What’s New in SAS 9.1 SAS
High-Performance Forecasting

Overview

New features have been added to the following SAS High-Performance Forecasting components:

- PROC HPF

Details

HPF Procedure

The HPF procedure has new options related to forecast model selection:

- HOLDOUTPCT= option
- SEASONTEST= option
- SELECT= option

The HPF procedure has new options related to forecast model initialization:

- NBACKCAST= option

The HPF procedure has new statement related to intermittent demand models:

- IDM statement

The HPF procedure has new options related to out-of-sample forecast performance statistics:

- BACK= option
- PRINT=PERFORMANCE option
- PRINT=PERFORMANCESUMMARY option
- PRINT=PERFORMANCEOVERALL option

The HPF procedure has new options related to the processing of the data:
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- NOTSORTED option
- REPLACEBACK option
- SORTNAMES option

The HPF procedure has new options related to printed and graphical output:

- PLOT= option
- PRINT=STATES option

The HPF procedure has a new option related to message log:

- MAXERROR= option
## Chapter 1
The HPF Procedure

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Chapter 1
The HPF Procedure
Overview

The HPF (High-Performance Forecasting) procedure provides a quick and automatic way to generate forecasts for many time series or transactional data in one step. The procedure can forecast millions of series at a time, with the series organized into separate variables or across BY groups.

- For typical time series, you can use the following smoothing models:
  - Simple
  - Double
  - Linear
  - Damped Trend
  - Seasonal
  - Winters Method (additive and multiplicative)

- Additionally, transformed versions of these models are provided:
  - Log
  - Square Root
  - Logistic
  - Box-Cox

- For intermittent time series (series where a large number of values are zero-valued), you can use an intermittent demand model such as Croston’s Method and Average Demand Model.

Experimental graphics are now available with the HPF procedure. For more information, see the “ODS Graphics” section on page 39.

All parameters associated with the forecast model are optimized based on the data. Optionally, the HPF procedure can select the appropriate smoothing model for you using holdout sample analysis based on one of several model selection criteria.

The HPF procedure writes the time series extrapolated by the forecasts, the series summary statistics, the forecasts and confidence limits, the parameter estimates, and the fit statistics to output data sets. The HPF procedure optionally produces printed output for these results utilizing the Output Delivery System (ODS).

The HPF procedure can forecast both time series data, whose observations are equally spaced by a specific time interval (e.g., monthly, weekly), or transactional data, whose observations are not spaced with respect to any particular time interval. Internet, inventory, sales, and similar data are typical examples of transactional data. For transactional data, the data is accumulated based on a specified time interval to form a
time series. The HPF procedure can also perform trend and seasonal analysis on transactional data.

Additionally, the Time Series Forecasting System of SAS/ETS software can be used to interactively develop forecasting models, estimate the model parameters, evaluate the models’ ability to forecast, and display these results graphically. Refer to Chapter 33, “Overview of the Time Series Forecasting System,” *(SAS/ETS User’s Guide)* for details.

Also, the EXPAND procedure can be used for the frequency conversion and transformations of time series. Refer to Chapter 16, “The EXPAND Procedure,” *(SAS/ETS User’s Guide)* for details.

### Getting Started

The HPF procedure is simple to use for someone who is new to forecasting, and yet at the same time it is powerful for the experienced professional forecaster who needs to generate a large number of forecasts automatically. It can provide results in output data sets or in other output formats using the Output Delivery System (ODS). The following examples are more fully illustrated in the “Examples” section on page 42.

Given an input data set that contains numerous time series variables recorded at a specific frequency, the HPF procedure can automatically forecast the series as follows:

```
PROC HPF DATA=<input-data-set> OUT=<output-data-set>;
  ID <time-ID-variable> INTERVAL=<frequency>;
  FORECAST <time-series-variables>;
RUN;
```

For example, suppose that the input data set `SALES` contains numerous sales data recorded monthly, the variable that represents time is `DATE`, and the forecasts are to be recorded in the output data set `NEXTYEAR`. The HPF procedure could be used as follows:

```
proc hpf data=sales out=nextyear;
  id date interval=month;
  forecast _ALL_
run;
```

The above statements automatically select the best fitting model, generate forecasts for every numeric variable in the input data set (`SALES`) for the next twelve months, and store these forecasts in the output data set (`NEXTYEAR`). Other output data sets can be specified to store the parameter estimates, forecasts, statistics of fit, and summary data.

If you want to print the forecasts using the Output Delivery System (ODS), then you need to add `PRINT=FORECASTS`: 
proc hpf data=sales out=nextyear print=forecasts;
   id date interval=month;
   forecast _ALL_;
run;

Other results can be specified to output the parameter estimates, forecasts, statistics of fit, and summary data using ODS.

The HPF procedure can forecast both time series data, whose observations are equally spaced by a specific time interval (e.g., monthly, weekly), or transactional data, whose observations are not spaced with respect to any particular time interval.

Given an input data set containing transactional variables not recorded at any specific frequency, the HPF procedure accumulates the data to a specific time interval and forecasts the accumulated series as follows:

PROC HPF DATA=<input-data-set> OUT=<output-data-set>;
   ID <time-ID-variable> INTERVAL=<frequency>
      ACCUMULATE=<accumulation>;
   FORECAST <time-series-variables>;
RUN;

For example, suppose that the input data set WEBSITES contains three variables (BOATS, CARS, PLANES), that are Internet data recorded on no particular time interval, and the variable that represents time is TIME, which records the time of the Web hit. The forecasts for the total daily values are to be recorded in the output data set NEXTWEEK. The HPF procedure could be used as follows:

proc hpf data=websites out=nextweek lead=7;
   id time interval=dtday accumulate=total;
   forecast boats cars planes;
run;

The above statements accumulate the data into a daily time series and automatically generate forecasts for the BOATS, CARS, and PLANES variables in the input data set (WEBSITES) for the next seven days and store the forecasts in the output data set (NEXTWEEK).

The HPF procedure can specify a particular forecast model or select from several candidate models based on a selection criterion. The HPF procedure also supports transformed models and holdout sample analysis.

Using the previous WEBSITES example, suppose that you want to forecast the BOATS variable using the best seasonal forecasting model that minimizes the mean absolute percent error (MAPE), the CARS variable using the best nonseasonal forecasting model that minimizes the mean square error (MSE) using holdout sample analysis on the last five days, and the PLANES variable using the Log Winters Method (additive). The HPF procedure could be used as follows:
Chapter 1. The HPF Procedure

```sas
proc hpf data=websites out=nextweek lead=7;
   id time interval=dtday accumulate=total;
   forecast boats / model=bests select=mape;
   forecast cars / model=bestn select=mse holdout=5;
   forecast planes / model=addwinters transform=log;
run;
```

The above statements demonstrate how each variable in the input data set can be modeled differently and how several candidate models can be specified and selected based on holdout sample analysis or the entire range of data.

The HPF procedure is also useful in extending independent variables in (auto) regression models where future values of the independent variable are needed to predict the dependent variable.

Using the WEBSITES example, suppose that you want to forecast the ENGINES variable using the BOATS, CARS, and PLANES variable as regressor variables. Since future values of the BOATS, CARS, and PLANES are needed, the HPF procedure can be used to extend these variables in the future:

```sas
proc hpf data=websites out=nextweek lead=7;
   id time interval=dtday accumulate=total;
   forecast engines / model=none;
   forecast boats / model=bests select=mape;
   forecast cars / model=bestn select=mse holdout=5;
   forecast planes / model=addwinters transform=log;
run;
```

```sas
proc autoreg data=nextweek;
   model engines = boats cars planes;
   output out=enginehits p=predicted;
run;
```

The above HPF procedure statements generate forecasts for BOATS, CARS, and PLANES in the input data set (WEBSITES) for the next seven days and extend the variable ENGINES with missing values. The output data set (NEXTWEEK) of the PROC HPF statement is used as an input data set for the PROC AUTOREG statement. The output data set of PROC AUTOREG contains the forecast of the variable ENGINE based on the regression model with the variables BOATS, CARS, and PLANES as regressors. See the AUTOREG procedure for details on autoregression.

The HPF procedure can also forecast intermittent time series (series where a large number of values are zero-valued). Typical time series forecasting techniques are less effective in forecasting intermittent time series.

For example, suppose that the input data set INVENTORY contains three variables (TIRES, HUBCAPS, LUGBOLTS) that are demand data recorded on no particular time interval, the variable that represents time is DATE, and the forecasts for the total weekly values are to be recorded in the output data set NEXTMONTH. The models requested are intermittent demand models, which can be specified as MODEL=IDM
option. Two intermittent demand models are compared, Croston Model and Average Demand Model. The HPF procedure could be used as follows:

```plaintext
proc hpf data=inventory out=nextmonth lead=4 print=forecasts;
   id date interval=week accumulate=total;
   forecast tires hubcaps lugbolts / model=idm;
run;
```

In the above example, the total demand for inventory items is accumulated on a weekly basis and forecasts are generated that recommend future stocking levels.

## Syntax

The following statements are used with the HPF procedure.

- **PROC HPF** *options*;
  - **BY** *variables*;
  - **FORECAST** *variable-list / options*;
  - **ID** *variable INTERVAL= interval options*;
  - **IDM** *options*;

## Functional Summary

The statements and options controlling the HPF procedure are summarized in the following table.

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<td>specify variables to forecast</td>
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<td>specify the time ID variable</td>
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<td>specify forecast output data set</td>
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<td>PROC HPF</td>
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<td>specify statistics output data set</td>
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<td>specify ending time ID value</td>
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The following options can be used in the PROC HPF statement.

**BACK= n**
- specifies the number of observations before the end of the data that the multistep forecasts are to begin. The default is BACK=0.

**DATA= SAS-data-set**
- names the SAS data set containing the input data for the procedure to forecast. If the DATA= option is not specified, the most recently created SAS data set is used.

**LEAD= n**
- specifies the number of periods ahead to forecast (forecast lead or horizon). The default is LEAD=12.

  The LEAD= value is relative to the last observation in the input data set and not to the last nonmissing observation of a particular series. Thus, if a series has missing values at the end, the actual number of forecasts computed for that series will be greater than the LEAD= value.

**MAXERROR= number**
- limits the number of warning and error messages produced during the execution of the procedure to the specified value. The default is MAXERRORS=50. This option is particularly useful in BY-group processing where it can be used to suppress the recurring messages.

**NOOUTALL**
- specifies that only forecasts are written to the OUT= and OUTFOR= data sets. The NOOUTALL option includes only the final forecast observations in the output data sets, not the one-step forecasts for the data before the forecast period.

  The OUT= and OUTFOR= data set will only contain the forecast results starting at the next period following the last observation to the forecast horizon specified by the LEAD= option.

**OUT= SAS-data-set**
- names the output data set to contain the forecasts of the variables specified in the subsequent FORECAST statements. If an ID variable is specified, it will also be included in the OUT= data set. The values are accumulated based on the ACCUMULATE=
option and forecasts are appended to these values based on the FORECAST statement USE= option. The OUT= data set is particularly useful in extending the independent variables when forecasting dependent series associated with (auto) regression models. If the OUT= option is not specified, a default output data set DATAn is created. If you do not want the OUT= data set created, then use OUT=_NULL_.

OUTEST= SAS-data-set
names the output data set to contain the model parameter estimates and the associated test statistics and probability values. The OUTEST= data set is particularly useful for evaluating the significance of the model parameters and understanding the model dynamics.

OUTFOR= SAS-data-set
names the output data set to contain the forecast time series components (actual, predicted, lower confidence limit, upper confidence limit, prediction error, prediction standard error). The OUTFOR= data set is particularly useful for displaying the forecasts in tabular or graphical form.

OUTSEASON= SAS-data-set
names the output data set to contain the seasonal statistics. The statistics are computed for each season as specified by the ID statement INTERVAL= option or the SEASONALITY= option. The OUTSEASON= data set is particularly useful when analyzing transactional data for seasonal variations.

OUTSTAT= SAS-data-set
names the output data set to contain the statistics of fit (or goodness-of-fit statistics). The OUTSTAT= data set is particularly useful for evaluating how well the model fits the series. The statistics of fit are based on the entire range of the time series regardless of whether the HOLDOUT= option is specified.

OUTSUM= SAS-data-set
names the output data set to contain the summary statistics and the forecast summation. The summary statistics are based on the accumulated time series when the ACCUMULATE= or SETMISSING= options are specified. The forecast summations are based on the LEAD=, STARTSUM=, and USE= options. The OUTSUM= data set is particularly useful when forecasting large numbers of series and a summary of the results are needed.

OUTTREND= SAS-data-set
names the output data set to contain the trend statistics. The statistics are computed for each time period as specified by the ID statement INTERVAL= option. The OUTTREND= data set is particularly useful when analyzing transactional data for trends.

PRINT= option | (options)
specifies the printed output desired. By default, the HPF procedure produces no printed output. The following printing options are available:

ESTIMATES               prints the results of parameter estimation. (OUTEST= data set)
FORECASTS               prints the forecasts. (OUTFOR= data set)
PROC HPF Statement

PERFORMANCE prints the performance statistics for each forecast.

PERFORMANCESUMMARY prints the performance summary for each BY group.

PERFORMANCEOVERALL prints the performance summary for all of the BY groups.

SEASONS prints the seasonal statistics. (OUTSEASON= data set)

STATISTICS prints the statistics of fit. (OUTSTAT= data set)

STATES prints the backcast, initial, and final states.

SUMMARY prints the summary statistics for the accumulated time series. (OUTSUM= data set)

TRENDS prints the trend statistics. (OUTTREND= data set)

ALL Same as PRINT=(ESTIMATES FORECASTS STATISTICS SUMMARY). PRINT=(ALL,TRENDS,SEASONS) prints all of the options listed above.

For example, PRINT=FORECASTS prints the forecasts, PRINT=(ESTIMATES, FORECASTS) prints the parameter estimates and the forecasts, and PRINT=ALL prints all of the above output.

The PRINT= option produces printed output for these results utilizing the Output Delivery System (ODS). The PRINT= option produces results similar to the data sets listed next to the above options in parenthesis.

PRINTDETAILS
specifies that output requested with the PRINT= option be printed in greater detail.

SEASONALITY= number
specifies the length of the seasonal cycle. For example, SEASONALITY=3 means that every group of three observations forms a seasonal cycle. The SEASONALITY= option is applicable only for seasonal forecasting models. By default, the length of the seasonal cycle is one (no seasonality) or the length implied by the INTERVAL= option specified in the ID statement. For example, INTERVAL=MONTH implies that the length of the seasonal cycle is twelve.

SORTNAMES
specifies that the variables specified in the FORECAST statements are processed in sorted order.

STARTSUM= n
specifies the starting forecast lead (or horizon) for which to begin summation of the forecasts specified by the LEAD= option. The STARTSUM= value must be less than the LEAD= value. The default is STARTSUM=1, that is, the sum from the one-step ahead forecast to the multistep forecast specified by the LEAD= option.

The prediction standard errors of the summation of forecasts takes into account the correlation between the multistep forecasts. The DETAILS section describes the STARTSUM= option in more detail.
**BY Statement**

```
BY variables;
```

A BY statement can be used with PROC HPF to obtain separate analyses for groups of observations defined by the BY variables.

**FORECAST Statement**

```
FORECAST variable-list / options;
```

The FORECAST statement lists the numeric variables in the DATA= data set whose accumulated values represent time series to be modeled and forecast. The options specify which forecast model is to be used or how the forecast model is selected from several possible candidate models.

A data set variable can be specified in only one FORECAST statement. Any number of FORECAST statements can be used. The following options can be used with the FORECAST statement.

**ACCUMULATE= option**

specifies how the data set observations are accumulated within each time period for the variables listed in the FORECAST statement. If the ACCUMULATE= option is not specified in the FORECAST statement, accumulation is determined by the ACCUMULATE= option of the ID statement. See the ID statement ACCUMULATE= option for more details.

**ALPHA= number**

specifies the significance level to use in computing the confidence limits of the forecast. The ALPHA=value must be between 0 and 1. The default is ALPHA=0.05, which produces 95% confidence intervals.

**HOLDOUT= n**

specifies the size of the holdout sample to be used for model selection. The holdout sample is a subset of actual time series ending at the last nonmissing observation. If the ACCUMULATE= option is specified, the holdout sample is based on the accumulated series. If the holdout sample is not specified, the full range of the actual time series is used for model selection.

For each candidate model specified, the holdout sample is excluded from the initial model fit and forecasts are made within the holdout sample time range. Then, for each candidate model specified, the statistic of fit specified by the SELECT= option is computed using only the observations in the holdout sample. Finally, the candidate model, which performs best in the holdout sample, based on this statistic, is selected to forecast the actual time series.

The HOLDOUT= option is only used to select the best forecasting model from a list of candidate models. After the best model is selected, the full range of the actual time series is used for subsequent model fitting and forecasting. It is possible that one model will outperform another model in the holdout sample but perform less well when the entire range of the actual series is used.
If MODEL=BESTALL and HOLDOUT= options are used together, the last one hundred observations are used to determine whether the series is intermittent. If the series determined not to be intermittent, holdout sample analysis will be used to select the smoothing model.

**HOLDOUTPCT= number**
specifies the size of the holdout sample as a percentage of the length of the time series. If HOLDOUT=5 and HOLDOUTPCT=10, the size of the holdout sample is \( \min(5, 0.1T) \) where \( T \) is the length of the time series with beginning and ending missing values removed. The default is 100 (100%).

**INTERMITTENT= number**
specifies a number greater than one which is used to determine whether or not a time series is intermittent. If the average demand interval is greater than this number then the series is assumed to be intermittent. This option is used with MODEL=BESTALL option. The default is INTERMITTENT=1.25.

**MEDIAN**
specifies that the median forecast values are to be estimated. Forecasts can be based on the mean or median. By default the mean value is provided. If no transformation is applied to the actual series using the TRANSFORM= option, the mean and median forecast values are identical.

**MODEL= model-name**
specifies the forecasting model to be used to forecast the actual time series. A single model can be specified or a group of candidate models can be specified. If a group of models is specified, the model used to forecast the accumulated time series is selected based on the SELECT= option and the HOLDOUT= option. The default is MODEL=BEST. The following forecasting models are provided:

- **NONE** No forecast. The accumulated time series is appended with missing values in the OUT= data set. This option is particularly useful when the results stored in the OUT= data set are subsequently used in (auto) regression analysis where forecasts of the independent variables are needed to forecast the dependent variable.
- **SIMPLE** Simple (Single) Exponential Smoothing
- **DOUBLE** Double (Brown) Exponential Smoothing
- **LINEAR** Linear (Holt) Exponential Smoothing
- **DAMPTREND** Damped Trend Exponential Smoothing
- **SEASONAL** Seasonal Exponential Smoothing
- **WINTERS** Winters Multiplicative Method
- **ADDWINTERS** Winters Additive Method
- **BEST** Best Candidate Smoothing Model (SIMPLE, DOUBLE, LINEAR, DAMPTREND, SEASONAL, WINTERS, ADDWINTERS)
- **BESTN** Best Candidate Nonseasonal Smoothing Model (SIMPLE, DOUBLE, LINEAR, DAMPTREND)
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BESTS Best Candidate Seasonal Smoothing Model (SEASONAL, WINTERS, ADDWINTERS)

IDM|CROSTON Intermittent Demand Model such as Croston’s Method or Average Demand Model. An intermittent time series is one whose values are mostly zero.

BESTALL Best Candidate Model (IDM, BEST)

The BEST, BESTN, and BESTS options specify a group of models by which the HOLDOUT= option and SELECT= option are used to select the model used to forecast the accumulated time series based on holdout sample analysis. Transformed versions of the above smoothing models can be specified using the TRANSFORM= option.

The BESTALL option specifies that if the series is intermittent, an intermittent demand model such as Croston’s Method or Average Demand Model (MODEL=IDM) is selected; otherwise, the best smoothing model is selected (MODEL=BEST). Intermittency is determined by the INTERMITTENT= option.

The documentation for Chapter 2, “Forecasting Process Details,” describes the above smoothing models and intermittent models in greater detail.

NBACKCAST= n specifies the number of observations used to initialize the backcast states. The default is the entire series.

REPLACEBACK specifies that actual values excluded by the BACK= option are replaced with one-step-ahead forecasts in the OUT= data set.

REPLACEMISSING specifies that embedded missing actual values are replaced with one-step-ahead forecasts in the OUT= data set.

SEASONTEST= option specifies the options related to the seasonality test. This option is used with MODEL=BEST and MODEL=BESTALL option.

The following options are provided:

SEASONTEST=NONE no test
SEASONTEST=(SIGLEVEL=number) Significance probability value.

Series with strong seasonality have small test probabilities. SEASONTEST=(SIGLEVEL=0) always implies seasonality. SEASONTEST=(SIGLEVEL=1) always implies no seasonality. The default is SEASONTEST=(SIGLEVEL=0.01).

SELECT= option specifies the model selection criterion (statistic of fit) to be used to select from several candidate models. This option would often be used in conjunction with the
HOLDOUT= option. The default is SELECT=RMSE. The following statistics of fit are provided:

- **SSE** Sum of Square Error
- **MSE** Mean Square Error
- **RMSE** Root Mean Square Error
- **UMSE** Unbiased Mean Square Error
- **URMSE** Unbiased Root Mean Square Error
- **MAXPE** Maximum Percent Error
- **MINPE** Minimum Percent Error
- **MPE** Mean Percent Error
- **MAPE** Mean Absolute Percent Error
- **MDAPE** Median Percent Error
- **GMAPE** Geometric Mean Percent Error
- **MINPPE** Minimum Predictive Percent Error
- **MAXPPE** Maximum Predictive Percent Error
- **MSPPE** Mean Predictive Percent Error
- **MAPPE** Median Predictive Percent Error
- **GMAPPE** Geometric Mean Predictive Percent Error
- **MINSPE** Minimum Symmetric Percent Error
- **MAXSPE** Maximum Symmetric Percent Error
- **MSPE** Mean Symmetric Percent Error
- **SMAPE** Symmetric Mean Absolute Percent Error
- **MDASPE** Median Symmetric Percent Error
- **GMASPE** Geometric Mean Symmetric Percent Error
- **MINRE** Minimum Relative Error
- **MAXRE** Maximum Relative Error
- **MRE** Mean Relative Error
- **MRAE** Mean Relative Absolute Error
- **MDRAE** Median Relative Absolute Error
- **GMRAE** Geometric Mean Relative Absolute Error
- **MAXERR** Maximum Error
- **MINERR** Minimum Error
- **ME** Mean Error
- **MAE** Mean Absolute Error
- **RSQUARE** R-Square
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ADJRSQ  Adjusted R-Square
AADJRSQ Amemiya’s Adjusted R-Square
RWRSQ  Random Walk R-Square
AIC    Akaike Information Criterion
SBC    Schwarz Bayesian Information Criterion
APC    Amemiya’s Prediction Criterion

**SETMISSING= option | number**
specifies how missing values (either actual or accumulated) are assigned in the accumulated time series for variables listed in the FORECAST statement. If the SETMISSING= option is not specified in the FORECAST statement, missing values are set based on the SETMISSING= option of the ID statement. See the ID statement SETMISSING= option for more details.

**TRANSFORM= option**
specifies the time series transformation to be applied to the actual time series. The following transformations are provided:

- **NONE** No transformation is applied. This option is the default.
- **LOG** Logarithmic transformation
- **SQRT** Square-root transformation
- **LOGISTIC** Logistic transformation
- **BOXCOX(n)** Box-Cox transformation with parameter number where number is between -5 and 5
- **AUTO** Automatically choose between NONE and LOG based on model selection criteria.

When the TRANSFORM= option is specified the time series must be strictly positive. Once the time series is transformed, the model parameters are estimated using the transformed series. The forecasts of the transformed series are then computed, and finally, the transformed series forecasts are inverse transformed. The inverse transform produces either mean or median forecasts depending on whether the MEDIAN option is specified.

The TRANSFORM= option is not applicable when MODEL=IDM is specified.

**USE= option**
specifies which forecast values are appended to the actual values in the OUT= and OUTSUM= data sets. The following USE= options are provided:

- **PREDICT** The predicted values are appended to the actual values. This option is the default.
- **LOWER** The lower confidence limit values are appended to the actual values.
UPPER  The upper confidence limit values are appended to the actual values.

Thus, the USE= option enables the OUT= and OUTSUM= data sets to be used for worst/best/average/median case decisions.

**ZEROMISS= option**

specifies how beginning and/or ending zero values (either actual or accumulated) are interpreted in the accumulated time series for variables listed in the FORECAST statement. If the ZEROMISS= option is not specified in the FORECAST statement, missing values are set based on the ZEROMISS= option of the ID statement. See the ID statement ZEROMISS= option for more details.

**ID Statement**

**ID variable INTERVAL= interval options;**

The ID statement names a numeric variable that identifies observations in the input and output data sets. The ID variable's values are assumed to be SAS date, time, or datetime values. In addition, the ID statement specifies the (desired) frequency associated with the actual time series. The ID statement options also specify how the observations are accumulated and how the time ID values are aligned to form the actual time series. The information specified affects all variables specified in subsequent FORECAST statements. If the ID statement is specified, the INTERVAL= option must also be specified. If an ID statement is not specified, the observation number, with respect to the BY group, is used as the time ID.

The following options can be used with the ID statement.

**ACCUMULATE= option**

specifies how the data set observations are accumulated within each time period. The frequency (width of each time interval) is specified by the INTERVAL= option. The ID variable contains the time ID values. Each time ID variable value corresponds to a specific time period. The accumulated values form the actual time series, which is used in subsequent model fitting and forecasting.

The ACCUMULATE= option is particularly useful when there are zero or more than one input observations coinciding with a particular time period (e.g., transactional data). The EXPAND procedure offers additional frequency conversions and transformations that can also be useful in creating a time series.

The following options determine how the observations are accumulated within each time period based on the ID variable and the frequency specified by the INTERVAL= option:

**NONE**  No accumulation occurs; the ID variable values must be equally spaced with respect to the frequency. This is the default option.
TOTAL Observations are accumulated based on the total sum of their values.

AVERAGE | AVG Observations are accumulated based on the average of their values.

MINIMUM | MIN Observations are accumulated based on the minimum of their values.

MEDIAN | MED Observations are accumulated based on the median of their values.

MAXIMUM | MAX Observations are accumulated based on the maximum of their values.

N Observations are accumulated based on the number of nonmissing observations.

NMISS Observations are accumulated based on the number of missing observations.

NOBS Observations are accumulated based on the number of observations.

FIRST Observations are accumulated based on the first of their values.

LAST Observations are accumulated based on the last of their values.

STDDEV | STD Observations are accumulated based on the standard deviation of their values.

CSS Observations are accumulated based on the corrected sum of squares of their values.

USS Observations are accumulated based on the uncorrected sum of squares of their values.

If the ACCUMULATE= option is specified, the SETMISSING= option is useful for specifying how accumulated missing values are treated. If missing values should be interpreted as zero, then SETMISSING=0 should be used. The DETAILS section describes accumulation in greater detail.

ALIGN= option
controls the alignment of SAS dates used to identify output observations. The ALIGN= option accepts the following values: BEGINNING | BEG | B, MIDDLE | MID | M, and ENDING | END | E. BEGINNING is the default.

END= option
specifies a SAS date, datetime, or time value that represents the end of the data. If the last time ID variable value is less than the END= value, the series is extended with missing values. If the last time ID variable value is greater than the END= value, the series is truncated. For example, END="&sysdate"D uses the automatic macro variable SYSDATE to extend or truncate the series to the current date. This option and the START= option can be used to ensure that data associated with each BY group contains the same number of observations.
**INTERVAL= interval**
specifies the frequency of the input time series. For example, if the input data set consists of quarterly observations, then INTERVAL=QTR should be used. If the SEASONALITY= option is not specified, the length of the seasonal cycle is implied from the INTERVAL= option. For example, INTERVAL=QTR implies a seasonal cycle of length 4. If the ACCUMULATE= option is also specified, the INTERVAL= option determines the time periods for the accumulation of observations.

The basic intervals are YEAR, SEMIYEAR, QTR, MONTH, SEMIMONTH, TENDAY, WEEK, WEEKDAY, DAY, HOUR, MINUTE, SECOND. Refer to SAS/ETS User’s Guide chapter on Date Interval, Formats, and Functions for the intervals that can be specified.

**NOTSORTED**
specifies that the time ID values are not in sorted order. The HPF procedure will sort the data with respect to the time ID prior to analysis.

**SETMISSING= option | number**
specifies how missing values (either actual or accumulated) are assigned in the accumulated time series. If a number is specified, missing values are set to number. If a missing value indicates an unknown value, this option should not be used. If a missing value indicates no value, a SETMISSING=0 should be used. You would typically use SETMISSING=0 for transactional data because no recorded data usually implies no activity. The following options can also be used to determine how missing values are assigned:

- **MISSING** Missing values are set to missing. This is the default option.
- **AVERAGE | AVG** Missing values are set to the accumulated average value.
- **MINIMUM | MIN** Missing values are set to the accumulated minimum value.
- **MEDIAN | MED** Missing values are set to the accumulated median value.
- **MAXIMUM | MAX** Missing values are set to the accumulated maximum value.
- **FIRST** Missing values are set to the accumulated first nonmissing value.
- **LAST** Missing values are set to the accumulated last nonmissing value.
- **PREVIOUS | PREV** Missing values are set to the previous accumulated nonmissing value. Missing values at the beginning of the accumulated series remain missing.
- **NEXT** Missing values are set to the next accumulated nonmissing value. Missing values at the end of the accumulated series remain missing.

If SETMISSING=MISSING is specified and the MODEL= option specifies a smoothing model, the missing observations are smoothed over. If MODEL=IDM
is specified, missing values are assumed to be periods of no demand, that is, SETMISSING=MISSING is equivalent to SETMISSING=0.

**START= option**
specifies a SAS date, datetime, or time value that represents the beginning of the data. If the first time ID variable value is greater than the START= value, the series is prepended with missing values. If the first time ID variable value is less than the START= value, the series is truncated. This option and the END= option can be used to ensure that data associated with each by group contains the same number of observations.

**ZEROMISS= option**
specifies how beginning and/or ending zero values (either actual or accumulated) are interpreted in the accumulated time series. The following options can also be used to determine how beginning and/or ending zero values are assigned:

- **NONE** Beginning and/or ending zeros unchanged. This is the default.
- **LEFT** Beginning zeros are set to missing.
- **RIGHT** Ending zeros are set to missing.
- **BOTH** Both beginning and ending zeros are set to missing.

If the accumulated series is all missing and/or zero the series is not changed.

### IDM Statement

**IDM options;**

The IDM statement is used to specify an intermittent demand model. An intermittent demand series can be analyzed in two ways: individually modeling both demand interval and size component or jointly modeling these components using the average demand component (demand size divided by demand interval). The IDM statement specifies the two smoothing models to be used to forecast the demand interval component (INTERVAL= option) and the demand size component (SIZE= option), or specifies the single smoothing model to be used to forecast the average demand component (AVERAGE= option). Therefore, two smoothing models (INTERVAL= and SIZE= options) must be specified or one smoothing model (AVERAGE= option) must be specified. Only one statement can be specified.

The following examples illustrate typical uses of the IDM statement:

```plaintext
/* default specification */
idm;

/* demand interval model only specification */
idm interval=(transform=log);

/* demand size model only specification */
idm size=(method=linear);
```
/* Croston’s Method */
  idm interval=(method=simple)
    size   =(method=simple);

/* Log Croston’s Method */
  idm interval=(method=simple transform=log)
    size   =(method=simple transform=log);

/* average demand model specification */
  idm average=(method=bestn);

The default specification uses both the INTERVAL= option and SIZE= option defaults for the decomposed (Croston’s) demand model and the AVERAGE= option defaults for the average demand model.

The following example illustrates how to automatically choose the decomposed demand model using MAPE as the model selection criterion:

  idm interval=(method=simple transform=auto select=mape)
    size   =(method=simple transform=auto select=mape);
  forecast sales / model=idm select=mape;

The preceding fits two forecast models (simple and log simple exponential smoothing) to both the demand interval and size components. The forecast model that results in the lowest in-sample MAPE for each component is used to forecast the component.

The following example illustrates how to automatically choose the average demand model using MAPE as the model selection criterion:

  idm average=(method=simple transform=auto select=mape);
  forecast sales / model=idm;

The preceding fits two forecast models (simple and log simple exponential smoothing) to the average demand component. The forecast model that results in the lowest in-sample MAPE is used to forecast the component.

Combining the above two examples, the following example illustrates how to automatically choose between the decomposed demand model and the average demand model using MAPE as the model selection criterion:

  idm interval=(method=simple transform=auto select=mape)
    size   =(method=simple transform=auto select=mape)
    average=(method=simple transform=auto select=mape);
  forecast sales / model=idm select=mape;

The preceding automatically selects between the decomposed demand model and the average demand model as described previously. The forecast model that results in the lowest in-sample MAPE is used to forecast the series.
The following options can be specified in the IDM statement:

**INTERVAL=(smoothing-model-options)**

specifies the smoothing model used to forecast the demand interval component. See smoothing model specification options described below.

**SIZE=(smoothing-model-options)**

specifies the smoothing model used to forecast the demand size component. See smoothing model specification options described below.

**AVERAGE=(smoothing-model-options)**

specifies the smoothing model used to forecast the demand average component. See smoothing model specification options described below.

**BASE=** AUTO | number

specifies the base value of the time series used to determine the demand series components. The demand series components are determined based on the departures from this base value. If a base value is specified, this value is used to determine the demand series components. If BASE=AUTO is specified, the time series properties are used to automatically adjust the time series. For the common definition of Croston’s Method use BASE=0 which defines departures based on zero. The default is BASE=0.

Given a time series, \( y_t \), and base value, \( b \), the time series is adjusted by the base value to create the base adjusted time series, \( x_t = y_t - b \). Demands are assumed to occur when the base adjusted series is nonzero (or when the time series, \( y_t \), departs from the base value, \( b \)).

When BASE=AUTO, the base value is automatically determined by the time series median, minimum, and maximum value and the INTERMITTENT= option value of the FORECAST statement.

---

**Smoothing Model Specification Options for IDM Statement**

The smoothing model options describe how to forecast the demand interval, size, and average demand components (INTERVAL= option, SIZE= option, and AVERAGE= option).

If the smoothing model options are not specified, the following are the defaults for the demand interval, size, and average components.

\[
\text{interval}=(\text{transform}=\text{auto} \ \text{method}=\text{bestn} \\
\quad \text{levelrest}=(0.0001 \ 0.9999) \\
\quad \text{trendrest}=(0.0001 \ 0.9999) \\
\quad \text{damprest}=(0.0001 \ 0.9999) \ \text{select}=\text{rmse} \ \text{bounds}=(1,.)
\]

\[
\text{size} \quad = (\text{transform}=\text{auto} \ \text{method}=\text{bestn} \\
\quad \text{levelrest}=(0.0001 \ 0.9999) \\
\quad \text{trendrest}=(0.0001 \ 0.9999) \\
\quad \text{damprest}=(0.0001 \ 0.9999) \ \text{select}=\text{rmse})
\]

\[
\text{average} \quad = (\text{transform}=\text{auto} \ \text{method}=\text{bestn} \\
\quad \text{levelrest}=(0.0001 \ 0.9999)
\]
Smoothing Model Specification Options for IDM Statement

\[
\text{trendrest}=(0.0001\ 0.9999) \\
damprest\ =\text{(0.0001\ 0.9999) select=rmse)};
\]

The above smoothing model options provide the typical automation in intermittent demand model selection.

The following describes the smoothing model options:

**TRANSFORM= option**

specifies the time series transformation to be applied to the demand component. The following transformations are provided:

- **NONE**: No transformation is applied.
- **LOG**: Logarithmic transformation
- **SQRT**: Square-root transformation
- **LOGISTIC**: Logistic transformation
- **BOXCOX(n)**: Box-Cox transformation with parameter number where number is between -5 and 5
- **AUTO**: Automatically choose between NONE and LOG based on model selection criteria. This option is the default.

When the TRANSFORM= option is specified, the demand component must be strictly positive. Once the demand component is transformed, the model parameters are estimated using the transformed component. The forecasts of the transformed component are then computed, and finally, the transformed component forecasts are inverse transformed. The inverse transform produces either mean or median forecasts depending on whether the MEDIAN option is specified.

**MEDIAN**

specifies that the median forecast values are to be estimated. Forecasts can be based on the mean or median. By default the mean value is provided. If no transformation is applied to the actual series using the TRANSFORM= option, the mean and median component forecast values are identical.

**METHOD= method-name**

specifies the forecasting model to be used to forecast the demand component. A single model can be specified or a group of candidate models can be specified. If a group of models is specified, the model used to forecast the accumulated time series is selected based on the SELECT= option of the IDM statement and the HOLDOUT= option of the FORECAST statement. The default is METHOD=BESTN. The following forecasting models are provided:

- **SIMPLE**: Simple (Single) Exponential Smoothing
- **DOUBLE**: Double (Brown) Exponential Smoothing
- **LINEAR**: Linear (Holt) Exponential Smoothing
- **DAMPTREND**: Damped Trend Exponential Smoothing
BESTN

Best Candidate Nonseasonal Smoothing Model (SIMPLE, DOUBLE, LINEAR, DAMPTREND)

NOSTABLE

specifies that the smoothing model parameters are not restricted to the additive invertible region of the parameter space. By default, the smoothing model parameters are restricted to be inside the additive invertible region of the parameter space.

LEVELPARM= number

specifies the level weight parameter initial value. See the smoothing model parameter specifications options below.

LEVELREST=(number,number)

specifies the level weight parameter restrictions. See the smoothing model parameter specifications options below.

TRENDPARM= number

specifies the trend weight parameter initial value. See the smoothing model parameter specifications options below.

TRENDREST=(number,number)

specifies the trend weight parameter restrictions. See the smoothing model parameter specifications options below.

DAMPPARM= number

specifies the damping weight parameter initial value. See the smoothing model parameter specifications options below.

DAMPREST=(number,number)

specifies the damping weight parameter restrictions. See the smoothing model parameter specifications options below.

NOEST

specifies that the smoothing model parameters are fixed values. To use this option, all of the smoothing model parameters must be explicitly specified. By default, the smoothing model parameters are optimized.

BOUNDS=(number,number)

Specifies the component forecast bound. See the smoothing model forecast bounds below.

SELECT= option

specifies the model selection criterion (statistic of fit) to be used to select from several candidate models. This option would often be used in conjunction with the HOLDOUT= option specified in the FORECAST statement. The default is SELECT=RMSE. The statistics of fit provided are the same as those provided in the FORECAST statement.
Smoothing Model Parameter Specification Options

The parameter options are used to specify smoothing model parameters. If the parameter restrictions are not specified the default is (0.0001 0.9999), which implies that the parameters are restricted between 0.0001 and 0.9999. Parameters and their restrictions are required to be greater than or equal to -1 and less than or equal to 2. Missing values indicate no lower and/or upper restriction. If the parameter initial values are not specified, the optimizer uses a grid search to find an appropriate initial value.

Smoothing Model Forecast Bounds Options

Specifies the demand component forecast bounds. The forecast bounds restrict the component forecasts. The default for demand interval forecasts is BOUNDS=1. The lower bound for the demand interval forecast must be greater than one. The default for demand size forecasts is BOUNDS=(..) or no bounds. The demand size forecasts bounds are applied after the forecast is adjusted by the base value.
Chapter 1. The HPF Procedure

Details

The HPF procedure can be used to perform trend and seasonal analysis on transactional data. For trend analysis, various sample statistics are computed for each time period defined by the time ID variable and INTERVAL= option. For seasonal analysis, various sample statistics are computed for each season defined by the INTERVAL= or the SEASONALITY= option. For example, suppose the transactional data ranges from June 1990 to January 2000, then the trend statistics are computed for every month: June 1990, July 1990, ..., January 2000. The seasonal statistics are computed for each season: January, February, ..., December.

The HPF procedure can be used to forecast time series data as well as transactional data. If the data is transactional, then the procedure must first accumulate the data into a time series before it can be forecast. The procedure uses the following sequential steps to produce forecasts, with the options that control the step listed to the right:

1. Accumulation ACCUMULATE= option
2. Missing Value Interpretation SETMISSING= option
3. Diagnostic Tests INTERMITTENT= and SEASONTEST= options
4. Model Selection MODEL=, HOLDOUT=, HOLDOUTPCT=, and SELECT= options
5. Transformations TRANSFORM= option
6. Parameter Estimation MODEL= option
7. Forecasting MODEL= and LEAD= options
8. Inverse Transformation TRANSFORM= and MEDIAN options
9. Statistics of Fit SELECT= option
10. Summation of Forecasts LEAD= and STARTSUM= options

Each of the above steps is described below.

Accumulation

If the ACCUMULATE= option is specified, data set observations are accumulated within each time period. The frequency (width of each time interval) is specified by the INTERVAL= option. The ID variable contains the time ID values. Each time ID value corresponds to a specific time period. Accumulation is particularly useful when the input data set contains transactional data, whose observations are not spaced with respect to any particular time interval. The accumulated values form the actual time series, which is used in subsequent analyses.

For example, suppose a data set contains the following observations:

```
19MAR1999 10
19MAR1999 30
11MAY1999 50
12MAY1999 20
23MAY1999 20
```
If the INTERVAL=MONTH is specified, all of the above observations fall within three time periods of March 1999, April 1999, and May 1999. The observations are accumulated within each time period as follows:

If the ACCUMULATE=NONE option is specified, an error is generated because the ID variable values are not equally spaced with respect to the specified frequency (MONTH).

If the ACCUMULATE=TOTAL option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>40</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>90</td>
</tr>
</tbody>
</table>

If the ACCUMULATE=AVERAGE option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>20</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>30</td>
</tr>
</tbody>
</table>

If the ACCUMULATE=MINIMUM option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>10</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>20</td>
</tr>
</tbody>
</table>

If the ACCUMULATE=MEDIAN option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>20</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>20</td>
</tr>
</tbody>
</table>

If the ACCUMULATE=MAXIMUM option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>30</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>50</td>
</tr>
</tbody>
</table>

If the ACCUMULATE=FIRST option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>10</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>50</td>
</tr>
</tbody>
</table>

If the ACCUMULATE=LAST option is specified:
If the ACCUMULATE=STDDEV option is specified:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>01MAR1999</td>
<td>14.14</td>
</tr>
<tr>
<td>01APR1999</td>
<td>.</td>
</tr>
<tr>
<td>01MAY1999</td>
<td>17.32</td>
</tr>
</tbody>
</table>

As can be seen from the above examples, even though the data set observations contained no missing values, the accumulated time series may have missing values.

**Missing Value Interpretation**

Sometimes missing values should be interpreted as unknown values. The forecasting models used by the HPF procedure can effectively handle missing values (see the “Missing Value Modeling Issues” section on page 31). But sometimes missing values are known, such as when missing values are created from accumulation and no observations should be interpreted as no (zero) value. In the former case, the SETMISSING= option can be used to interpret how missing values are treated. The SETMISSING=0 option should be used when missing observations are to be treated as no (zero) values. In other cases, missing values should be interpreted as global values, such as minimum or maximum values of the accumulated series. The accumulated and interpreted time series is used in subsequent analyses.

**Diagnostic Tests**

The INTERMITTENT= option set the thresholds for categorizing a series as intermittent or non-intermittent. The SEASONTEST= option set the thresholds for categorizing a series as seasonal or non-seasonal.

**Model Selection**

When more than one candidate model is specified, forecasts for each candidate model are compared using the model selection criterion specified by the SELECT= option. The selection criterion is computed using the multistep forecasts in the holdout sample range if the HOLDOUT= or HOLDOUTPCT= options are specified, or the one-step ahead forecasts for the full range of the time series if the HOLDOUT= and HOLDOUTPCT= options are not specified. The candidate model with the best selection criterion is selected to forecast the time series.
Transformations

If the TRANSFORM= option is specified, the time series is transformed prior to model parameter estimation and forecasting. Only strictly positive series can be transformed. An error is generated when the TRANSFORM= option is used with a nonpositive series.

Parameter Estimation

All parameters associated with the model are optimized based on the data with the default parameter restrictions imposed. If the TRANSFORM= option is specified, the transformed time series data are used to estimate the model parameters.

Missing Value Modeling Issues

The treatment of missing values varies with the forecasting model. For the smoothing models, missing values after the start of the series are replaced with one-step-ahead predicted values, and the predicted values are applied to the smoothing equations. See Chapter 2, “Forecasting Process Details,” for greater detail on how missing values are treated in the smoothing models. For MODEL=IDM, specified missing values are assumed to be periods of no demand.

The treatment of missing values can also be specified by the user with the SETMISSING= option, which changes the missing values prior to modeling.

Even though all of the observed data are nonmissing, using the ACCUMULATE= option can create missing values in the accumulated series.

Forecasting

Once the model parameters are estimated, one-step ahead forecasts are generated for the full range of the actual (optionally transformed) time series data, and multistep forecasts are generated from the end of the observed time series to the future time period after the last observation specified by the LEAD= option. If there are missing values at the end of the time series, the forecast horizon will be greater than that specified by the LEAD= option.

Inverse Transformations

If the TRANSFORM= option is specified, the forecasts of the transformed time series are inverse transformed. By default, the mean (expected) forecasts are generated. If the MEDIAN option is specified, the median forecasts are generated.
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Statistics of Fit

The statistics of fit (or goodness-of-fit statistics) are computed by comparing the actual time series data and the generated forecasts. If the TRANSFORM= option was specified, the statistics of fit are based on the inverse transformed forecasts.

Forecast Summation

The multistep forecasts generated by the above steps are summed from the STARTSUM= number to the LEAD= number. For example, if STARTSUM=4 and LEAD=6, the 4-step through 6-step ahead forecasts are summed. The predictions are simply summed. However, the prediction error variance of this sum is computed taking into account the correlation between the individual predictions. The upper and lower confidence limits for the sum of the predictions is then computed based on the prediction error variance of the sum.

The forecast summation is particularly useful when it is desirable to model in one frequency yet the forecast of interest is another frequency. For example, if a time series has a monthly frequency (INTERVAL=MONTH) and you want a forecast for the third and fourth future months, a forecast summation for the third and fourth month can be obtained by specifying STARTSUM=3 and LEAD=4.

Variance-related computations are only computed when no transformation is specified (TRANSFORM=NONE).

Comparison to the Time Series Forecasting System

With the exception of Model Selection, the techniques used in the HPF procedure are identical to the Time Series Forecasting System of SAS/ETS software. For Model Parameter Estimation, the default parameter restrictions are imposed.

Data Set Output

The HPF procedure can create the OUT=, OUTTEST=, OUTFOR=, OUTSTAT=, OUTSUM=, OUTSEASON=, and OUTTREND= data sets. In general, these data sets will contain the variables listed in the BY statement. In general, if a forecasting step related to an output data step fails, the values of this step are not recorded or are set to missing in the related output data set, and appropriate error and/or warning messages are recorded in the log.

OUT= Data Set

The OUT= data set contains the variables specified in the BY, ID, and FORECAST statements. If the ID statement is specified, the ID variable values are aligned and extended based on the ALIGN= and INTERVAL= options. The values of the variables specified in the FORECAST statements are accumulated based on the ACCUMULATE= option and missing values are interpreted based on the SETMISSING= option. If the REPLACEMISSING option is specified, missing values embedded missing values are replaced by the one step-ahead forecasts.
These variable values are then extrapolated based on their forecasts or extended with missing values when the MODEL=NONE option is specified. If USE=LOWER is specified, the variable is extrapolated with the lower confidence limits; if USE=UPPER, the variable is extrapolated using the upper confidence limits; otherwise, the variable values are extrapolated with the predicted values. If the TRANSFORM= option is specified, the predicted values will contain either mean or median forecasts depending on whether or not the MEDIAN option is specified.

If any of the forecasting steps fail for particular variable, the variable values are extended by missing values.

**OUTEST= Data Set**

The OUTEST= data set contains the variables specified in the BY statement as well as the variables listed below. For variables listed in FORECAST statements where the option MODEL=NONE is specified, no observations are recorded for these variables. For variables listed in FORECAST statements where the option MODEL=NONE is not specified, the following variables contain observations related to the parameter estimation step:

- `_NAME_` Variable name
- `_MODEL_` Forecasting Model
- `_TRANSFORM_` Transformation
- `_PARM_` Parameter Name
- `_EST_` Parameter Estimate
- `_STDERR_` Standard Errors
- `_TVALUE_` $t$-Values
- `_PV ALUE_` Probability Values

If the parameter estimation step fails for a particular variable, no observations are recorded.

**OUTFOR= Data Set**

The OUTFOR= data set contains the variables specified in the BY statement as well as the variables listed below. For variables listed in FORECAST statements where the option MODEL=NONE is specified, no observations are recorded for these variables. For variables listed in FORECAST statements where the option MODEL=NONE is not specified, the following variables contain observations related to the forecasting step:

- `_NAME_` Variable name
- `_TIMEID_` Time ID values
- `PREDICT` Predicted Values
- `STD` Prediction Standard Errors
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- LOWER Lower Confidence Limits
- UPPER Upper Confidence Limits
- ERROR Prediction Errors

If the forecasting step fails for a particular variable, no observations are recorded. If the TRANSFORM= option is specified, the values in the variables listed above are the inverse transform forecasts. If the MEDIAN option is specified, the median forecasts are stored; otherwise, the mean forecasts are stored.

**OUTSTAT= Data Set**

The OUTSTAT= data set contains the variables specified in the BY statement as well as the variables listed below. For variables listed in FORECAST statements where the option MODEL=NONE is specified, no observations are recorded for these variables. For variables listed in FORECAST statements where the option MODEL=NONE is not specified, the following variables contain observations related to the statistics of fit step:

- _NAME_ Variable name
- SSE Sum of Square Error
- MSE Mean Square Error
- UMSE Unbiased Mean Square Error
- RMSE Root Mean Square Error
- URMSE Unbiased Root Mean Square Error
- MAPE Mean Absolute Percent Error
- MAE Mean Absolute Error
- RSQUARE R-Square
- ADJRSQ Adjusted R-Square
- AADJRSQ Amemiya’s Adjusted R-Square
- RWRSQ Random Walk R-Square
- AIC Akaike Information Criterion
- SBC Schwarz Bayesian Information Criterion
- APC Amemiya’s Prediction Criterion
- MAXERR Maximum Error
- MINERR Minimum Error
- MINPE Maximum Percent Error
- MAXPE Minimum Percent Error
- ME Mean Error
- MPE Mean Percent Error
If the statistics of fit step fails for particular variable, no observations are recorded. If the TRANSFORM= option is specified, the values in the variables listed above are computed based on the inverse transform forecasts. If the MEDIAN option is specified, the median forecasts are the basis; otherwise, the mean forecasts are the basis.

**OUTSUM= Data Set**

The OUTSUM= data set contains the variables specified in the BY statement as well as the variables listed below. The OUTSUM= data set records the summary statistics for each variable specified in a FORECAST statement. For variables listed in FORECAST statements where the option MODEL=NONE is specified, the values related to forecasts are set to missing. For variables listed in FORECAST statements where the option MODEL=NONE is not specified, the forecast values are set based on the USE= option.

Variables related to summary statistics are based on the ACCUMULATE= and SETMISSING= options:

- `_NAME_` Variable name
- `_STATUS_` Forecasting Status. Nonzero values imply that no forecast was generated for the series.
- NOBS Number of Observations
- N Number of Nonmissing Observations
- NMISS Number of Missing Observations
- MIN Minimum Value
- MAX Maximum Value
- MEAN Mean Value
- STDDEV Standard Deviation

Variables related to forecast summation are based on the LEAD= and STARTSUM= options:

- PREDICT Forecast Summation Predicted Values
- STD Forecast Summation Prediction Standard Errors
- LOWER Forecast Summation Lower Confidence Limits
- UPPER Forecast Summation Upper Confidence Limits

Variance-related computations are only computed when no transformation is specified (TRANSFORM=NONE).

Variables related to multistep forecast based on the LEAD= and USE= options:
**_LEADn_**  Multistep Forecast (n ranges from one to LEAD=number). If USE=LOWER, this variable will contain the lower confidence limits; if USE=UPPER, this variable will contain the upper confidence limits; otherwise, this variable will contain the predicted values.

If the forecast step fails for a particular variable, the variables related to forecasting are set to missing. The OUTSUM= data set contains both a summary of the (accumulated) time series and optionally its forecasts for all series.

**OUTSEASON= Data Set**

The OUTSEASON= data set contains the variables specified in the BY statement as well as the variables listed below. The OUTSEASON= data set records the seasonal statistics for each variable specified in a FORECAST statement.

Variables related to seasonal statistics are based on the INTERVAL= or SEASONALITY= options:

- **_NAME_**  Variable name
- **_TIMEID_**  Time ID values
- **_SEASON_**  Seasonal index
- **NOBS**  Number of Observations
- **N**  Number of Nonmissing Observations
- **NMISS**  Number of Missing Observations
- **MIN**  Minimum Value
- **MAX**  Maximum Value
- **RANGE**  Range Value
- **SUM**  Summation Value
- **MEAN**  Mean Value
- **STDDEV**  Standard Deviation
- **CSS**  Corrected Sum of Squares
- **USS**  Uncorrected Sum of Squares
- **MEDIAN**  Median Value

The above statistics are computed for each season.
OUTTREND= Data Set

The OUTTREND= data set contains the variables specified in the BY statement as well as the variables listed below. The OUTTREND= data set records the trend statistics for each variable specified in a FORECAST statement.

Variables related to trend statistics are based on the INTERVAL= and SEASONALITY= options:

- NAME-_      Variable name
- TIMEID_     Time ID values
- SEASON_     Seasonal index
- NOBS       Number of Observations
- N          Number of Nonmissing Observations
- NMISS      Number of Missing Observations
- MIN        Minimum Value
- MAX        Maximum Value
- RANGE      Range Value
- SUM        Summation Value
- MEAN       Mean Value
- STDDEV     Standard Deviation
- CSS        Corrected Sum of Squares
- USS        Uncorrected Sum of Squares
- MEDIAN     Median Value

The above statistics are computed for each time period.

Printed Output

The HPF procedure optionally produces printed output for these results utilizing the Output Delivery System (ODS). By default, the procedure produces no printed output. All output is controlled by the PRINT= and PRINTDETAILS options associated with the PROC HPF statement. In general, if a forecasting step related to printed output fails, the values of this step are not printed and appropriate error and/or warning messages are recorded in the log. The printed output is similar to the output data set and these similarities are described below.

PRINT=SUMMARY

prints the summary statistics and forecast summaries similar to the OUTSUM= data set.
**PRINT=ESTIMATES**

prints the parameter estimates similar to the OUTEST= data set.

**PRINT=FORECASTS**

prints the forecasts similar to the OUTFOR= data set. For MODEL=IDM, a table containing demand series is also printed.

If the MODEL=IDM option is specified, the demand series predictions table is also printed. This table is based on the demand index (when demands occurred).

**PRINT=PERFORMANCE**

prints the performance statistics.

**PRINT=PERFORMANCESUMMARY**

prints the performance summary for each BY group.

**PRINT=PERFORMANCEOVERALL**

prints the performance summary for all BY groups.

**PRINT=STATES**

prints the backcast, initial, and final smoothed states.

**PRINT=SEASONS**

prints the seasonal statistics similar to the OUTSEASON= data set.

**PRINT=STATISTICS**

prints the statistics of fit similar to the OUTSTAT= data set.

**PRINT=TRENDS**

Prints the trend statistics similar to the OUTTREND= data set.

**PRINTDETAILS**

The PRINTDETAILS option is the opposite of the NOOUTALL option.

Specifically, if PRINT=FORECASTS and the PRINTDETAILS options are specified, the one-step ahead forecasts, throughout the range of the data, are printed as well as the information related to a specific forecasting model such as the smoothing states. If the PRINTDETAILS option is not specified, only the multistep forecasts are printed.
**ODS Table Names**

The table below relates the PRINT= options to ODS tables:

<table>
<thead>
<tr>
<th>ODS Table Name</th>
<th>Description</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ODS Tables Created by the PRINT=SUMMARY option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DescStats</td>
<td>Descriptive Statistics</td>
<td></td>
</tr>
<tr>
<td>DemandSummary</td>
<td>Demand Summary</td>
<td>MODEL=IDM option only</td>
</tr>
<tr>
<td>ForecastSummary</td>
<td>Forecast Summary</td>
<td></td>
</tr>
<tr>
<td>ForecastSummation</td>
<td>Forecast Summation</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=ESTIMATES option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ModelSelection</td>
<td>Model Selection</td>
<td></td>
</tr>
<tr>
<td>ParameterEstimates</td>
<td>Parameter Estimates</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=FORECASTS option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forecasts</td>
<td>Forecast</td>
<td></td>
</tr>
<tr>
<td>Demands</td>
<td>Demands</td>
<td>MODEL=IDM option only</td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=PERFORMANCE option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>Performance Statistics</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=PERFORMANCESUMMARY option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PerformanceSummary</td>
<td>Performance Summary</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=PERFORMANCEOVERALL option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PerformanceSummary</td>
<td>Performance Overall</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=SEASONS option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SeasonStatistics</td>
<td>Seasonal Statistics</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=STATES option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SmoothedStates</td>
<td>Smoothed States</td>
<td></td>
</tr>
<tr>
<td>DemandStates</td>
<td>Demand States</td>
<td>MODEL=IDM option only</td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=STATISTICS option</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FitStatistics</td>
<td>Statistics of Fit</td>
<td></td>
</tr>
<tr>
<td><strong>ODS Tables Created by the PRINT=TRENDS option</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ODS table ForecastSummary is related to all time series within a BY group. The other tables are related to a single series within a BY group.

**ODS Graphics (Experimental)**

This section describes the use of ODS for creating graphics with the HPF procedure. These graphics are experimental in this release, meaning that both the graphical results and the syntax for specifying them are subject to change in a future release.

To request these graphs, you must specify the ODS GRAPHICS statement. In addition, you can specify the PLOT= option in the HPF statement according to the

**PLOT= option | (options)**

specifies the graphical output desired. By default, the HPF procedure produces no graphical output. The following printing options are available:

- **ERRORS** plots prediction error time series graphics.
- **ACF** plots prediction error autocorrelation function graphics.
- **PACF** plots prediction error partial autocorrelation function graphics.
- **IACF** plots prediction error inverse autocorrelation function graphics.
- **WN** plots white noise graphics.
- **MODELS** plots model graphics.
- **FORECASTS** plots forecast graphics.
- **MODELFORECASTSONLY** plots forecast graphics with confidence limits in the data range.
- **FORECASTSONLY** plots the forecast in the forecast horizon only.
- **LEVELS** plots smoothed level component graphics.
- **SEASONS** plots smoothed seasonal component graphics.
- **TRENDS** plots smoothed trend (slope) component graphics.
- **ALL** Same as specifying all of the above PLOT= options.

For example, PLOT=FORECASTS plots the forecasts for each series. The PLOT= option produces printed output for these results utilizing the Output Delivery System (ODS). The PLOT= statement is experimental for this release of SAS.

**ODS Graph Names**

PROC HPF assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when using ODS. The names are listed in Table 1.2.

To request these graphs, you must specify the ODS GRAPHICS statement. In addition, you can specify the PLOT= option in the HPF statement. For more information on the ODS GRAPHICS statement, refer to Chapter 9, “Statistical Graphics Using ODS” (SAS/ETS User’s Guide).

**Table 1.2.** ODS Graphics Produced by PROC HPF

<table>
<thead>
<tr>
<th>ODS Graph Name</th>
<th>Plot Description</th>
<th>Statement</th>
<th>PLOT= Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>DemandErrorsPlot</td>
<td>Average Errors</td>
<td>Demand</td>
<td>PROC HPF</td>
</tr>
<tr>
<td>DemandForecastsPlot</td>
<td>Average Forecasts</td>
<td>Demand</td>
<td>PROC HPF</td>
</tr>
<tr>
<td>DemandIntervalHistogram</td>
<td>Demand Interval</td>
<td>Interval</td>
<td>PROC HPF</td>
</tr>
<tr>
<td>DemandIntervalPlot</td>
<td>Demand Forecast Plot</td>
<td>Interval</td>
<td>PROC HPF</td>
</tr>
</tbody>
</table>
Table 1.2. (continued)

<table>
<thead>
<tr>
<th>ODS Graph Name</th>
<th>Plot Description</th>
<th>Statement</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>DemandSizeHistogram</td>
<td>Demand Size Histogram</td>
<td>PROC HPF</td>
<td>PLOT=MODELS</td>
</tr>
<tr>
<td>DemandSizePlot</td>
<td>Demand Size Forecast Plot</td>
<td>PROC HPF</td>
<td>PLOT=MODELS</td>
</tr>
<tr>
<td>ErrorACFNORMPlot</td>
<td>Standardized autocorrelation of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=ACF</td>
</tr>
<tr>
<td>ErrorACFPlot</td>
<td>Autocorrelation of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=ACF</td>
</tr>
<tr>
<td>ErrorHistogram</td>
<td>Prediction Error Histogram</td>
<td>PROC HPF</td>
<td>PLOT=ERRORS</td>
</tr>
<tr>
<td>ErrorIACFNORMPlot</td>
<td>Standardized inverse autocorrelation of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=IACF</td>
</tr>
<tr>
<td>ErrorIACFPlot</td>
<td>Inverse autocorrelation of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=IACF</td>
</tr>
<tr>
<td>ErrorPACFNORMPlot</td>
<td>Standardized partial autocorrelation of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=PACF</td>
</tr>
<tr>
<td>ErrorPACFPlot</td>
<td>Partial autocorrelation of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=ERRORS</td>
</tr>
<tr>
<td>ErrorPlot</td>
<td>Plot of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=ERRORS</td>
</tr>
<tr>
<td>ErrorWhiteNoiseLogProbPlot</td>
<td>White noise log probability plot of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=WN</td>
</tr>
<tr>
<td>ErrorWhiteNoiseProbPlot</td>
<td>White noise probability plot of Prediction Errors</td>
<td>PROC HPF</td>
<td>PLOT=WN</td>
</tr>
<tr>
<td>ForecastsOnlyPlot</td>
<td>Forecasts Only Plot</td>
<td>PROC HPF</td>
<td>PLOT=FORECASTONLY</td>
</tr>
<tr>
<td>ForecastsPlot</td>
<td>Forecasts Plot</td>
<td>PROC HPF</td>
<td>PLOT=FORECAST</td>
</tr>
<tr>
<td>LevelStatePlot</td>
<td>Smoothed Level State Plot</td>
<td>PROC HPF</td>
<td>PLOT=LEVELS</td>
</tr>
<tr>
<td>ModelForecastsPlot</td>
<td>Model and Forecasts Plot</td>
<td>PROC HPF</td>
<td>PLOT=MODELFORECAST</td>
</tr>
<tr>
<td>ModelPlot</td>
<td>Model Plot</td>
<td>PROC HPF</td>
<td>PLOT=MODELS</td>
</tr>
<tr>
<td>SeasonStatePlot</td>
<td>Smoothed Season State Plot</td>
<td>PROC HPF</td>
<td>PLOT=SEASONS</td>
</tr>
<tr>
<td>StockingAveragePlot</td>
<td>Stocking Average Plot</td>
<td>PROC HPF</td>
<td>PLOT=FORECASTS</td>
</tr>
<tr>
<td>StockingLevelPlot</td>
<td>Stocking Level Plot</td>
<td>PROC HPF</td>
<td>PLOT=FORECASTS</td>
</tr>
<tr>
<td>TrendStatePlot</td>
<td>Smoothed Trend State Plot</td>
<td>PROC HPF</td>
<td>PLOT=TRENDS</td>
</tr>
</tbody>
</table>
Examples

Example 1.1. Automatic Forecasting of Time Series Data

This example illustrates how the HPF procedure can be used for the automatic forecasting of time series data. Retail sales data is used for this illustration.

The following DATA step creates a data set from data recorded monthly at numerous points of sales. The data set, SALES, will contain a variable DATE that represents time and a variable for each sales item. Each value of the DATE variable is recorded in ascending order and the values of each of the other variables represent a single time series:

``` SAS
data sales;
  format date date9. ;
  input date date9. shoes socks laces dresses coats shirts ties belts hats blouses;
  datalines;
  ... data lines omitted ... 
  ;
run;
```

The following HPF procedure statements automatically forecast each of the monthly time series.

``` SAS
proc hpf data=sales out=nextyear;
  id date interval=month;
  forecast _ALL_;
run;
```

The above statements automatically select the best fitting model and generate forecasts for every numeric variable in the input data set (SALES) for the next twelve months, and stores these forecasts in the output data set (NEXTYEAR).

The following GPLOT procedure statements plot the forecasts related to shoe sales:

``` SAS
title1 "Shoe Department Sales";
axis2 label=(a=-90 r=90 "items" );
symbol1 v = dot i = join l = 1;
symbol2 v = star i = join l = 2;
symbol3 v = circle i = join l = 2;
proc gplot data=nextyear;
  plot shoes * date = 1
    socks * date = 2
    laces * date = 3 / overlay
    haxis= ’01JAN1994’d to ’01DEC2000’d by year
    href= ’01JAN1999’d
    vaxis=axis2;
run;
```
The GPLOT procedure results are shown in Figure 1.1. The historical data is shown left the horizontal reference line and the forecasts for the next twelve monthly periods is shown to the right.

**Figure 1.1. Retail Sales Forecast Plots**

The following HPF procedure statements are identical to the statements above with the exception that the PRINT=FORECASTS option is specified:

```plaintext
proc hpf data=sales out=nextyear print=forecasts;
  id date interval=month;
  forecast _ALL_
run;
```

In addition to automatically forecasting each of the monthly time series, the above statements print the forecasts using the Output Delivery System (ODS), which is partially shown in Output 1.1.1. This output shows the predictions, prediction standard errors and the upper and lower confidence limits for the next twelve monthly periods.
Output 1.1.1. Forecast Tables

Forecasts for Variable shoes

<table>
<thead>
<tr>
<th>Obs</th>
<th>Time</th>
<th>Forecasts</th>
<th>Standard Error</th>
<th>95% Confidence Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>FEB1999</td>
<td>7548.0041</td>
<td>607.5238</td>
<td>6357.2792</td>
</tr>
<tr>
<td>63</td>
<td>MAR1999</td>
<td>7177.1472</td>
<td>699.4400</td>
<td>5806.2701</td>
</tr>
<tr>
<td>64</td>
<td>APR1999</td>
<td>5497.5595</td>
<td>780.7609</td>
<td>3967.2964</td>
</tr>
<tr>
<td>65</td>
<td>MAY1999</td>
<td>4838.2001</td>
<td>854.5169</td>
<td>3163.3778</td>
</tr>
<tr>
<td>66</td>
<td>JUN1999</td>
<td>6728.4521</td>
<td>922.5244</td>
<td>4920.3375</td>
</tr>
<tr>
<td>67</td>
<td>JUL1999</td>
<td>6786.1094</td>
<td>985.9738</td>
<td>4853.6362</td>
</tr>
<tr>
<td>68</td>
<td>AUG1999</td>
<td>5853.9650</td>
<td>1045.6953</td>
<td>3804.4399</td>
</tr>
<tr>
<td>69</td>
<td>SEP1999</td>
<td>7517.0144</td>
<td>1102.2949</td>
<td>5356.5561</td>
</tr>
<tr>
<td>70</td>
<td>OCT1999</td>
<td>7100.2489</td>
<td>1156.2315</td>
<td>4834.0769</td>
</tr>
<tr>
<td>71</td>
<td>NOV1999</td>
<td>7224.6449</td>
<td>1207.8618</td>
<td>4857.2793</td>
</tr>
<tr>
<td>72</td>
<td>DEC1999</td>
<td>6357.1556</td>
<td>1257.4701</td>
<td>3892.5594</td>
</tr>
<tr>
<td>73</td>
<td>JAN2000</td>
<td>6492.2657</td>
<td>1305.2871</td>
<td>3933.9500</td>
</tr>
</tbody>
</table>

Example 1.2. Automatic Forecasting of Transactional Data

This example illustrates how the HPF procedure can be used to automatically forecast transactional data. Internet data is used for this illustration.

The following DATA step creates a data set from data recorded at several Internet Web sites. The data set, WEBSITES, will contain a variable TIME that represents time and the variables ENGINE, BOATS, CARS, and PLANES that represent Internet Web site data. Each value of the TIME variable is recorded in ascending order, and the values of each of the other variables represent a transactional data series.

```plaintext
data websites;
  format time datetime.;
  input time datetime. boats cars planes engines;
  datalines;
  ... data lines omitted ...
;
run;
```

The following HPF procedure statements automatically forecast each of the transactional data series:

```plaintext
proc hpf data=websites out=nextweek lead=7;
  id time interval=dtday accumulate=total;
  forecast boats cars planes;
run;
```

The above statements accumulate the data into a daily time series and automatically generate forecasts for the BOATS, CARS, and PLANES variables in the input data set (WEBSITES) for the next week and stores the forecasts in the output data set (NEXTWEEK).

The following GPLOT procedure statements plot the forecasts related to the Internet data:
Example 1.3. Specifying the Forecasting Model

In the previous example, the HPF procedure was used to automatically select the appropriate forecasting model using the root mean square error (RMSE) as the default selection criterion. This example illustrates how the HPF procedure can be used to more narrowly specify the possible candidate models. Internet data from the previous example are used for this illustration.
In this example, we will forecast the \textit{BOATS} variable using the best seasonal forecasting model (BESTS) that minimizes the mean absolute percent error (MAPE), the \textit{CARS} variable using the best nonseasonal forecasting model (BESTN) that minimizes the mean square error (MSE) using holdout sample analysis, and the \textit{PLANES} variable using Log Winters (additive). The following HPF procedure statements forecast each of the transactional data series based on these requirements:

\begin{verbatim}
proc hpf data=websites out=nextweek lead=7;
  id time interval=dtday accumulate=total;
  forecast boats / model=bests select=mape;
  forecast cars / model=bestn select=mse holdout=5;
  forecast planes / model=addwinters transform=log;
run;
\end{verbatim}

\section*{Example 1.4. Extending the Independent Variables for Multivariate Forecasts}

In the previous example, the HPF procedure was used to forecast several transactional series variables using univariate models. This example illustrates how the HPF procedure can be used to extend the independent variables associated with a multiple regression forecasting problem. Specifically, PROC HPF is used to extend the independent variables for use in forecasting a regression model.

In this example, we will accumulate and forecast the \textit{BOATS}, \textit{CARS}, and \textit{PLANES} variables as illustrated in the previous example. In addition, we will accumulate the \textit{ENGINES} variable to form a time series that is then extended with missing values within the forecast horizon with the specification of MODEL=NONE.

\begin{verbatim}
proc hpf data=websites out=nextweek lead=7;
  id time interval=dtday accumulate=total;
  forecast engines / model=none;
  forecast boats / model=bests select=mape;
  forecast cars / model=bestn select=mse holdout=5;
  forecast planes / model=winters transform=log;
run;
\end{verbatim}

The following AUTOREG procedure statements are used to forecast the \textit{ENGINES} variable by regressing on the independent variables (\textit{BOATS}, \textit{CARS}, and \textit{PLANES}).

\begin{verbatim}
proc autoreg data= nextweek;
  model engines = boats cars planes / noprint;
  output out=enginehits p=predicted;
run;
\end{verbatim}

The output data set (NEXTWEEK) of the PROC HPF statement is used as an input data set for the PROC AUTOREG statement. The output data set of PROC AUTOREG contains the forecast of the variable ENGINES based on the regression model with the variables \textit{BOATS}, \textit{CARS}, and \textit{PLANES} as regressors. See the AUTOREG procedure for details on autoregression models.
Example 1.5. Forecasting Intermittent Time Series Data

The following GPLOT procedure statements plot the forecasts related to the ENGINES variable:

```
proc gplot data=enginehits;
plot boats * time = 1
   cars * time = 2
   planes * time = 3
   predicted * time = 4 / overlay
   haxis= '13MAR2000:00:00:00'dt to
       '18APR2000:00:00:00'dt by dtweek
   href= '11APR2000:00:00:00'dt
   vaxis=axis2;
run;
```

The GPLOT procedure results are shown in Figure 1.3. The historical data is shown left the horizontal reference line and the forecasts for the next four weekly periods is shown to the right.

![Internet Forecast for Engines](image)

**Figure 1.3.** Internet Data Forecast Plots

Example 1.5. Forecasting Intermittent Time Series Data

This example illustrates how the HPF procedure can be used to forecast intermittent time series data. Inventory demand is used for this illustration.

The following DATA step creates a data set from inventory data recorded at no particular frequency. The data set, INVENTORY, will contain a variable DATE that represents time and the demand variables (TIRES, HUBCAPS, and LUGBOLTS), which represent inventory items. Each value of the DATE variable is recorded in ascending order, and the values of each of the other variables represent a transactional data series.
data inventory;
  format date date9.;
  input date date9. tires hubcaps lugbolts;
  datalines;
  ... data lines omitted ...
  ;
run;

The following HPF procedure statements forecast each of the transactional data series using an intermittent demand model:

    proc hpf data=inventory out=nextmonth lead=4 print=forecasts;
      id date interval=week accumulate=total;
      forecast tires hubcaps lugbolts / model=idm;
    run;

The above statements accumulate the data into a weekly time series, and generate forecasts for the TIRES, HUBCAPS, and LUGBOLTS variables in the input data set (INVENTORY) for the four weekly periods, and store the forecasts in the output data set (NEXTMONTH). The PRINT=FORECAST option produces the results partially shown in Output 1.5.1. The first table records the demand series and predictions. The second table represents forecasts or recommended stocking levels.
Example 1.6. Illustration of ODS Graphics (Experimental)

This example illustrates the use of experimental ODS graphics.

The following statements utilize the SASHELP.AIR data set to automatically forecast the time series of international airline travel.

The graphical displays are requested by specifying the experimental ODS GRAPHICS statement and the experimental PLOT= option in the PROC HPF statement. In this case, all plots are requested. Output 1.6.1 through Output 1.6.4 show a selection of the plots created.


### Output 1.5.1. Forecast Tables

**Demands for Variable tires**

<table>
<thead>
<tr>
<th>Index</th>
<th>Demand</th>
<th>Demand Intervals</th>
<th>Demand Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Actual Predicted</td>
<td>Actual Predicted</td>
</tr>
<tr>
<td>1</td>
<td>Sun, 31 Aug 1997</td>
<td>14</td>
<td>14.0000</td>
</tr>
<tr>
<td>2</td>
<td>Sun, 26 Oct 1997</td>
<td>8</td>
<td>14.0000</td>
</tr>
<tr>
<td>3</td>
<td>Sun, 1 Mar 1998</td>
<td>18</td>
<td>12.8920</td>
</tr>
<tr>
<td>4</td>
<td>Sun, 26 Apr 1998</td>
<td>8</td>
<td>13.8353</td>
</tr>
<tr>
<td>5</td>
<td>Sun, 31 May 1998</td>
<td>5</td>
<td>12.7577</td>
</tr>
<tr>
<td>6</td>
<td>Sun, 27 Sep 1998</td>
<td>17</td>
<td>11.3251</td>
</tr>
<tr>
<td>7</td>
<td>Sun, 3 Jan 1999</td>
<td>14</td>
<td>12.3731</td>
</tr>
</tbody>
</table>

Error = (Interval Actual)*(Predict) - (Size Actual)

### Output 1.5.2. Forecast Tables

**Estimate of Mean Demand per Period**

<table>
<thead>
<tr>
<th>Index</th>
<th>Demand</th>
<th>Predict Std Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>95% Confidence</td>
</tr>
<tr>
<td>1</td>
<td>Sun, 31 Aug 1997</td>
<td>0.31582 0.11634 0.08780 0.54384 0.57849</td>
</tr>
<tr>
<td>2</td>
<td>Sun, 26 Oct 1997</td>
<td>0.33873 0.11634 0.11071 0.56674 0.290193</td>
</tr>
<tr>
<td>3</td>
<td>Sun, 1 Mar 1998</td>
<td>0.35614 0.11634 0.12813 0.58416 4.140578</td>
</tr>
<tr>
<td>4</td>
<td>Sun, 26 Apr 1998</td>
<td>0.29381 0.11634 0.06579 0.52183 0.350503</td>
</tr>
<tr>
<td>5</td>
<td>Sun, 31 May 1998</td>
<td>0.28575 0.11634 0.05773 0.51377 0.571254</td>
</tr>
<tr>
<td>6</td>
<td>Sun, 27 Sep 1998</td>
<td>0.29238 0.11634 0.06436 0.52040 0.1029543</td>
</tr>
<tr>
<td>7</td>
<td>Sun, 3 Jan 1999</td>
<td>0.27553 0.11634 0.04751 0.50355</td>
</tr>
</tbody>
</table>

Error = (Interval Actual)*(Predict) - (Size Actual)
Chapter 1. The HPF Procedure

ods html;
ods graphics on;

proc hpf data=sashelp.air out=_null_ lead=20 back=20 print=all plot=all;
   id date interval=month;
   forecast air / model=best transform=auto select=mape;
run;

ods graphics off;
ods html close;

Output 1.6.1. Smoothed Trend Plot (Experimental)
Output 1.6.2. Prediction Error Plot (Experimental)

Output 1.6.3. Prediction Error Standardized ACF Plot (Experimental)
Output 1.6.4. Forecast Plot (Experimental)

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Chapter 2
Forecasting Process Details

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<td>SERIES TRANSFORMATIONS</td>
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<td>SERIES DIAGNOSTIC TESTS</td>
<td>74</td>
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<tr>
<td>STATISTICS OF FIT</td>
<td>75</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>77</td>
</tr>
</tbody>
</table>
Chapter 2
Forecasting Process Details

This chapter provides computational details on several aspects of the SAS High Performance Forecasting.

Forecasting Process Summary

This section summarizes the forecasting process.

Parameter Estimation

Computational details for the forecasting models are provided in the following sections "Smoothing Models" and "Intermittent Models." The results of the parameter estimation process are printed in the Parameter Estimates table or stored in the OUTTEST= data set.

Model Evaluation

Model evaluation is based on the one-step-ahead prediction errors for observations within the period of evaluation. The one-step-ahead predictions are generated from the model specification and parameter estimates. The predictions are inverse transformed (median or mean) and adjustments are removed. The prediction errors (the difference of the dependent series and the predictions) are used to compute the statistics of fit, which are described in section "Statistics of Fit." The results generated by the evaluation process are printed in the Statistics of Fit table or stored in the OUTSTAT= data set.

Forecasting

The forecasting process is similar to the model evaluation process described in the preceding section, except that \( k \)-step-ahead predictions are made from the end of the data through the specified forecast horizon, and prediction standard errors and confidence limits are calculated. The forecasts and confidence limits are printed in the Forecast table or stored in the OUTFOR= data set.

Smoothing Models

This section details the computations performed for the exponential smoothing and Winters method forecasting models.
Chapter 2. Forecasting Process Details

Smoothing Model Calculations

The descriptions and properties of various smoothing methods can be found in Gardner (1985), Chatfield (1978), and Bowerman and O’Connell (1979). The following section summarizes the smoothing model computations.

Given a time series \( \{ Y_t : 1 \leq t \leq n \} \), the underlying model assumed by the smoothing models has the following (additive seasonal) form:

\[
Y_t = \mu_t + \beta_t t + s_p(t) + \epsilon_t
\]

where

- \( \mu_t \) represents the time-varying mean term.
- \( \beta_t \) represents the time-varying slope.
- \( s_p(t) \) represents the time-varying seasonal contribution for one of the \( p \) seasons
- \( \epsilon_t \) are disturbances.

For smoothing models without trend terms, \( \beta_t = 0 \); and for smoothing models without seasonal terms, \( s_p(t) = 0 \). Each smoothing model is described in the following sections.

At each time \( t \), the smoothing models estimate the time-varying components described above with the smoothing state. After initialization, the smoothing state is updated for each observation using the smoothing equations. The smoothing state at the last nonmissing observation is used for predictions.

Smoothing State and Smoothing Equations

Depending on the smoothing model, the smoothing state at time \( t \) will consist of the following:

- \( L_t \) is a smoothed level that estimates \( \mu_t \).
- \( T_t \) is a smoothed trend that estimates \( \beta_t \).
- \( S_{t-j}, j = 0, \ldots, p - 1 \), are seasonal factors that estimate \( s_p(t) \).

The smoothing process starts with an initial estimate of the smoothing state, which is subsequently updated for each observation using the smoothing equations.

The smoothing equations determine how the smoothing state changes as time progresses. Knowledge of the smoothing state at time \( t - 1 \) and that of the time-series value at time \( t \) uniquely determine the smoothing state at time \( t \). The smoothing weights determine the contribution of the previous smoothing state to the current smoothing state. The smoothing equations for each smoothing model are listed in the following sections.
Smoothing State Initialization

Given a time series \( \{Y_t : 1 \leq t \leq n\} \), the smoothing process first computes the smoothing state for time \( t = 1 \). However, this computation requires an initial estimate of the smoothing state at time \( t = 0 \), even though no data exists at or before time \( t = 0 \).

An appropriate choice for the initial smoothing state is made by backcasting from time \( t = n \) to \( t = 1 \) to obtain a prediction at \( t = 0 \). The initialization for the backcast is obtained by regression with constant and linear terms and seasonal dummies (additive or multiplicative) as appropriate for the smoothing model. For models with linear or seasonal terms, the estimates obtained by the regression are used for initial smoothed trend and seasonal factors; however, the initial smoothed level for backcasting is always set to the last observation, \( Y_n \).

The smoothing state at time \( t = 0 \) obtained from the backcast is used to initialize the smoothing process from time \( t = 1 \) to \( t = n \) (refer to Chatfield and Yar 1988).

For models with seasonal terms, the smoothing state is normalized so that the seasonal factors \( S_{t-j} \) for \( j = 0, \ldots, p-1 \) sum to zero for models that assume additive seasonality and average to one for models (such as Winters method) that assume multiplicative seasonality.

Missing Values

When a missing value is encountered at time \( t \), the smoothed values are updated using the error-correction form of the smoothing equations with the one-step-ahead prediction error, \( e_t \), set to zero. The missing value is estimated using the one-step-ahead prediction at time \( t - 1 \), that is, \( \hat{Y}_{t-1}(1) \) (refer to Aldrin 1989). The error-correction forms of each of the smoothing models are listed in the following sections.

Predictions and Prediction Errors

Predictions are made based on the last known smoothing state. Predictions made at time \( t \) for \( k \) steps ahead are denoted \( \hat{Y}_t(k) \) and the associated prediction errors are denoted \( e_t(k) = Y_{t+k} - \hat{Y}_t(k) \). The prediction equation for each smoothing model is listed in the following sections.

The one-step-ahead predictions refer to predictions made at time \( t - 1 \) for one time unit into the future, that is, \( \hat{Y}_{t-1}(1) \), and the one-step-ahead prediction errors are more simply denoted \( e_t = e_{t-1}(1) = Y_t - \hat{Y}_{t-1}(1) \). The one-step-ahead prediction errors are also the model residuals, and the sum of squares of the one-step-ahead prediction errors is the objective function used in smoothing weight optimization.

The variance of the prediction errors are used to calculate the confidence limits (refer to Sweet 1985, McKenzie 1986, Yar and Chatfield 1990, and Chatfield and Yar 1991). The equations for the variance of the prediction errors for each smoothing model are listed in the following sections.

Note: \( \text{var}(e_t) \) is estimated by the mean square of the one-step-ahead prediction errors.
Smoothing Weights

Depending on the smoothing model, the smoothing weights consist of the following:

\[ \alpha \] is a level smoothing weight.
\[ \gamma \] is a trend smoothing weight.
\[ \delta \] is a seasonal smoothing weight.
\[ \phi \] is a trend damping weight.

Larger smoothing weights (less damping) permit the more recent data to have a
greater influence on the predictions. Smaller smoothing weights (more damping)
give less weight to recent data.

Specifying the Smoothing Weights

Typically the smoothing weights are chosen to be from zero to one. (This is intuitive
because the weights associated with the past smoothing state and the value of cur-
rent observation would normally sum to one.) However, each smoothing model (ex-
cept Winters Method – Multiplicative Version) has an ARIMA equivalent. Weights
chosen to be within the ARIMA additive-invertible region will guarantee stable pre-
dictions (refer to Archibald 1990 and Gardner 1985). The ARIMA equivalent and
the additive-invertible region for each smoothing model are listed in the following
sections.

Optimizing the Smoothing Weights

Smoothing weights are determined so as to minimize the sum of squared one-step-
head prediction errors. The optimization is initialized by choosing from a predeter-
nined grid the initial smoothing weights that result in the smallest sum of squared,
one-step-ahead prediction errors. The optimization process is highly dependent on
this initialization. It is possible that the optimization process will fail due to the in-
ability to obtain stable initial values for the smoothing weights (refer to Greene 1993
and Judge et al 1980), and it is possible for the optimization to result in a local min-
ima.

The optimization process can result in weights to be chosen outside both the zero-
to-one range and the ARIMA additive-invertible region. By restricting weight opti-
mization to additive-invertible region, you can obtain a local minimum with stable
predictions. Likewise, weight optimization can be restricted to the zero-to-one range
or other ranges.

Standard Errors

The standard errors associated with the smoothing weights are calculated from the
Hessian matrix of the sum of squared, one-step-ahead prediction errors with respect
to the smoothing weights used in the optimization process.
Weights Near Zero or One

Sometimes the optimization process results in weights near zero or one.

For Simple or Double (Brown) Exponential Smoothing, a level weight near zero implies that simple differencing of the time series may be appropriate.

For Linear (Holt) Exponential Smoothing, a level weight near zero implies that the smoothed trend is constant and that an ARIMA model with deterministic trend may be a more appropriate model.

For Damped-Trend Linear Exponential Smoothing, a damping weight near one implies that Linear (Holt) Exponential Smoothing may be a more appropriate model.

For Winters Method and Seasonal Exponential Smoothing, a seasonal weight near one implies that a nonseasonal model may be more appropriate and a seasonal weight near zero implies that deterministic seasonal factors may be present.

Equations for the Smoothing Models

Simple Exponential Smoothing

The model equation for simple exponential smoothing is

\[ Y_t = \mu_t + \epsilon_t \]

The smoothing equation is

\[ L_t = \alpha Y_t + (1 - \alpha)L_{t-1} \]

The error-correction form of the smoothing equation is

\[ L_t = L_{t-1} + \alpha \epsilon_t \]

(Note: For missing values, \( \epsilon_t = 0 \).)

The \( k \)-step prediction equation is

\[ \hat{Y}_t(k) = L_t \]

The ARIMA model equivalency to simple exponential smoothing is the ARIMA(0,1,1) model

\[(1 - B)Y_t = (1 - \theta B)\epsilon_t \]

\[ \theta = 1 - \alpha \]
The moving-average form of the equation is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} \alpha \epsilon_{t-j} \]

For simple exponential smoothing, the additive-invertible region is

\[ \{ 0 < \alpha < 2 \} \]

The variance of the prediction errors is estimated as

\[ \text{var}(\epsilon_t(k)) = \text{var}(\epsilon_t) \left[ 1 + \sum_{j=1}^{k-1} \alpha^2 \right] = \text{var}(\epsilon_t)(1 + (k - 1)\alpha^2) \]

**Double (Brown) Exponential Smoothing**

The model equation for double exponential smoothing is

\[ Y_t = \mu_t + \beta_t t + \epsilon_t \]

The smoothing equations are

\[ L_t = \alpha Y_t + (1 - \alpha)L_{t-1} \]
\[ T_t = \alpha (L_t - L_{t-1}) + (1 - \alpha)T_{t-1} \]

This method may be equivalently described in terms of two successive applications of simple exponential smoothing:

\[ S_t^{[1]} = \alpha Y_t + (1 - \alpha)S_{t-1}^{[1]} \]
\[ S_t^{[2]} = \alpha S_t^{[1]} + (1 - \alpha)S_{t-1}^{[2]} \]

where \( S_t^{[1]} \) are the smoothed values of \( Y_t \), and \( S_t^{[2]} \) are the smoothed values of \( S_t^{[1]} \).

The prediction equation then takes the form:

\[ \hat{Y}_t(k) = (2 + \alpha k/(1 - \alpha))S_t^{[1]} - (1 + \alpha k/(1 - \alpha))S_t^{[2]} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T_{t-1} + \alpha \epsilon_t \]
\[ T_t = T_{t-1} + \alpha^2 e_t \]

(Note: For missing values, \( e_t = 0 \).)

The \( k \)-step prediction equation is

\[ \hat{Y}_t(k) = L_t + ((k - 1) + 1/\alpha)T_t \]

The ARIMA model equivalency to double exponential smoothing is the ARIMA\((0,2,2)\) model

\[ (1 - B)^2 Y_t = (1 - \theta B)^2 \epsilon_t \]

\[ \theta = 1 - \alpha \]

The moving-average form of the equation is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} (2\alpha + (j - 1)\alpha^2)\epsilon_{t-j} \]

For double exponential smoothing, the additive-invertible region is

\[ \{0 < \alpha < 2\} \]

The variance of the prediction errors is estimated as

\[ \text{var}(e_t(k)) = \text{var}(\epsilon_t) \left[ 1 + \sum_{j=1}^{k-1} (2\alpha + (j - 1)\alpha^2)^2 \right] \]

**Linear (Holt) Exponential Smoothing**

The model equation for linear exponential smoothing is

\[ Y_t = \mu_t + \beta_t t + \epsilon_t \]

The smoothing equations are

\[ L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1}) \]

\[ T_t = \gamma (L_t - L_{t-1}) + (1 - \gamma)T_{t-1} \]
The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T_{t-1} + \alpha e_t \]
\[ T_t = T_{t-1} + \alpha \gamma e_t \]

(Note: For missing values, \( e_t = 0 \).)

The \( k \)-step prediction equation is

\[ \hat{Y}_t(k) = L_t + kT_t \]

The ARIMA model equivalency to linear exponential smoothing is the ARIMA(0,2,2) model

\[ (1 - B)^2 Y_t = (1 - \theta_1 B - \theta_2 B^2) e_t \]
\[ \theta_1 = 2 - \alpha - \alpha \gamma \]
\[ \theta_2 = \alpha - 1 \]

The moving-average form of the equation is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} (\alpha + j\alpha \gamma) \epsilon_{t-j} \]

For linear exponential smoothing, the additive-invertible region is

\[ \{0 < \alpha < 2\} \]
\[ \{0 < \gamma < 4/\alpha - 2\} \]

The variance of the prediction errors is estimated as

\[ \text{var}(e_t(k)) = \text{var}(\epsilon_t) \left[ 1 + \sum_{j=1}^{k-1} (\alpha + j\alpha \gamma)^2 \right] \]
Damped-Trend Linear Exponential Smoothing

The model equation for damped-trend linear exponential smoothing is

\[ Y_t = \mu_t + \beta_t t + \epsilon_t \]

The smoothing equations are

\[ L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + \phi T_{t-1}) \]

\[ T_t = \gamma (L_t - L_{t-1}) + (1 - \gamma) \phi T_{t-1} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + \phi T_{t-1} + \alpha \epsilon_t \]

\[ T_t = \phi T_{t-1} + \alpha \gamma \epsilon_t \]

(Note: For missing values, \( \epsilon_t = 0 \).)

The \( k \)-step prediction equation is

\[ \hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^i T_t \]

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA(1,1,2) model

\[ (1 - \phi B)(1 - B)Y_t = (1 - \theta_1 B - \theta_2 B^2)\epsilon_t \]

\[ \theta_1 = 1 + \phi - \alpha - \alpha \gamma \phi \]

\[ \theta_2 = (\alpha - 1)\phi \]

The moving-average form of the equation (assuming \(|\phi| < 1\)) is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} \left( \alpha + \alpha \gamma \phi (\phi^j - 1)/(\phi - 1) \right) \epsilon_{t-j} \]

For damped-trend linear exponential smoothing, the additive-invertible region is

\[ \{0 < \alpha < 2\} \]
The variance of the prediction errors is estimated as

\[ \text{var}(e_t(k)) = \text{var}(e_t) \left[ 1 + \sum_{j=1}^{k-1} (\alpha + \alpha \gamma \phi (\phi^j - 1)/(\phi - 1))^2 \right] \]

**Seasonal Exponential Smoothing**

The model equation for seasonal exponential smoothing is

\[ Y_t = \mu_t + s_p(t) + \epsilon_t \]

The smoothing equations are

\[ L_t = \alpha(Y_t - S_{t-p}) + (1 - \alpha)L_{t-1} \]
\[ S_t = \delta(Y_t - L_t) + (1 - \delta)S_{t-p} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + \alpha \epsilon_t \]
\[ S_t = S_{t-p} + \delta(1 - \alpha) \epsilon_t \]

(Note: For missing values, \( \epsilon_t = 0 \).)

The \( k \)-step prediction equation is

\[ \hat{Y}_t(k) = L_t + S_{t-p+k} \]

The ARIMA model equivalency to seasonal exponential smoothing is the ARIMA(0,1,p+1)(0,1,0)_p model

\[ (1 - B)(1 - B^p)Y_t = (1 - \theta_1 B - \theta_2 B^p - \theta_3 B^{p+1})\epsilon_t \]

\[ \theta_1 = 1 - \alpha \]
\[ \theta_2 = 1 - \delta(1 - \alpha) \]
\[ \theta_3 = (1 - \alpha)(\delta - 1) \]
The moving-average form of the equation is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} \psi_j \epsilon_{t-j} \]

\[ \psi_j = \begin{cases} \alpha & \text{for } j \mod p \neq 0 \\ \alpha + \delta(1 - \alpha) & \text{for } j \mod p = 0 \end{cases} \]

For seasonal exponential smoothing, the additive-invertible region is

\[ \{ \max(-p\alpha, 0) < \delta(1 - \alpha) < (2 - \alpha) \} \]

The variance of the prediction errors is estimated as

\[ \text{var}(e_t(k)) = \text{var}(\epsilon_t) \left[ 1 + \sum_{j=1}^{k-1} \psi_j^2 \right] \]

**Winters Method – Additive Version**

The model equation for the additive version of Winters method is

\[ Y_t = \mu_t + \beta_t t + s_p(t) + \epsilon_t \]

The smoothing equations are

\[ L_t = \alpha(Y_t - S_{t-p}) + (1 - \alpha)(L_{t-1} + T_{t-1}) \]

\[ T_t = \gamma(L_t - L_{t-1}) + (1 - \gamma)T_{t-1} \]

\[ S_t = \delta(Y_t - L_t) + (1 - \delta)S_{t-p} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T_{t-1} + \alpha e_t \]

\[ T_t = T_{t-1} + \alpha \gamma e_t \]

\[ S_t = S_{t-p} + \delta(1 - \alpha)e_t \]

(Note: For missing values, \( e_t = 0 \).)
The \( k \)-step prediction equation is

\[
\hat{Y}_t(k) = L_t + kT_t + S_{t-p+k}
\]

The ARIMA model equivalency to the additive version of Winters method is the ARIMA\((0,1,p+1)(0,1,0)\) model

\[
(1 - B)(1 - B^p)Y_t = \left[ 1 - \sum_{i=1}^{p+1} \theta_i B^i \right] \epsilon_t
\]

\[
\theta_j = \begin{cases} 
1 - \alpha - \alpha \gamma & j = 1 \\
-\alpha \gamma & 2 \leq j \leq p - 1 \\
1 - \alpha \gamma - \delta(1 - \alpha) & j = p \\
(1 - \alpha)(\delta - 1) & j = p + 1
\end{cases}
\]

The moving-average form of the equation is

\[
Y_t = \epsilon_t + \sum_{j=1}^{\infty} \psi_j \epsilon_{t-j}
\]

\[
\psi_j = \begin{cases} 
\alpha + j \alpha \gamma & \text{for } j \mod p \neq 0 \\
\alpha + j \alpha \gamma + \delta(1 - \alpha), & \text{for } j \mod p = 0
\end{cases}
\]

For the additive version of Winters method (see Archibald 1990), the additive-invertible region is

\[
\{ \max(-p\alpha, 0) < \delta(1 - \alpha) < (2 - \alpha) \}
\]

\[
\{ 0 < \alpha \gamma < 2 - \alpha - \delta(1 - \alpha)(1 - \cos(\vartheta)) \}
\]

where \( \vartheta \) is the smallest nonnegative solution to the equations listed in Archibald (1990).

The variance of the prediction errors is estimated as

\[
var(\epsilon_t(k)) = var(\epsilon_t) \left[ 1 + \sum_{j=1}^{k-1} \psi_j^2 \right]
\]
Winters Method – Multiplicative Version

In order to use the multiplicative version of Winters method, the time series and all predictions must be strictly positive.

The model equation for the multiplicative version of Winters method is

\[ Y_t = (\mu_t + \beta_t) s_p(t) + \epsilon_t \]

The smoothing equations are

\[ L_t = \alpha \left( \frac{Y_t}{S_{t-p}} \right) + (1 - \alpha)(L_{t-1} + T_{t-1}) \]

\[ T_t = \gamma (L_t - L_{t-1}) + (1 - \gamma) T_{t-1} \]

\[ S_t = \delta \left( \frac{Y_t}{L_t} \right) + (1 - \delta) S_{t-p} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T_{t-1} + \alpha e_t / S_{t-p} \]

\[ T_t = T_{t-1} + \alpha \gamma e_t / S_{t-p} \]

\[ S_t = S_{t-p} + \delta (1 - \alpha) e_t / L_t \]

(Note: For missing values, \( e_t = 0 \).)

The k-step prediction equation is

\[ \hat{Y}_t(k) = (L_t + kT_t) S_{t-p+k} \]

The multiplicative version of Winters method does not have an ARIMA equivalent; however, when the seasonal variation is small, the ARIMA additive-invertible region of the additive version of Winters method described in the preceding section can approximate the stability region of the multiplicative version.

The variance of the prediction errors is estimated as

\[ var(e_t(k)) = var(e_t) \sum_{i=0}^{\infty} \sum_{j=0}^{p-1} (\psi_{j+ip} S_{t+k} / S_{t+k-j})^2 \]

where \( \psi_j \) are as described for the additive version of Winters method, and \( \psi_j = 0 \) for \( j \geq k \).
Intertmittent Models

This section details the computations performed for the intermittent forecasting models.

Interruption Time Series

Interruption time series have a large number of values that are zero. These types of series commonly occur in internet, inventory, sales, and other data where the demand for a particular item is intermittent. Typically, when the value of the series associated with a particular time period is nonzero, demand occurs; and, when the value is zero (or missing), no demand occurs. Since it is entirely possible that the number of time periods for which no demand occurs is large, many of the series values will be zero. Typical time series models (for example, smoothing models) are inadequate in the case of intermittent time series because many of the series values are zero. Since these models are based on weighted-summations of past values, they bias forecasts away from zero. Unlike the smoothing models that provide forecasts for future time periods, interruption forecasting models provide recommended stocking levels or estimated demand per period that are used to satisfy future demand.

Interruption Series Decomposition and Analysis

An intermittent time series (demand series) can be decomposed into two components: a demand interval series and a demand size series. Both of these component series are indexed based on when a demand occurred (demand index) rather than each time period (time index). The demand interval series is constructed based on the number of time periods between demands. The demand size series is constructed based on the size (or value) of the demands excluding zero (or base) demand values. Using these two component series, the average demand series is computed by dividing the size component values by the interval component values.

When a demand occurs is subjective; but typically depends on a base value. Typically, the base value is zero (default), but it can be any constant value and can be automatically determined based on the characteristics of the demand series.

Given a time series $y_t$, for $t = 1$ to $T$, where $t$ is the time index, suppose that there are $N$ nonzero demands occurring at times $t = t_i$, where $t_{i-1} < t_i$, for $i = 1$ to $N$. The time series is dissected into the demand interval series and the demand size series as follows:

(Demand Interval Series) $q_i = t_i - t_{i-1}$ for $i = 2$ to $N$

(Demand Size Series) $d_i = y_{t_i} - base$ for $i = 1$ to $N$

(Average Demand Series) $a_i = d_i / q_i$ for $i = 2$ to $N$

For the beginning of the demand series, $q_1$ is assigned to $t_1$, which assumes that a demand just occurred prior to the first recorded observation. For the end of the demand series, $q_{N+1}$ is assigned to $(T + 1 - t_N)$, which assumes that demand will occur just after the last recorded observation. The next future demand size, $d_{N+1}$, is always set to missing.
After decomposition, descriptive (summary) statistics can be computed to gain a greater understanding of the demand series including those statistics based on the season index.

For statistical analysis and model fitting, \( q_i \) and \( a_i \) for \( i = 2 \) to \( N \) and \( d_i \) for \( i = 1 \) to \( N \) are used. For forecasting, \( q_i \) for \( i = 1 \) to \( N + 1 \), \( a_i \) for \( i = 1 \) to \( N \), and \( d_i \) for \( i = 1 \) to \( N \) are used.

**Croston’s Method**

Croston’s Method models and forecasts each component independently, then combines the two forecasts. The following provides a description of how Croston’s Method is used in SAS High Performance Forecasting. More detailed information on this method can be found in Croston (1972) and Willemain, Smart, and Shocker (1994). The following description of Croston’s Method is based on the perspective of a person familiar with typical time series modeling techniques such as smoothing models.

By treating each component of the demand series as a time series based on the demand index, optimal smoothing parameters can be estimated and predictions for each component can be computed using nonseasonal exponential smoothing methods (simple, double, linear, damped-trend) as well as their transformed versions (log, square-root, logistic, Box-Cox).

For example, the following simple smoothing equations are used to generate predictions for demand size and interval components:

\[
\begin{align*}
\text{(Smoothed demand interval series)} & \quad L^q_{i+1} = L^q_i + \alpha_q(q_i - L^q_{i-1}) \\
\text{(Smoothed demand size series)} & \quad L^d_i = L^d_{i-1} + \alpha_d(d_i - L^d_{i-1})
\end{align*}
\]

The demand interval parameter, \( \alpha_q \), and demand size parameter, \( \alpha_d \), and the starting, intermediate, and final smoothing level states, \( L^q_i \) and \( L^d_i \), are estimated from the data using simple exponential smoothing parameter estimation. For the starting state at \( i = 1 \), \( L^q_1 = \max(q_1, L^q_0) \) where \( L^q_0 \) is the final backcast level state. For \( i > 1 \), the one-step-ahead prediction for demand interval \( q_i \) is \( \hat{q}_i = L^q_{i-1} \). For \( i > 0 \), the one-step-ahead prediction for demand size \( d_i \) is \( \hat{d}_i = L^d_{i-1} \).

Other (transformed) nonseasonal exponential smoothing methods can be used in a similar fashion. For linear smoothing, \( L^q_1 = \max(q_1 - T^q_0, L^q_0) \) where \( T^q_0 \) is the final backcast trend state. For damp-trend smoothing, \( L^q_1 = \max(q_1 - \phi_q T^q_0, L^q_0) \) where \( \phi_q \) is the damping parameter and \( T^q_0 \) is the final backcast trend state. For double smoothing, \( L^q_1 = \max(q_1 - T^q_0 / \alpha_q, L^q_0) \) where \( \alpha_q \) is the weight parameter and \( T^q_0 \) is the final backcast trend state.

Using these predictions based on the demand index, predictions of the average demand per period can be estimated. Predicted demand per period is also known as “stocking level,” assuming that disturbances affecting \( q_i \) are independent of \( d_i \). (This assumption is quite significant.)
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(Estimated demand per period) \[ y_i^* = \hat{d}_i / \hat{q}_i \]

\[ E[y_i^*] = E[d_i] / E[q_i] = E[z_{i-1}] / E[p_{i-1}] = \mu_d / \mu_q \]

(Variance) \[ \text{Var}(y_i^*) = (\hat{d}_i / \hat{q}_i)^2 \left( \text{Var}(d_i) / \hat{d}_i^2 + \text{Var}(q_i) / \hat{q}_i^2 \right) \]

where \( \mu_d, \hat{d}, \) and \( s_d \) are the mean, sample average, and standard deviation of the non-zero demands, and \( \mu_q, \hat{q}, \) and \( s_q \) are the mean, sample average, and standard deviation of the number of time periods between demands.

For the beginning of the series, the denominator of \( y_1^* \) is assigned \( q_i \) or the starting smoothing state \( p_0 \), whichever is greater. For the end of the series, the denominator of \( y_{N+1}^* \) is assigned \( q_{N+1} = (T + 1 - t_N) \) or the final smoothing state \( p_N \), whichever is greater.

Once the average demand per period has been estimated, a stocking level can be recommended:

(Recommended stocking level) \[ \hat{y}_t = y_i^* \quad \text{when} \quad t_i \leq t < t_{i+1} \]

(Variance) \[ \text{Var}(\hat{y}_t) = \text{Var}(y_i^*) \quad \text{when} \quad t_i \leq t < t_{i+1} \]

Since the predicted demand per period will be different than typical time series forecasts, the usual way of computing statistics of fit should also be different. The statistics of fit are based on the difference between the recommended stocking levels between demands and the demands:

(Accumulated recommended stocks) \[ s_t = \sum_{i=0}^{t} (\hat{y}_t - y_i^*) \]

(Estimate - Demand) \[ e_{t_i} = \hat{d}_i q_i - d_i \quad \text{when time} \ t_{i+1} \text{has demand} \]

Croston’s Method produces the same forecasts as simple exponential smoothing when demand occurs in every time period, \( q_i = 1 \) for all \( i \), but different (lower) prediction error variances. Croston’s Method is recommended for intermittent time series only.

**Average Demand Method**

Similar to Croston’s Method, the Average Demand Method is used to forecast intermittent time series; however, the Average Demand Method forecasts the average demand series directly, whereas Croston’s Method forecasts average demand series indirectly using the inverse decomposition of the demand interval and size series forecasts.

By treating the average demand series as a time series based on the demand index, optimal smoothing parameters can be estimated and predictions for average demand can be computed using nonseasonal exponential smoothing methods (simple, double, linear, damped-trend) as well as their transformed versions (log, square-root, logistic, Box-Cox).

For example, the following simple smoothing equations are used to generate predictions for the average demand series:
Time Indexed Holdout Samples versus Demand Indexed Holdout Samples

(Smoothed Average Demand Series) \[ L^a_i = L^a_{i-1} + \alpha (a_{i-1} - L^a_{i-1}) \]

The average demand level parameter, \( \alpha \), and the starting, intermediate, and final smoothing level states, \( L^a_i \), are estimated from the data using simple exponential smoothing parameter estimation. For the starting state at \( i = 1 \), \( L^a_1 = \max(a_1, L^a_0) \) where \( L^a_0 \) is the final backcast level state. For \( i > 1 \), the one-step-ahead prediction for \( a_i \) is \( \hat{a}_i = L^a_{i-1} \).

Other (transformed) nonseasonal exponential smoothing methods can be used in a similar fashion. For linear smoothing, \( L^a_1 = \max(a_1 - T^0_1, L^a_0) \) where \( T^0_0 \) is the final backcast trend state. For damp-trend smoothing, \( L^a_1 = \max(a_1 - \phi a T^a_0, L^a_0) \) where \( \phi \) is the damping parameter and \( T^a_0 \) is the final backcast trend state. For double smoothing, \( L^a_1 = \max(a_1 - T^0_1 / \alpha, L^a_0) \) where \( \alpha \) is the weight parameter and \( T^a_0 \) is the final backcast trend state.

Using these predictions based on the demand index, predictions of the average demand per period are provided directly, unlike Croston’s Method where the average demand is predicted using a ratio of predictions of the demand interval and size components.

\[
\text{(Estimated demand per period)} \quad y^*_i = \hat{a}_i + \text{base} \\
E[y^*_i] = E[a_i] \\
\text{(Variance)} \quad \text{see the exponential smoothing models}
\]

For the beginning of the series, \( \hat{a}_1 \) is derived from the starting level smoothing state and starting trend smoothing state (if applicable).

Once the average demand per period has been estimated, a stocking level can be recommended similar to Croston’s Method.

The Average Demand Method produces the same forecasts as exponential smoothing when demand occurs in every time period, \( q_i = 1 \) for all \( i \), but different (lower) prediction error variances. The Average Demand Method is recommended for intermittent time series only.

Time Indexed Holdout Samples versus Demand Indexed Holdout Samples

Holdout samples are typically specified based on the time index. For example, “holdout the last six months data.” For a demand series, the demand indexed holdout refers to the “demands that have occurred in the last six months.” If there are four demands in the last six months, the demand indexed holdout is four for a time indexed holdout of six. If there are no demands in the time indexed holdout, the demand indexed holdout is zero and in-sample analysis is used. For intermittent demand model selection, demand indexed-based holdouts are used for model selection.
Automatic Intermittent Demand Model Selection

The exponential smoothing method to be used to forecast the intermittent demand series can be specified, or it can be selected automatically using a model selection criterion and either in-sample or holdout sample analysis. The exponential smoothing method for each demand series component (interval, size, and average) can be automatically selected as well as the choice between Croston’s Method and the Average Demand Method.

For Croston’s Method, the exponential smoothing methods used to forecast the demand interval and size components are automatically selected independently. For the Average Demand Method, the exponential smoothing methods used to forecast the average demand component are automatically selected, again independently. Based on the model selection criterion, the selection is based on how well the method fits (in sample) or predicts (holdout sample) the demand series component by treating the demand index as a time index. The following equations describe the component prediction errors associated with each of the demand series components that are used in component model selection:

\[
\begin{align*}
\text{(Demand Interval Series)} & \quad e^q_i = q_i - \hat{q}_i \quad \text{for } i = 2 \text{ to } N \\
\text{(Demand Size Series)} & \quad e^d_i = d_i - \hat{d}_i \quad \text{for } i = 1 \text{ to } N \\
\text{(Average Demand Series)} & \quad e^a_i = a_i - \hat{a}_i \quad \text{for } i = 2 \text{ to } N
\end{align*}
\]

Once the exponential smoothing methods are selected for each demand series component, the predictions for either Croston’s Method, \((\hat{d}_i / \hat{q}_i)\), the Average Demand Method, \(\hat{a}_i\), or both are computed based on the selected method for each component.

When choosing between Croston’s Method and the Average Demand Method, the model is selected by considering how well the model predicts average demand with respect to the time. The following equations describe the average prediction errors associated with the predicted average demand that are used in model selection:

\[
\begin{align*}
\text{(Croston’s Method)} & \quad e^c_i = (d_i / q_i) - (\hat{d}_i / \hat{q}_i) \quad \text{for } i = 2 \text{ to } N \\
\text{(Average Demand Method)} & \quad e^a_i = a_i - \hat{a}_i \quad \text{for } i = 2 \text{ to } N
\end{align*}
\]

Series Transformations

For forecasting models, transforming the time series may aid in improving forecasting accuracy.

There are four transformations available, for strictly positive series only. Let \(y_t > 0\) be the original time series, and let \(w_t\) be the transformed series. The transformations are defined as follows:

\( \text{Log} \quad \quad \text{is the logarithmic transformation} \)

\[ w_t = \ln(y_t) \]
Logistic is the logistic transformation

\[ w_t = \ln\left(\frac{c y_t}{1 - c y_t}\right) \]

where the scaling factor \( c \) is

\[ c = (1 - e^{-6})10^{-\text{ceil} (\log_{10} (\max(y_t)))} \]

and \( \text{ceil}(x) \) is the smallest integer greater than or equal to \( x \).

Square Root is the square root transformation

\[ w_t = \sqrt{y_t} \]

Box Cox is the Box-Cox transformation

\[ w_t = \begin{cases} 
\frac{y_t^{\lambda} - 1}{\lambda}, & \lambda \neq 0 \\
\ln(y_t), & \lambda = 0 
\end{cases} \]

Parameter estimation is performed using the transformed series. The transformed model predictions and confidence limits are then obtained from the transformed time-series and these parameter estimates.

The transformed model predictions \( \hat{w}_t \) are used to obtain either the minimum mean absolute error (MMAE) or minimum mean squared error (MMSE) predictions \( \hat{y}_t \), depending on the setting of the forecast options. The model is then evaluated based on the residuals of the original time series and these predictions. The transformed model confidence limits are inverse-transformed to obtain the forecast confidence limits.

**Predictions for Transformed Models**

Since the transformations described in the previous section are monotonic, applying the inverse-transformation to the transformed model predictions results in the median of the conditional probability density function at each point in time. This is the minimum mean absolute error (MMAE) prediction.

If \( w_t = F(y_t) \) is the transform with inverse-transform \( y_t = F^{-1}(w_t) \), then

\[ \text{median}(\hat{y}_t) = F^{-1}(\mathbb{E}[w_t]) = F^{-1}(\hat{w}_t) \]

The minimum mean squared error (MMSE) predictions are the mean of the conditional probability density function at each point in time. Assuming that the prediction errors are normally distributed with variance \( \sigma_t^2 \), the MMSE predictions for each of the transformations are as follows:

Log is the conditional expectation of inverse-logarithmic transformation.

\[ \hat{y}_t = E[e^{w_t}] = \exp(\hat{w}_t + \sigma_t^2/2) \]
Logistic is the conditional expectation of inverse-logistic transformation.

\[ \hat{y}_t = E \left[ \frac{1}{c(1 + e^{w_t})} \right] \]

where the scaling factor \( c = (1 - 10^{-6})10^{-\text{ceil}(\log_{10}(\max(y_t)))} \).

Square Root is the conditional expectation of the inverse-square root transformation.

\[ \hat{y}_t = E \left[ w_t^2 \right] = \hat{w}_t^2 + \sigma_t^2 \]

Box Cox is the conditional expectation of the inverse Box-Cox transformation.

\[ \hat{y}_t = \begin{cases} E \left[ (\lambda w_t + 1)^{1/\lambda} \right], & \lambda \neq 0 \\ E[e^{w_t}] = \exp(\hat{w}_t + \frac{1}{2}\sigma_t^2), & \lambda = 0 \end{cases} \]

The expectations of the inverse logistic and Box-Cox (\( \lambda \neq 0 \)) transformations do not generally have explicit solutions and are computed using numerical integration.

**Series Diagnostic Tests**

This section describes the diagnostic tests that are used to determine the kinds of forecasting models appropriate for a series.

The series diagnostics are a set of heuristics that provide recommendations on whether or not the forecasting model should contain a log transform, trend terms, and seasonal terms or whether or not the time series is intermittent. These recommendations are used by the automatic model selection process to restrict the model search to a subset of the model selection list.

The tests that are used by the series diagnostics will not always produce the correct classification of the series. They are intended to accelerate the process of searching for a good forecasting model for the series, but you should not rely on them if finding the very best model is important to you.

The series diagnostics tests are intended as a heuristic tool only, and no statistical validity is claimed for them. These tests may be modified and enhanced in future releases of the SAS High Performance Forecasting. The testing strategy is as follows:

1. **Intermittent test.** Compute the average time interval between demands. If the time average time interval is greater than a preset limit, an intermittent forecasting model is used.

2. **Seasonality test.** The resultant series is tested for seasonality. A seasonal dummy model with AR(1) errors is fit and the joint significance of the seasonal dummy estimates is tested. If the seasonal dummies are significant, the AIC statistic for this model is compared to the AIC for and AR(1) model without seasonal dummies. Nonseasonal model, a seasonal forecasting model is used.
Statistics of Fit

This section explains the goodness-of-fit statistics reported to measure how well different models fit the data. The statistics of fit for the various forecasting models can be printed or stored in a data set.

The various statistics of fit reported are as follows. In these formula, \( n \) is the number of nonmissing observations and \( k \) is the number of fitted parameters in the model.

**Number of Nonmissing Observations.**
The number of nonmissing observations used to fit the model.

**Number of Observations.**
The total number of observations used to fit the model, including both missing and nonmissing observations.

**Number of Missing Actuals.**
The number of missing actual values.

**Number of Missing Predicted Values.**
The number of missing predicted values.

**Number of Model Parameters.**
The number of parameters fit to the data. For combined forecast, this is the number of forecast components.

**Total Sum of Squares (Uncorrected).**
The total sum of squares for the series, SST, uncorrected for the mean: \( \sum_{t=1}^{n} y_t^2 \).

**Total Sum of Squares (Corrected).**
The total sum of squares for the series, SST, corrected for the mean: \( \sum_{t=1}^{n} (y_t - \bar{y})^2 \), where \( \bar{y} \) is the series mean.

**Sum of Square Errors.**
The sum of the squared prediction errors, SSE. \( SSE = \sum_{t=1}^{n} (y_t - \hat{y}_t)^2 \), where \( \hat{y} \) is the one-step predicted value.

**Mean Square Error.**
The mean squared prediction error, MSE, calculated from the one-step-ahead forecasts. \( MSE = \frac{1}{n} SSE \). This formula enables you to evaluate small holdout samples.

**Root Mean Square Error.**
The root mean square error (RMSE), \( \sqrt{MSE} \).

**Mean Absolute Percent Error.**
The mean absolute percent prediction error (MAPE), \( \frac{100}{n} \sum_{t=1}^{n} \frac{|y_t - \hat{y}_t|}{y_t} \).
The summation ignores observations where \( y_t = 0 \).
Mean Absolute Error.
The mean absolute prediction error, $\frac{1}{n} \sum_{t=1}^{n} |y_t - \hat{y}_t|$.

R-Square.
The $R^2$ statistic, $R^2 = 1 - \frac{SSE}{SST}$. If the model fits the series badly, the model error sum of squares, $SSE$, may be larger than $SST$ and the $R^2$ statistic will be negative.

Adjusted R-Square.
The adjusted $R^2$ statistic, $1 - \left(\frac{n-1}{n-k}\right)(1 - R^2)$.

Amemiya’s Adjusted R-Square.
Amemiya’s adjusted $R^2$, $1 - \left(\frac{n+k}{n-k}\right)(1 - R^2)$.

Random Walk R-Square.
The random walk $R^2$ statistic (Harvey’s $R^2$ statistic using the random walk model for comparison), $1 - \left(\frac{n-1}{n}\right)\frac{SSE}{RWSSE}$, where $RWSSE = \sum_{t=2}^{n} (y_t - y_{t-1} - \mu)^2$, and $\mu = \frac{1}{n-1} \sum_{t=2}^{n} (y_t - y_{t-1})$.

Akaike’s Information Criterion.
Akaike’s information criterion (AIC), $n \ln(SSE/n) + 2k$.

Schwarz Bayesian Information Criterion.
Schwarz Bayesian information criterion (SBC or BIC), $n \ln(SSE/n) + k \ln(n)$.

Amemiya’s Prediction Criterion.
Amemiya’s prediction criterion, $\frac{1}{n} \frac{SST(n+k)}{n-k} (1 - R^2) = \left(\frac{n+k}{n-k}\right) \frac{1}{n} SSE$.

Maximum Error.
The largest prediction error.

Minimum Error.
The smallest prediction error.

Maximum Percent Error.
The largest percent prediction error, $100 \max \left(\frac{(y_t - \hat{y}_t)}{y_t}\right)$. The summation ignores observations where $y_t = 0$.

Minimum Percent Error.
The smallest percent prediction error, $100 \min \left(\frac{(y_t - \hat{y}_t)}{y_t}\right)$. The summation ignores observations where $y_t = 0$.

Mean Error.
The mean prediction error, $\frac{1}{n} \sum_{t=1}^{n} (y_t - \hat{y}_t)$.

Mean Percent Error.
The mean percent prediction error, $\frac{100}{n} \sum_{t=1}^{n} \frac{(y_t - \hat{y}_t)}{y_t}$. The summation ignores observations where $y_t = 0$. 
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