUT statements.

The residuals are computed as actual values minus predicted values. Predicted values never use lags of other predicted values, as would be desirable for dynamic simulation. For these applications, PROC SIMLIN is available to predict or simulate values from the estimated equations.

OUTEST= Data Set

The OUTEST= option produces a TYPE=EST output SAS data set that contains estimates from the regressions. The variables in the OUTEST= data set are as follows:

BY variables — identifies the BY statement variables that are included in the OUTEST= data set.

TYPE — identifies the estimation type for the observations. The _TYPE_ value INST indicates first-stage regression estimates. Other values indicate the estimation method used: 2SLS indicates two-stage least squares results, 3SLS indicates three-stage least squares results, LIML indicates limited information maximum likelihood results, and so forth. Observations added by IDENTITY statements have the _TYPE_ value IDENTITY.

STATUS — identifies the convergence status of the estimation. _STATUS_ equals 0 when convergence criteria are met. Otherwise, _STATUS_ equals 1 when the estimation converges with a note, 2 when it converges with a warning, or 3 when it fails to converge.

MODEL — identifies the model label. The model label is the label specified in the MODEL statement or the dependent variable name if no label is specified. For first-stage regression estimates, _MODEL_ has the value FIRST.

DEPVAR — identifies the name of the dependent variable for the model.

NAME — identifies the names of the regressors for the rows of the covariance matrix, if the COVOUT option is specified. _NAME_ has a blank value for the parameter estimates observations. The _NAME_ variable is not included in the OUTEST= data set unless the COVOUT option is used to output the covariance of parameter estimates matrix.

SIGMA — contains the root mean squared error for the model, which is an estimate of the standard deviation of the error term. The _SIGMA_ variable contains the same values reported as Root MSE in the printed output.

INTERCEPT — identifies the intercept parameter estimates.

regressors — identifies the regressor variables from all the MODEL statements that are included in the OUTEST= data set. Variables used in IDENTIFY statements are also included in the OUTEST= data set.

The parameter estimates are stored under the names of the regressor variables. The intercept parameters are stored in the variable INTERCEPT. The dependent variable of the model is given a coefficient of –1. Variables that are not in a model have missing values for the OUTEST= observations for that model.

Some estimation methods require computation of preliminary estimates. All estimates computed are output to the OUTEST= data set. For each BY group and each estimation, the OUTEST= data set contains one observation for each MODEL or IDENTITY statement. Results for different estimations are identified by the _TYPE_ variable.

For example, consider the following statements:

```
proc syslin data=a outest=est 3sls;
   by b;
   endogenous y1 y2;
   instruments x1-x4;
   model y1 = y2 x1 x2;
   model y2 = y1 x3 x4;
   identity x1 = x3 + x4;
run;
```

The 3SLS method requires both a preliminary 2SLS stage and preliminary first-stage regressions for the endogenous variable. The OUTEST= data set thus contains three different kinds of estimates. The observations for the first-stage regression estimates have the _TYPE_ value INST. The observations for the 2SLS estimates have the _TYPE_ value 2SLS. The observations for the final 3SLS estimates have the _TYPE_ value 3SLS.

Since there are two endogenous variables in this example, there are two first-stage regressions and two _TYPE_=INST observations in the OUTEST= data set. Since there are two model statements, there are two OUTEST= observations with _TYPE_=2SLS and two observations with _TYPE_=3SLS. In addition, the OUTEST= data set contains an observation with the _TYPE_ value IDENTITY that contains the coefficients specified by the IDENTITY statement. All these observations are repeated for each BY group in the input data set defined by the values of the BY variable B.

When the COVOUT option is specified, the estimated covariance matrix for the parameter estimates is included in the OUTEST= data set. Each observation for parameter estimates is followed by observations that contain the rows of the parameter covariance matrix for that model. The row of the covariance matrix is identified by the variable _NAME_. For observations that contain parameter estimates, _NAME_ is blank. For covariance observations, _NAME_ contains the regressor name for the row of the covariance matrix and the regressor variables contain the covariances.

See Example 36.1 for an example of the OUTEST= data set.

OUTSSCP= Data Set

The OUTSSCP= option produces a TYPE=SSCP output SAS data set that contains sums of squares and cross products. The data set contains all variables used in the MODEL, IDENTITY, and VAR statements. Observations are identified by the variable _NAME_.

The OUTSSCP= data set can be useful when a large number of observations are to be explored in many different SYSLIN runs. The sum-of-squares-and-crossproducts matrix can be saved with the OUTSSCP=

option and used as the DATA= data set on subsequent SYSLIN runs. This is much less expensive computationally because PROC SYSLIN never reads the original data again. In the step that creates the OUTSSCP= data set, include in the VAR statement all the variables you expect to use.

Printed Output

The printed output produced by the SYSLIN procedure is as follows:

1. If the SIMPLE option is used, a table of descriptive statistics is printed that shows the sum, mean, sum of squares, variance, and standard deviation for all the variables used in the models.
2. If the FIRST option is specified and an instrumental variables method is used, first-stage regression results are printed. The results show the regression of each endogenous variable on the variables in the INSTRUMENTS list.
3. The results of the second-stage regression are printed for each model. (See the following section "Printed Output for Each Model" on page 2587 for details.)
4. If a systems method like 3SLS, SUR, or FIML is used, the cross-equation error covariance matrix is printed. This matrix is shown four ways: the covariance matrix itself, the correlation matrix form, the inverse of the correlation matrix, and the inverse of the covariance matrix.
5. If a systems method like 3SLS, SUR, or FIML is used, the system weighted mean squared error and system weighted R^2 statistics are printed. The system weighted MSE and R^2 measure the fit of the joint model obtained by stacking all the models together and performing a single regression with the stacked observations weighted by the inverse of the model error variances.
6. If a systems method like 3SLS, SUR, or FIML is used, the final results are printed for each model.
7. If the REDUCED option is used, the reduced form coefficients are printed. These consist of the structural coefficient matrix for the endogenous variables, the structural coefficient matrix for the exogenous variables, the inverse of the endogenous coefficient matrix, and the reduced form coefficient matrix. The reduced form coefficient matrix is the product of the inverse of the endogenous coefficient matrix and the exogenous structural coefficient matrix.

Printed Output for Each Model

The results printed for each model include the analysis-of-variance table, the "Parameter Estimates" table, and optional items requested by TEST statements or by options in the MODEL statement.

The printed output produced for each model is described in the following.

The analysis-of-variance table includes the following:

- the model degrees of freedom, sum of squares, and mean square
- the error degrees of freedom, sum of squares, and mean square. The error mean square is computed by dividing the error sum of squares by the error degrees of freedom and is not affected by the VARDEF= option.

- the corrected total degrees of freedom and total sum of squares. Note that for instrumental variables methods, the model and error sums of squares do not add to the total sum of squares.
- the F ratio, labeled "F Value," and its significance, labeled "PROB>F," for the test of the hypothesis that all the nonintercept parameters are 0
- the root mean squared error. This is the square root of the error mean square.
- the dependent variable mean
- the coefficient of variation (CV) of the dependent variable
- the R^2 statistic. This R^2 is computed consistently with the calculation of the F statistic. It is valid for hypothesis tests but might not be a good measure of fit for models estimated by instrumental variables methods.
- the R^2 statistic adjusted for model degrees of freedom, labeled "Adj R-SQ"

The "Parameter Estimates" table includes the following:

- estimates of parameters for regressors in the model and the Lagrangian parameter for each restriction specified
- a degrees of freedom column labeled DF. Estimated model parameters have 1 degree of freedom. Restrictions have a DF of –1. Regressors or restrictions dropped from the model due to collinearity have a DF of 0.
- the standard errors of the parameter estimates
- the t statistics, which are the parameter estimates divided by the standard errors
- the significance of the t tests for the hypothesis that the true parameter is 0, labeled "Pr > |t|." As previously noted, the significance tests are strictly valid in finite samples only for OLS estimates but are asymptotically valid for the other methods.
- the standardized regression coefficients, if the STB option is specified. This is the parameter estimate multiplied by the ratio of the standard deviation of the regressor to the standard deviation of the dependent variable.
- the labels of the regressor variables or restriction labels

In addition to the analysis-of-variance table and the "Parameter Estimates" table, the results printed for each model can include the following:

- If TEST statements are specified, the test results are printed.
- If the DW option is specified, the Durbin-Watson statistic and first-order autocorrelation coefficient are printed.
- If the OVERID option is specified, the results of Basmann's test for overidentifying restrictions are printed.

- If the PLOT option is used, plots of residual against each regressor are printed.
- If the COVB or CORRB options are specified, the results for each model also include the covariance or correlation matrix of the parameter estimates. For systems methods like 3SLS and FIML, the COVB and CORB output is printed for the whole system after the output for the last model, instead of separately for each model.

The third-stage output for 3SLS, SUR, IT3SLS, ITSUR, and FIML does not include the analysis-of-variance table. When a systems method is used, the second-stage output does not include the optional output, except for the COVB and CORRB matrices.

ODS Table Names

PROC SYSLIN assigns a name to each table it creates. You can use these names to reference the table when you use the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the following table. If the estimation method used is 3SLS, IT3SLS, ITSUR or SUR, you can obtain tables by specifying ODS OUTPUT CorrResiduals, InvCorrResiduals, InvCovResiduals.

Table 36.2 ODS Tables Produced in PROC SYSLIN

ODS Table Name	Description	Option
ANOVA	Summary of the SSE, MSE for the equations	default
AugXPXMat	Model crossproducts	XPX or USSCP
AutoCorrStat	Autocorrelation statistics	DW
ConvergenceStatus	Convergence status	default
CorrB	Correlations of parameters	CORRB
CorrResiduals	Correlations of residuals	
CovB	Covariance of parameters	COVB
CovResiduals	Covariance of residuals	
EndoMat	Endogenous variables	REDUCED
ExogMat	Exogenous variables	REDUCED
FitStatistics	Statistics of fit	default
InvCorrResiduals	Inverse correlations of residuals	
InvCovResiduals	Inverse covariance of residuals	
InvEndoMat	Inverse endogenous variables	REDUCED
InvXPX	$X'X$ inverse for system	I
IterHistory	Iteration printing	ITPRINT
MissingValues	Missing values generated by the program	default
ModelVars	Name and label for the model	default
ParameterEstimates	Parameter estimates	default
RedMat	Reduced form	REDUCED
SimpleStatistics	Descriptive statistics	SIMPLE
SSCP	Model crossproducts	XPX or USSCP
TestResults	Test for overidentifying restrictions	
Weight	Weighted model statistics	

ODS Graphics

This section describes the use of ODS for creating graphics with the SYSLIN procedure.

ODS Graph Names

PROC SYSLIN assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when you use ODS. The names are listed in Table 36.3.

To request these graphs, you must specify the ODS GRAPHICS statement.

Table 36.3 ODS Graphics Produced by PROC SYSLIN

ODS Graph Name	Plot Description
DiagnosticsPanel	All applicable plots listed below
ActualByPredicted	Predicted versus actual plot
QQPlot	Q-Q plot of residuals
ResidualHistogram	Histogram of the residuals
ResidualPlot	Residual plot

Examples: SYSLIN Procedure

Example 36.1: Klein's Model I Estimated with LIML and 3SLS

This example uses PROC SYSLIN to estimate the classic Klein Model I. For a discussion of this model, see Theil (1971). The following statements read the data.

```
*---------------------------Klein's Model I----------------------------*
| By L.R. Klein, Economic Fluctuations in the United States, 1921-1941 |
| (1950), NY: John Wiley.   A macro-economic model of the U.S. with    |
| three behavioral equations, and several identities. See Theil, p.456.|
*----------------------------------------------------------------------*;
data klein;
input year c p w i x wp g t k wsum;
   date=mdy(1,1,year);
   format date monyy.;
   y   =c+i+g-t;
   yr  =year-1931;
   klag=lag(k);
   plag=lag(p);
   xlag=lag(x);
   label year='Year'
         date='Date'
         c   ='Consumption'
         p   ='Profits'
         w   ='Private Wage Bill'
         i   ='Investment'
```

```
         k   ='Capital Stock'
         y   ='National Income'
         x   ='Private Production'
         wsum='Total Wage Bill'
         wp  ='Govt Wage Bill'
         g   ='Govt Demand'
         i   ='Taxes'
         klag='Capital Stock Lagged'
         plag='Profits Lagged'
         xlag='Private Product Lagged'
         yr  ='YEAR-1931';
datalines;
1920     .  12.7     .    .  44.9    .     .     .  182.8     .
1921  41.9  12.4  25.5 -0.2  45.6  2.7   3.9   7.7  182.6  28.2
1922  45.0  16.9  29.3  1.9  50.1  2.9   3.2   3.9  184.5  32.2
1923  49.2  18.4  34.1  5.2  57.2  2.9   2.8   4.7  189.7  37.0
1924  50.6  19.4  33.9  3.0  57.1  3.1   3.5   3.8  192.7  37.0
1925  52.6  20.1  35.4  5.1  61.0  3.2   3.3   5.5  197.8  38.6
1926  55.1  19.6  37.4  5.6  64.0  3.3   3.3   7.0  203.4  40.7
1927  56.2  19.8  37.9  4.2  64.4  3.6   4.0   6.7  207.6  41.5
1928  57.3  21.1  39.2  3.0  64.5  3.7   4.2   4.2  210.6  42.9
1929  57.8  21.7  41.3  5.1  67.0  4.0   4.1   4.0  215.7  45.3
1930  55.0  15.6  37.9  1.0  61.2  4.2   5.2   7.7  216.7  42.1

   ... more lines ...
```

The following statements estimate the Klein model using the limited information maximum likelihood method. In addition, the parameter estimates are written to a SAS data set with the OUTEST= option.

```
proc syslin data=klein outest=b liml;
   endogenous c p w i x wsum k y;
   instruments klag plag xlag wp g t yr;
   consume: model c = p plag  wsum;
   invest:  model i = p plag  klag;
   labor:   model w = x xlag  yr;
run;

proc print data=b;
run;
```

The PROC SYSLIN estimates are shown in Output 36.1.1 through Output 36.1.3.

Output 36.1.1 LIML Estimates for Consumption

The SYSLIN Procedure
Limited-Information Maximum Likelihood Estimation

Model	CONSUME
Dependent Variable	c
Label	Consumption

Output 36.1.1 *continued*

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	854.3541	284.7847	118.42	<.0001
Error	17	40.88419	2.404952		
Corrected Total	20	941.4295			

Root MSE	1.55079	R-Square	0.95433
Dependent Mean	53.99524	Adj R-Sq	0.94627
Coeff Var	2.87209		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	17.14765	2.045374	8.38	<.0001	Intercept
p	1	-0.22251	0.224230	-0.99	0.3349	Profits
plag	1	0.396027	0.192943	2.05	0.0558	Profits Lagged
wsum	1	0.822559	0.061549	13.36	<.0001	Total Wage Bill

Output 36.1.2 LIML Estimates for Investments

The SYSLIN Procedure
Limited-Information Maximum Likelihood Estimation

Model	INVEST
Dependent Variable	i
Label	Taxes

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	210.3790	70.12634	34.06	<.0001
Error	17	34.99649	2.058617		
Corrected Total	20	252.3267			

Root MSE	1.43479	R-Square	0.85738
Dependent Mean	1.26667	Adj R-Sq	0.83221
Coeff Var	113.27274		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	22.59083	9.498146	2.38	0.0294	Intercept
p	1	0.075185	0.224712	0.33	0.7420	Profits
plag	1	0.680386	0.209145	3.25	0.0047	Profits Lagged
klag	1	-0.16826	0.045345	-3.71	0.0017	Capital Stock Lagged

Output 36.1.3 LIML Estimates for Labor

The SYSLIN Procedure
Limited-Information Maximum Likelihood Estimation

Model	LABOR
Dependent Variable	w
Label	Private Wage Bill

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	696.1485	232.0495	393.62	<.0001
Error	17	10.02192	0.589525		
Corrected Total	20	794.9095			

Root MSE	0.76781	**R-Square**	0.98581
Dependent Mean	36.36190	**Adj R-Sq**	0.98330
Coeff Var	2.11156		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.526187	1.320838	1.16	0.2639	Intercept
x	1	0.433941	0.075507	5.75	<.0001	Private Production
xlag	1	0.151321	0.074527	2.03	0.0583	Private Product Lagged
yr	1	0.131593	0.035995	3.66	0.0020	YEAR-1931

The OUTEST= data set is shown in part in Output 36.1.4. Note that the data set contains the parameter estimates and root mean squared errors, _SIGMA_, for the first-stage instrumental regressions as well as the parameter estimates and σ for the LIML estimates for the three structural equations.

Output 36.1.4 The OUTEST= Data Set

Obs	_TYPE_	_STATUS_	_MODEL_	_DEPVAR_	_SIGMA_	Intercept	klag	plag	xlag	wp
1	LIML	0 Converged	CONSUME	c	1.55079	17.1477	.	0.39603	.	.
2	LIML	0 Converged	INVEST	i	1.43479	22.5908	-0.16826	0.68039	.	.
3	LIML	0 Converged	LABOR	w	0.76781	1.5262	.	.	0.15132	.

Obs	g	t	yr	c	p	w	i	x	wsum	k	y
1	.	.	.	-1	-0.22251	.	.	.	0.82256	.	.
2	.	.	.	.	0.07518	.	-1	.	.	.	.
3	.	.	0.13159	.	.	-1	.	0.43394	.	.	.

The following statements estimate the model using the 3SLS method. The reduced form estimates are produced by the REDUCED option; IDENTITY statements are used to make the model complete.

```
proc syslin data=klein 3sls reduced;
   endogenous c p w i x wsum k y;
   instruments klag plag xlag wp g t yr;
   consume: model    c = p plag wsum;
   invest:  model    i = p plag klag;
```

```
    labor:   model    w = x xlag yr;
    product: identity x = c + i + g;
    income:  identity y = c + i + g - t;
    profit:  identity p = y - w;
    stock:   identity k = klag + i;
    wage:    identity wsum = w + wp;
run;
```

The preliminary 2SLS results and estimated cross-model covariance matrix are not shown. The 3SLS estimates are shown in Output 36.1.5 through Output 36.1.7. The reduced form estimates are shown in Output 36.1.8 through Output 36.1.11.

Output 36.1.5 3SLS Estimates for Consumption

The SYSLIN Procedure
Three-Stage Least Squares Estimation

System Weighted MSE	5.9342
Degrees of freedom	51
System Weighted R-Square	0.9550

Model	CONSUME
Dependent Variable	c
Label	Consumption

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	16.44079	1.449925	11.34	<.0001	Intercept
p	1	0.124890	0.120179	1.04	0.3133	Profits
plag	1	0.163144	0.111631	1.46	0.1621	Profits Lagged
wsum	1	0.790081	0.042166	18.74	<.0001	Total Wage Bill

Output 36.1.6 3SLS Estimates for Investments

Model	INVEST
Dependent Variable	i
Label	Taxes

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	28.17785	7.550853	3.73	0.0017	Intercept
p	1	-0.01308	0.179938	-0.07	0.9429	Profits
plag	1	0.755724	0.169976	4.45	0.0004	Profits Lagged
klag	1	-0.19485	0.036156	-5.39	<.0001	Capital Stock Lagged

Output 36.1.7 3SLS Estimates for Labor

Model	LABOR
Dependent Variable	w
Label	Private Wage Bill

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	1.797218	1.240203	1.45	0.1655	Intercept
x	1	0.400492	0.035359	11.33	<.0001	Private Production
xlag	1	0.181291	0.037965	4.78	0.0002	Private Product Lagged
yr	1	0.149674	0.031048	4.82	0.0002	YEAR-1931

Output 36.1.8 Reduced Form Estimates

Endogenous Variables

	c	p	w	i	x	wsum	k	y
CONSUME	1	-0.12489	0	0	0	-0.79008	0	0
INVEST	0	0.013079	0	1	0	0	0	0
LABOR	0	0	1	0	-0.40049	0	0	0
PRODUCT	-1	0	0	-1	1	0	0	0
INCOME	-1	0	0	-1	0	0	0	1
PROFIT	0	1	1	0	0	0	0	-1
STOCK	0	0	0	-1	0	0	1	0
WAGE	0	0	-1	0	0	1	0	0

Output 36.1.9 Reduced Form Estimates

Exogenous Variables

	Intercept	plag	klag	xlag	yr	g	t	wp
CONSUME	16.44079	0.163144	0	0	0	0	0	0
INVEST	28.17785	0.755724	-0.19485	0	0	0	0	0
LABOR	1.797218	0	0	0.181291	0.149674	0	0	0
PRODUCT	0	0	0	0	0	1	0	0
INCOME	0	0	0	0	0	1	-1	0
PROFIT	0	0	0	0	0	0	0	0
STOCK	0	0	1	0	0	0	0	0
WAGE	0	0	0	0	0	0	0	1

Output 36.1.10 Reduced Form Estimates

Inverse Endogenous Variables								
	CONSUME	INVEST	LABOR	PRODUCT	INCOME	PROFIT	STOCK	WAGE
c	1.634654	0.634654	1.095657	0.438802	0.195852	0.195852	0	1.291509
p	0.972364	0.972364	-0.34048	-0.13636	1.108721	1.108721	0	0.768246
w	0.649572	0.649572	1.440585	0.576943	0.072629	0.072629	0	0.513215
i	-0.01272	0.987282	0.004453	0.001783	-0.0145	-0.0145	0	-0.01005
x	1.621936	1.621936	1.10011	1.440585	0.181351	0.181351	0	1.281461
wsum	0.649572	0.649572	1.440585	0.576943	0.072629	0.072629	0	1.513215
k	-0.01272	0.987282	0.004453	0.001783	-0.0145	-0.0145	1	-0.01005
y	1.621936	1.621936	1.10011	0.440585	1.181351	0.181351	0	1.281461

Output 36.1.11 Reduced Form Estimates

Reduced Form								
	Intercept	plag	klag	xlag	yr	g	t	wp
c	46.7273	0.746307	-0.12366	0.198633	0.163991	0.634654	-0.19585	1.291509
p	42.77363	0.893474	-0.18946	-0.06173	-0.05096	0.972364	-1.10872	0.768246
w	31.57207	0.596871	-0.12657	0.261165	0.215618	0.649572	-0.07263	0.513215
i	27.6184	0.744038	-0.19237	0.000807	0.000667	-0.01272	0.014501	-0.01005
x	74.3457	1.490345	-0.31603	0.19944	0.164658	1.621936	-0.18135	1.281461
wsum	31.57207	0.596871	-0.12657	0.261165	0.215618	0.649572	-0.07263	1.513215
k	27.6184	0.744038	0.80763	0.000807	0.000667	-0.01272	0.014501	-0.01005
y	74.3457	1.490345	-0.31603	0.19944	0.164658	1.621936	-1.18135	1.281461

Example 36.2: Grunfeld's Model Estimated with SUR

The following example was used by Zellner in his classic 1962 paper on seemingly unrelated regressions. Different stock prices often move in the same direction at a given point in time. The SUR technique might provide more efficient estimates than OLS in this situation.

The following statements read the data. (The prefix GE stands for General Electric and WH stands for Westinghouse.)

```
*---------Zellner's Seemingly Unrelated Technique------------*
| A. Zellner, "An Efficient Method of Estimating Seemingly   |
| Unrelated Regressions and Tests for Aggregation Bias,"     |
| JASA 57(1962) pp.348-364                                   |
|                                                            |
| J.C.G. Boot, "Investment Demand: an Empirical Contribution |
| to the Aggregation Problem," IER 1(1960) pp.3-30.          |
|                                                            |
| Y. Grunfeld, "The Determinants of Corporate Investment,"   |
| Unpublished thesis, Chicago, 1958                          |
*------------------------------------------------------------*;

data grunfeld;
   input year ge_i ge_f ge_c wh_i wh_f wh_c;
   label ge_i = 'Gross Investment, GE'
```

```
         ge_c = 'Capital Stock Lagged, GE'
         ge_f = 'Value of Outstanding Shares Lagged, GE'
         wh_i = 'Gross Investment, WH'
         wh_c = 'Capital Stock Lagged, WH'
         wh_f = 'Value of Outstanding Shares Lagged, WH';
datalines;
1935     33.1      1170.6    97.8      12.93     191.5     1.8

   ... more lines ...
```

The following statements compute the SUR estimates for the Grunfeld model.

```
proc syslin data=grunfeld sur;
   ge:      model ge_i = ge_f ge_c;
   westing: model wh_i = wh_f wh_c;
run;
```

The PROC SYSLIN output is shown in Output 36.2.1 through Output 36.2.5.

Output 36.2.1 PROC SYSLIN Output for SUR

The SYSLIN Procedure
Ordinary Least Squares Estimation

Model	GE
Dependent Variable	ge_i
Label	Gross Investment, GE

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	31632.03	15816.02	20.34	<.0001
Error	17	13216.59	777.4463		
Corrected Total	19	44848.62			

Root MSE	27.88272	**R-Square**	0.70531
Dependent Mean	102.29000	**Adj R-Sq**	0.67064
Coeff Var	27.25850		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-9.95631	31.37425	-0.32	0.7548	Intercept
ge_f	1	0.026551	0.015566	1.71	0.1063	Value of Outstanding Shares Lagged, GE
ge_c	1	0.151694	0.025704	5.90	<.0001	Capital Stock Lagged, GE

Output 36.2.2 PROC SYSLIN Output for SUR

The SYSLIN Procedure
Ordinary Least Squares Estimation

Model	WESTING
Dependent Variable	wh_i
Label	Gross Investment, WH

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	5165.553	2582.776	24.76	<.0001
Error	17	1773.234	104.3079		
Corrected Total	19	6938.787			

Root MSE	10.21312	**R-Square**	0.74445
Dependent Mean	42.89150	**Adj R-Sq**	0.71438
Coeff Var	23.81153		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-0.50939	8.015289	-0.06	0.9501	Intercept
wh_f	1	0.052894	0.015707	3.37	0.0037	Value of Outstanding Shares Lagged, WH
wh_c	1	0.092406	0.056099	1.65	0.1179	Capital Stock Lagged, WH

Output 36.2.3 PROC SYSLIN Output for SUR

The SYSLIN Procedure
Seemingly Unrelated Regression Estimation

Cross Model Covariance

	GE	WESTING
GE	777.446	207.587
WESTING	207.587	104.308

Cross Model Correlation

	GE	WESTING
GE	1.00000	0.72896
WESTING	0.72896	1.00000

Cross Model Inverse Correlation

	GE	WESTING
GE	2.13397	-1.55559
WESTING	-1.55559	2.13397

Output 36.2.3 *continued*

Cross Model Inverse Covariance	GE	WESTING
GE	0.002745	-.005463
WESTING	-.005463	0.020458

Output 36.2.4 PROC SYSLIN Output for SUR

System Weighted MSE	0.9719
Degrees of freedom	34
System Weighted R-Square	0.6284

Model	GE
Dependent Variable	ge_i
Label	Gross Investment, GE

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-27.7193	29.32122	-0.95	0.3577	Intercept
ge_f	1	0.038310	0.014415	2.66	0.0166	Value of Outstanding Shares Lagged, GE
ge_c	1	0.139036	0.024986	5.56	<.0001	Capital Stock Lagged, GE

Output 36.2.5 PROC SYSLIN Output for SUR

Model	WESTING
Dependent Variable	wh_i
Label	Gross Investment, WH

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variable Label
Intercept	1	-1.25199	7.545217	-0.17	0.8702	Intercept
wh_f	1	0.057630	0.014546	3.96	0.0010	Value of Outstanding Shares Lagged, WH
wh_c	1	0.063978	0.053041	1.21	0.2443	Capital Stock Lagged, WH

Example 36.3: Illustration of ODS Graphics

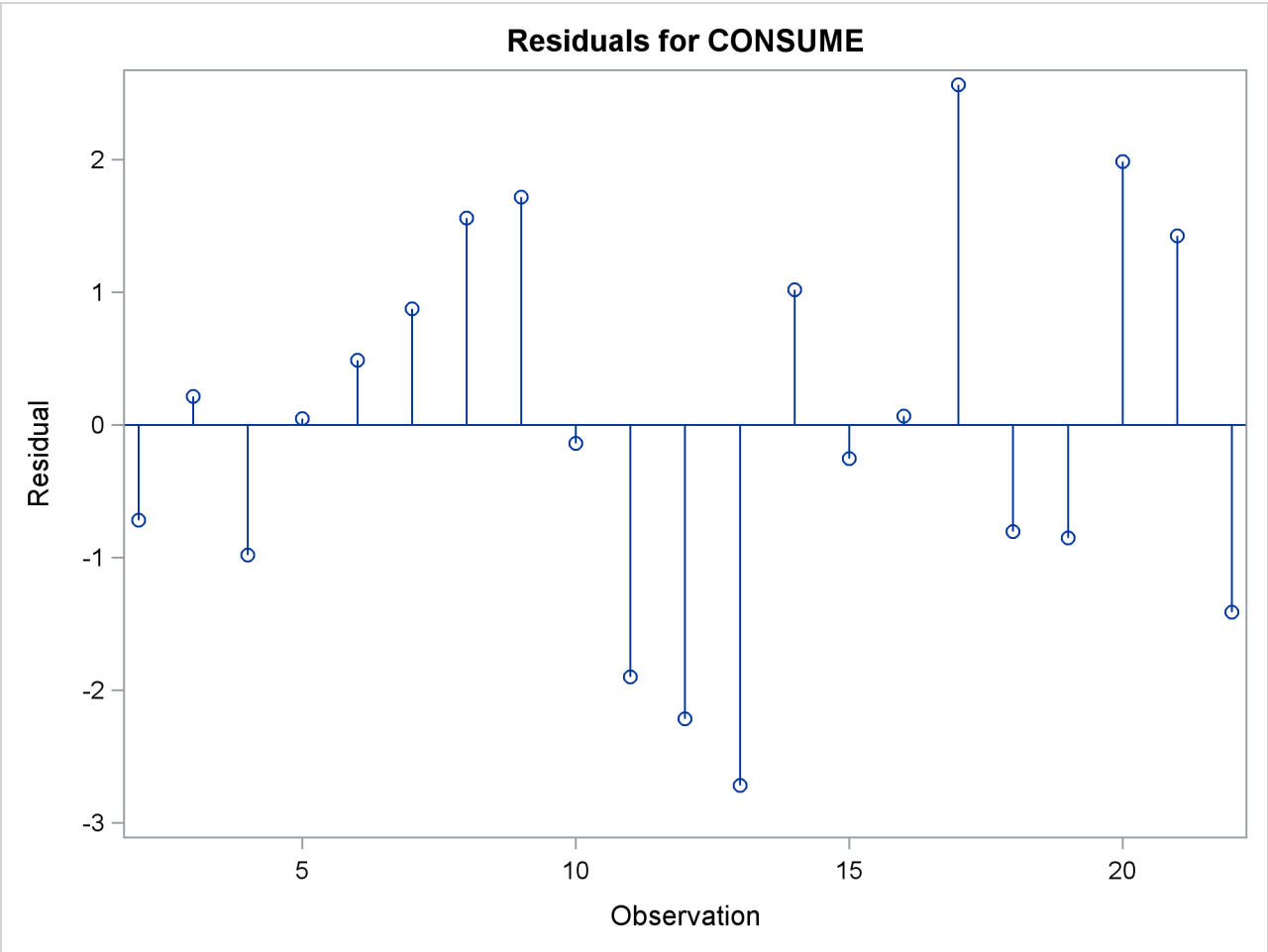
This example illustrates the use of ODS graphics. This is a continuation of the section "Example 36.1: Klein's Model I Estimated with LIML and 3SLS" on page 2590. These graphical displays are requested by specifying the ODS GRAPHICS statement before running PROC SYSLIN. For information about the graphics available in the SYSLIN procedure, see the section "ODS Graphics" on page 2590.

The following statements show how to generate ODS graphics plots with the SYSLIN procedure. The plots of residuals for each one of the equations in the model are displayed in Figure 36.3.1 through Figure 36.3.3.

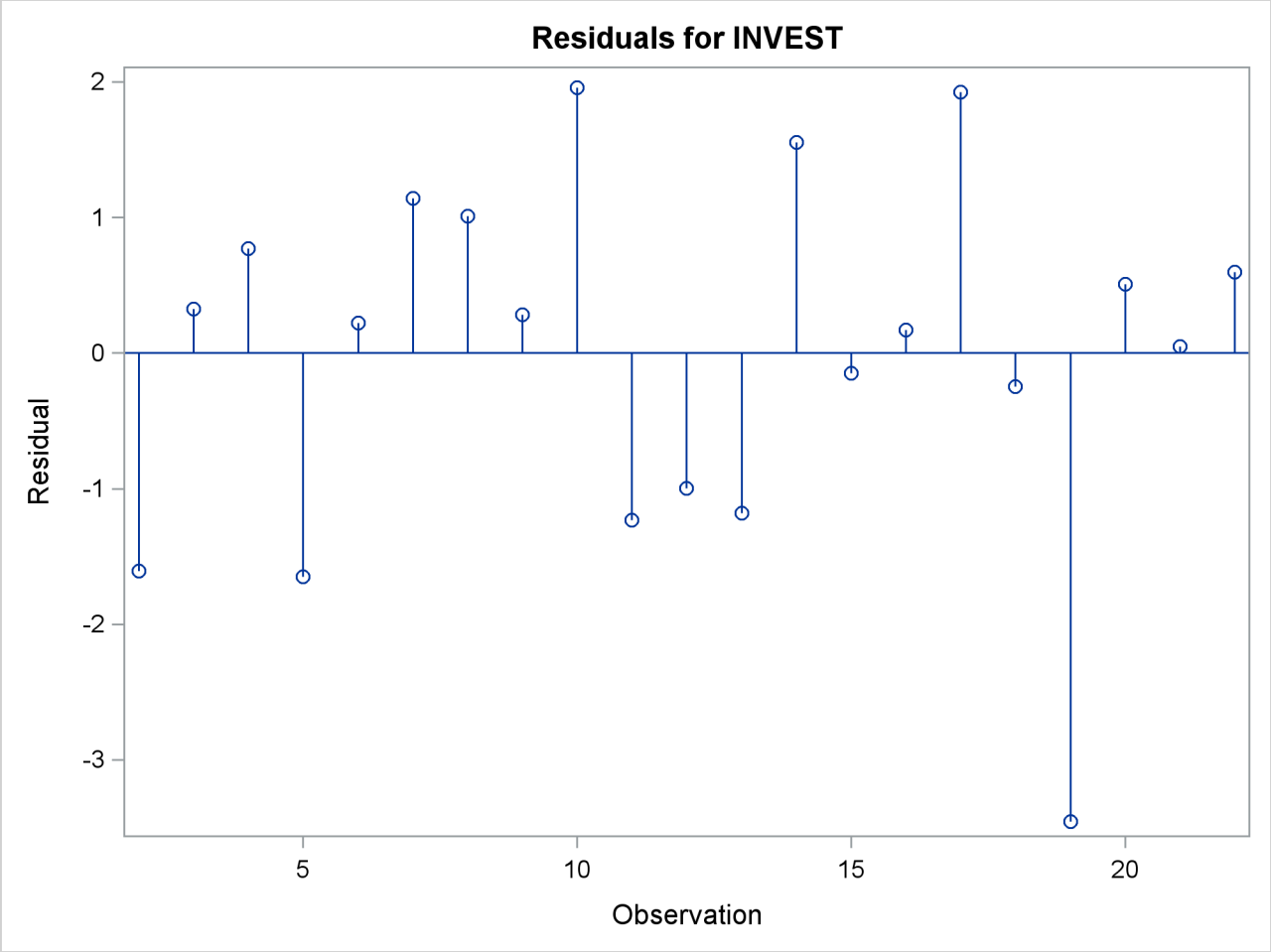
```
*---------------------------Klein's Model I----------------------------*
| By L.R. Klein, Economic Fluctuations in the United States, 1921-1941 |
| (1950), NY: John Wiley.   A macro-economic model of the U.S. with    |
| three behavioral equations, and several identities. See Theil, p.456.|
*----------------------------------------------------------------------*;
data klein;
input year c p w i x wp g t k wsum;
   date=mdy(1,1,year);
   format date monyy.;
   y   =c+i+g-t;
   yr  =year-1931;
   klag=lag(k);
   plag=lag(p);
   xlag=lag(x);
   label year='Year'
         date='Date'
         c   ='Consumption'
         p   ='Profits'
         w   ='Private Wage Bill'
         i   ='Investment'
         k   ='Capital Stock'
         y   ='National Income'
         x   ='Private Production'
         wsum='Total Wage Bill'
         wp  ='Govt Wage Bill'
         g   ='Govt Demand'
         i   ='Taxes'
         klag='Capital Stock Lagged'
         plag='Profits Lagged'
         xlag='Private Product Lagged'
         yr  ='YEAR-1931';
datalines;
1920     .  12.7     .    .  44.9    .     .     .  182.8     .
1921  41.9  12.4  25.5 -0.2  45.6  2.7   3.9   7.7  182.6  28.2
1922  45.0  16.9  29.3  1.9  50.1  2.9   3.2   3.9  184.5  32.2
1923  49.2  18.4  34.1  5.2  57.2  2.9   2.8   4.7  189.7  37.0
1924  50.6  19.4  33.9  3.0  57.1  3.1   3.5   3.8  192.7  37.0
1925  52.6  20.1  35.4  5.1  61.0  3.2   3.3   5.5  197.8  38.6
1926  55.1  19.6  37.4  5.6  64.0  3.3   3.3   7.0  203.4  40.7
1927  56.2  19.8  37.9  4.2  64.4  3.6   4.0   6.7  207.6  41.5
1928  57.3  21.1  39.2  3.0  64.5  3.7   4.2   4.2  210.6  42.9
1929  57.8  21.7  41.3  5.1  67.0  4.0   4.1   4.0  215.7  45.3
1930  55.0  15.6  37.9  1.0  61.2  4.2   5.2   7.7  216.7  42.1

   ... more lines ...
```

```
ods graphics on;

proc syslin data=klein outest=b liml plots(unpack only)=residual ;
   endogenous c p w i x wsum k y;
   instruments klag plag xlag wp g t yr;
   consume: model c = p plag  wsum;
   invest:  model i = p plag  klag;
   labor:   model w = x xlag  yr;
run;
```

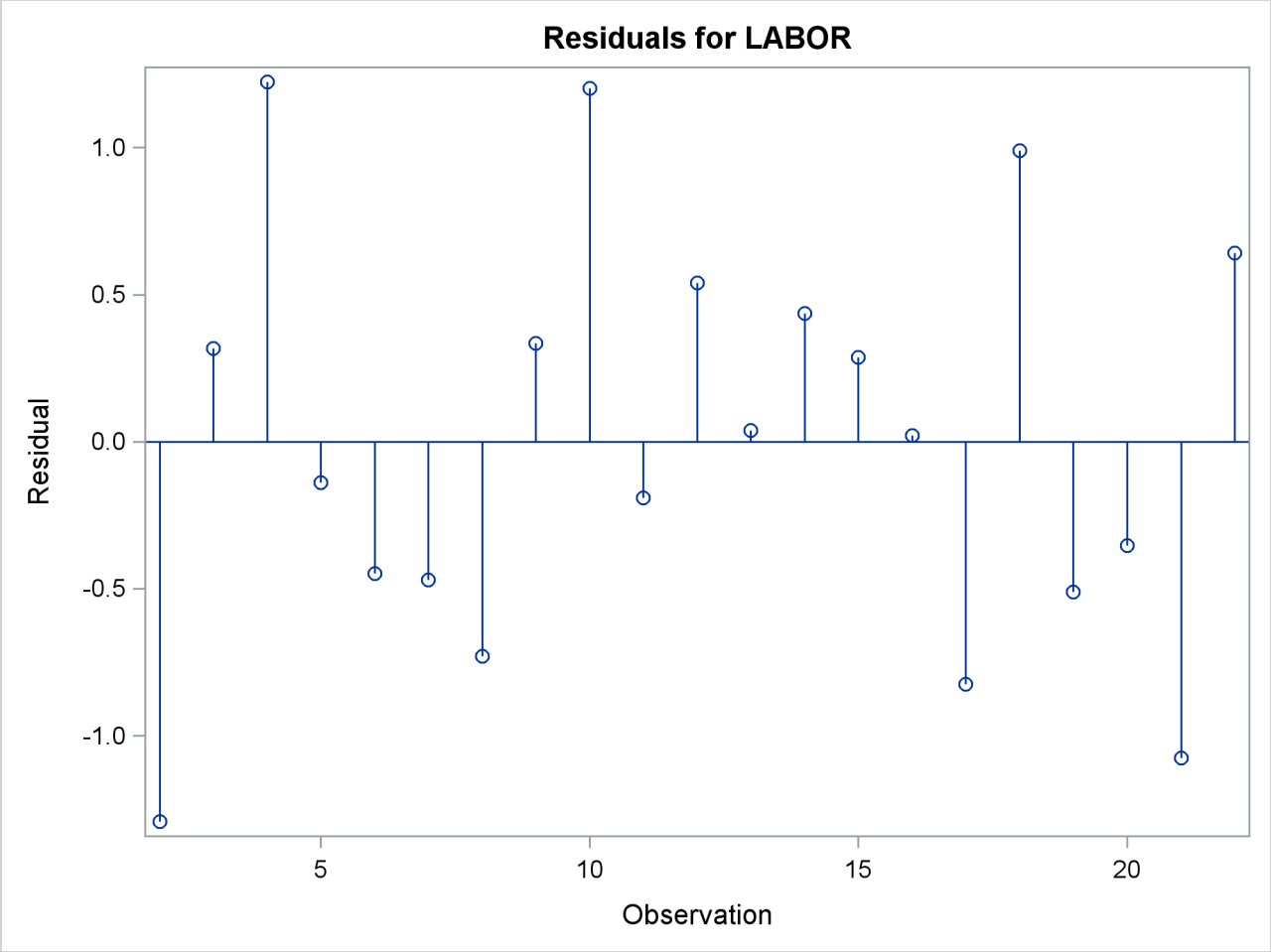
Output 36.3.1 Residuals Diagnostic Plots for Consumption



Output 36.3.2 Residuals Diagnostic Plots for Investments



Output 36.3.3 Residuals Diagnostic Plots for Labor



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