SAS/STAT® 9.22 User's Guide

The MI Procedure

(Book Excerpt)

SAS® Documentation

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Chapter 54

The MI Procedure

Contents

Overview: MI Procedure

The MI procedure performs multiple imputation of missing data. Missing values are an issue in a substantial number of statistical analyses. Most SAS statistical procedures exclude observations with any missing variable values from the analysis. These observations are called incomplete cases. Although analyzing only complete cases has the advantage of simplicity, the information contained in the incomplete cases is lost. This approach also ignores possible systematic differences between the complete cases and the incomplete cases, and the resulting inference might not be applicable to the population of all cases, especially with a small number of complete cases.

Some SAS procedures use all the available cases in an analysis—that is, cases with useful information. For example, the CORR procedure estimates a variable mean by using all cases with nonmissing values for this variable, ignoring the possible missing values in other variables. PROC CORR also estimates a correlation by using all cases with nonmissing values for this pair of variables. This makes better use of the available data than using only the complete cases does, but the resulting correlation matrix might not be positive definite.

Another strategy for handling missing data is single imputation, which substitutes a value for each missing value. Standard statistical procedures for complete data analysis can then be used with the filled-in data set. For example, each missing value can be imputed with the variable mean of the complete cases, or it can be imputed with the mean conditional on observed values of other variables. This approach treats missing values as if they were known in the complete-data analysis. However, single imputation does not reflect the uncertainty about the predictions of the unknown missing values, and the resulting estimated variances of the parameter estimates will be biased toward zero (Rubin 1987, p. 13).

Instead of filling in a single value for each missing value, multiple imputation (Rubin 1976, 1987) replaces each missing value with a set of plausible values that represent the uncertainty about the right value to impute. The multiply imputed data sets are then analyzed by using standard procedures for complete data and combining the results from these analyses. No matter which complete-data analysis is used, the process of combining results from different data sets is essentially the same.

Multiple imputation does not attempt to estimate each missing value through simulated values. Instead, it draws a random sample of the missing values from its distribution. This process results in valid statistical inferences that properly reflect the uncertainty due to missing values—for example, confidence intervals with the correct probability coverage.

Multiple imputation inference involves three distinct phases:

1. The missing data are filled in m times to generate m complete data sets.
2. The m complete data sets are analyzed using standard statistical analyses.
3. The results from the m complete data sets are combined to produce inferential results.

The MI procedure creates multiply imputed data sets for incomplete multivariate data. It uses methods that incorporate appropriate variability across the m imputations. The method of choice depends on the patterns of missingness.

A data set with variables Y_1, Y_2, ..., Y_p (in that order) is said to have a *monotone missing pattern* when the event that a variable Y_j is missing for a particular individual implies that all subsequent variables Y_k, $k > j$, are missing for that individual.

For data sets with monotone missing patterns, either a parametric method that assumes multivariate normality or a nonparametric method is appropriate to impute missing values for a continuous variable. Parametric methods available include the regression method (Rubin 1987, pp. 166–167) and the predictive mean matching method (Heitjan and Little 1991; Schenker and Taylor 1996). The nonparametric method is the propensity score method (Rubin 1987, pp. 124, 158).

To impute missing values for a classification variable in data sets with monotone missing patterns, you can use the logistic regression method when the classification variable has a binary or ordinal response, and the discriminant function method when the classification variable has a binary or nominal response.

For data sets with arbitrary missing patterns, a Markov chain Monte Carlo (MCMC) method (Schafer 1997) that assumes multivariate normality is used to impute all missing values or just enough missing values for continuous variables to make the imputed data sets have monotone missing patterns. When an imputed data set has a monotone missing pattern, methods for data sets with monotone missing patterns can then be used to impute remaining missing values.

Once the m complete data sets are analyzed using standard SAS procedures, the MIANALYZE procedure can be used to generate valid statistical inferences about these parameters by combining results from the m analyses.

Often, as few as three to five imputations are adequate in multiple imputation (Rubin 1996, p. 480). The relative efficiency of the small m imputation estimator is high for cases with little missing information (Rubin 1987, p. 114). (Also see the section "Multiple Imputation Efficiency" on page 4419.)

Multiple imputation inference assumes that the model (variables) you used to analyze the multiply imputed data (the analyst's model) is the same as the model used to impute missing values in multiple imputation (the imputer's model). But in practice, the two models might not be the same. The consequences for different scenarios (Schafer 1997, pp. 139–143) are discussed in the section "Imputer's Model Versus Analyst's Model" on page 4420.

When an MCMC method is used to used to impute missing values, the trace (time series) and autocorrelation function plots for parameters such as variable means and covariances can be displayed to check for convergence of the MCMC method. See the section "Checking Convergence in MCMC" on page 4412 for a detailed description of these plots. If the ODS GRAPHICS ON statement is specified, these statistical graphics are created via the Output Delivery System (ODS). Otherwise, the traditional graphics are created.

Getting Started: MI Procedure

Consider the following Fitness data set that has been altered to contain an arbitrary pattern of missingness:

```
*----------------- Data on Physical Fitness -----------------*
| These measurements were made on men involved in a physical |
| fitness course at N.C. State University.                   |
| Only selected variables of                                 |
| Oxygen (oxygen intake, ml per kg body weight per minute),   |
| Runtime (time to run 1.5 miles in minutes), and            |
| RunPulse (heart rate while running) are used.              |
| Certain values were changed to missing for the analysis.   |
*------------------------------------------------------------*;
data FitMiss;
   input Oxygen RunTime RunPulse @@;
   datalines;
44.609  11.37  178     45.313  10.07  185
54.297   8.65  156     59.571    .      .
49.874   9.22    .     44.811  11.63  176
  .     11.95  176          .  10.85    .
39.442  13.08  174     60.055   8.63  170
50.541    .      .     37.388  14.03  186
44.754  11.12  176     47.273    .      .
51.855  10.33  166     49.156   8.95  180
40.836  10.95  168     46.672  10.00    .
46.774  10.25    .     50.388  10.08  168
39.407  12.63  174     46.080  11.17  156
45.441   9.63  164       .      8.92    .
45.118  11.08    .     39.203  12.88  168
45.790  10.47  186     50.545   9.93  148
48.673   9.40  186     47.920  11.50  170
47.467  10.50  170
;
```

Suppose that the data are multivariate normally distributed and the missing data are missing at random (MAR). That is, the probability that an observation is missing can depend on the observed variable values of the individual, but not on the missing variable values of the individual. See the section "Statistical Assumptions for Multiple Imputation" on page 4395 for a detailed description of the MAR assumption.

The following statements invoke the MI procedure and impute missing values for the FitMiss data set:

```
proc mi data=FitMiss seed=501213 mu0=50 10 180 out=outmi;
   var Oxygen RunTime RunPulse;
run;
```

The "Model Information" table in Figure 54.1 describes the method used in the multiple imputation process. By default, the procedure uses the Markov chain Monte Carlo (MCMC) method with a single chain to create five imputations. The posterior mode, the highest observed-data posterior density, with a noninformative prior, is computed from the expectation-maximization (EM) algorithm and is used as the starting value for the chain.

Figure 54.1 Model Information

```
                              The MI Procedure

                             Model Information

          Data Set                              WORK.FITMISS
          Method                                MCMC
          Multiple Imputation Chain             Single Chain
          Initial Estimates for MCMC            EM Posterior Mode
          Start                                 Starting Value
          Prior                                 Jeffreys
          Number of Imputations                 5
          Number of Burn-in Iterations          200
          Number of Iterations                  100
          Seed for random number generator      501213
```

The MI procedure takes 200 burn-in iterations before the first imputation and 100 iterations between imputations. In a Markov chain, the information in the current iteration influences the state of the next iteration. The burn-in iterations are iterations in the beginning of each chain that are used both to eliminate the series of dependence on the starting value of the chain and to achieve the stationary distribution. The between-imputation iterations in a single chain are used to eliminate the series of dependence between the two imputations.

The "Missing Data Patterns" table in Figure 54.2 lists distinct missing data patterns with corresponding frequencies and percents. Here, an "X" means that the variable is observed in the corresponding group and a "." means that the variable is missing. The table also displays group-specific variable means. The MI procedure sorts the data into groups based on whether the analysis variables are observed or missing. For a detailed description of missing data patterns, see the section "Missing Data Patterns" on page 4396.

Figure 54.2 Missing Data Patterns

```
                         Missing Data Patterns

                                Run       Run
           Group      Oxygen    Time      Pulse        Freq      Percent

               1      X         X         X              21        67.74
               2      X         X         .               4        12.90
               3      X         .         .               3         9.68
               4      .         X         X               1         3.23
               5      .         X         .               2         6.45

                        Missing Data Patterns

                   -----------------Group Means----------------
            Group            Oxygen          RunTime         RunPulse

                1         46.353810        10.809524       171.666667
                2         47.109500        10.137500                .
                3         52.461667                .                .
                4                 .        11.950000       176.000000
                5                 .         9.885000                .
```

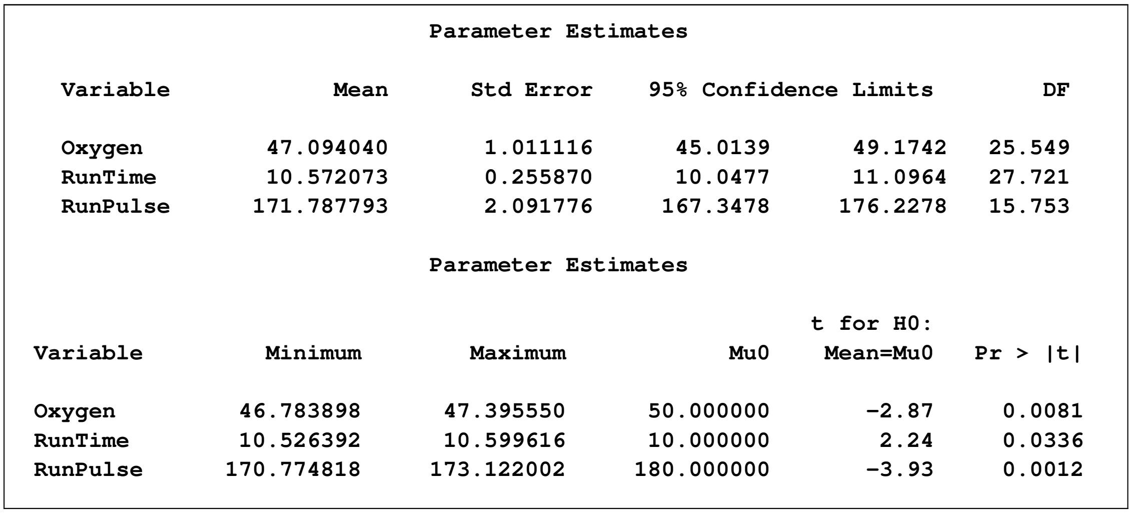
After the completion of *m* imputations, the "Variance Information" table in Figure 54.3 displays the between-imputation variance, within-imputation variance, and total variance for combining complete-data inferences. It also displays the degrees of freedom for the total variance. The relative increase in variance due to missing values, the fraction of missing information, and the relative efficiency (in units of variance) for each variable are also displayed. A detailed description of these statistics is provided in the section "Combining Inferences from Multiply Imputed Data Sets" on page 4418.

Figure 54.3 Variance Information

```
                        Variance Information

                 -----------------Variance-----------------
        Variable          Between          Within           Total       DF

        Oxygen           0.056930        0.954041        1.022356   25.549
        RunTime          0.000811        0.064496        0.065469   27.721
        RunPulse         0.922032        3.269089        4.375528   15.753

                         Variance Information

                        Relative        Fraction
                        Increase         Missing        Relative
             Variable    in Variance     Information      Efficiency

             Oxygen         0.071606        0.068898        0.986408
             RunTime        0.015084        0.014968        0.997015
             RunPulse       0.338455        0.275664        0.947748
```

The “Parameter Estimates” table in Figure 54.4 displays the estimated mean and standard error of the mean for each variable. The inferences are based on the *t* distribution. The table also displays a 95% confidence interval for the mean and a *t* statistic with the associated *p*-value for the hypothesis that the population mean is equal to the value specified with the MU0= option. A detailed description of these statistics is provided in the section “Combining Inferences from Multiply Imputed Data Sets” on page 4418.

Figure 54.4 Parameter Estimates



```
                              Parameter Estimates

  Variable            Mean      Std Error    95% Confidence Limits         DF

  Oxygen         47.094040       1.011116      45.0139      49.1742    25.549
  RunTime        10.572073       0.255870      10.0477      11.0964    27.721
  RunPulse      171.787793       2.091776     167.3478     176.2278    15.753

                              Parameter Estimates

                                                           t for H0:
Variable           Minimum         Maximum          Mu0     Mean=Mu0   Pr > |t|

Oxygen           46.783898       47.395550    50.000000        -2.87     0.0081
RunTime          10.526392       10.599616    10.000000         2.24     0.0336
RunPulse        170.774818      173.122002   180.000000        -3.93     0.0012
```

In addition to the output tables, the procedure also creates a data set with imputed values. The imputed data sets are stored in the outmi data set, with the index variable _Imputation_ indicating the imputation numbers. The data set can now be analyzed using standard statistical procedures with _Imputation_ as a BY variable.

The following statements list the first 10 observations of data set outmi:

```
proc print data=outmi (obs=10);
  title 'First 10 Observations of the Imputed Data Set';
run;
```

The table in Figure 54.5 shows that the precision of the imputed values differs from the precision of the observed values. You can use the ROUND= option to make the imputed values consistent with the observed values.

Figure 54.5 Imputed Data Set

```
                 First 10 Observations of the Imputed Data Set

                                                                 Run
              Obs     _Imputation_      Oxygen     RunTime      Pulse

                1           1          44.6090     11.3700     178.000
                2           1          45.3130     10.0700     185.000
                3           1          54.2970      8.6500     156.000
                4           1          59.5710      8.0747     155.925
                5           1          49.8740      9.2200     176.837
                6           1          44.8110     11.6300     176.000
                7           1          42.8857     11.9500     176.000
                8           1          46.9992     10.8500     173.099
                9           1          39.4420     13.0800     174.000
               10           1          60.0550      8.6300     170.000
```

Syntax: MI Procedure

The following statements are available in PROC MI:

PROC MI < *options* > ;
  BY *variables* ;
  CLASS *variables* ;
  EM < *options* > ;
  FREQ *variable* ;
  MCMC < *options* > ;
  MONOTONE < *options* > ;
  TRANSFORM *transform (variables < / options >)* < . . . *transform (variables < / options >)*
    > ;
  VAR *variables* ;

The BY statement specifies groups in which separate multiple imputation analyses are performed.

The CLASS statement lists the classification variables in the VAR statement. Classification variables can be either character or numeric.

The EM statement uses the EM algorithm to compute the maximum likelihood estimate (MLE) of the data with missing values, assuming a multivariate normal distribution for the data.

The FREQ statement specifies the variable that represents the frequency of occurrence for other values in the observation.

The MCMC statement uses a Markov chain Monte Carlo method to impute values for a data set with an arbitrary missing pattern, assuming a multivariate normal distribution for the data.

The MONOTONE statement specifies monotone methods to impute continuous and classification variables for a data set with a monotone missing pattern. Note that you can use either an MCMC statement or a MONOTONE statement, but not both. When neither of these two statements is specified, the MCMC method with its default options is used.

The TRANSFORM statement lists the variables to be transformed before the imputation process. The imputed values of these transformed variables are reverse-transformed to the original forms before the imputation.

The VAR statement lists the numeric variables to be analyzed. If you omit the VAR statement, all numeric variables not listed in other statements are used.

The PROC MI statement is the only required statement for the MI procedure. The rest of this section provides detailed syntax information for each of these statements, beginning with the PROC MI statement. The remaining statements are presented in alphabetical order.

PROC MI Statement

PROC MI < *options* > **;**

Table 54.1 summarizes the options available in the PROC MI statement.

Table 54.1 Summary of PROC MI Options

Option	Description
Data Sets	
DATA=	Specifies the input data set
OUT=	Specifies the output data set with imputed values
Imputation Details	
NIMPUTE=	Specifies the number of imputations
SEED=	Specifies the seed to begin random number generator
ROUND=	Specifies units to round imputed variable values
MAXIMUM=	Specifies maximum values for imputed variable values
MINIMUM=	Specifies minimum values for imputed variable values
MINMAXITER=	Specifies the maximum number of iterations to impute values in the specified range
SINGULAR=	Specifies the singularity criterion
Statistical Analysis	
ALPHA=	Specifies the level for the confidence interval, $(1 - \alpha)$
MU0=	Specifies means under the null hypothesis
Printed Output	
NOPRINT	Suppresses all displayed output
SIMPLE	Displays univariate statistics and correlations

The following options can be used in the PROC MI statement. They are listed in alphabetical order.

ALPHA=α

specifies that confidence limits be constructed for the mean estimates with confidence level $100(1-\alpha)\%$, where $0 < \alpha < 1$. The default is ALPHA=0.05.

DATA=*SAS-data-set*

names the SAS data set to be analyzed by PROC MI. By default, the procedure uses the most recently created SAS data set.

MAXIMUM=*numbers*

specifies maximum values for imputed variables. When an intended imputed value is greater than the maximum, PROC MI redraws another value for imputation. If only one number is specified, that number is used for all variables. If more than one number is specified, you must use a VAR statement, and the specified numbers must correspond to variables in the VAR statement. The default number is a missing value, which indicates no restriction on the maximum for the corresponding variable

The MAXIMUM= option is related to the MINIMUM= and ROUND= options, which are used to make the imputed values more consistent with the observed variable values. These options are applicable only if you use the MCMC method or the monotone regression method.

When specifying a maximum for the first variable only, you must also specify a missing value after the maximum. Otherwise, the maximum is used for all variables.
For example, the “MAXIMUM= 100 .” option sets a maximum of 100 for the first analysis variable only and no maximum for the remaining variables. The “MAXIMUM= . 100” option sets a maximum of 100 for the second analysis variable only and no maximum for the other variables.

MINIMUM=*numbers*

specifies the minimum values for imputed variables. When an intended imputed value is less than the minimum, PROC MI redraws another value for imputation. If only one number is specified, that number is used for all variables. If more than one number is specified, you must use a VAR statement, and the specified numbers must correspond to variables in the VAR statement. The default number is a missing value, which indicates no restriction on the minimum for the corresponding variable

MINMAXITER=*number*

specifies the maximum number of iterations for imputed values to be in the specified range when the option MINIMUM or MAXIMUM is also specified. The default is MINMAXITER=100.

MU0=*numbers*

THETA0=*numbers*

specifies the parameter values $\boldsymbol{\mu}_0$ under the null hypothesis $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ for the population means corresponding to the analysis variables. Each hypothesis is tested with a t test. If only one number is specified, that number is used for all variables. If more than one number is specified, you must use a VAR statement, and the specified numbers must correspond to variables in the VAR statement. The default is MU0=0.

If a variable is transformed as specified in a TRANSFORM statement, then the same transformation for that variable is also applied to its corresponding specified MU0= value in the t test. If the parameter values $\boldsymbol{\mu}_0$ for a transformed variable are not specified, then a value of zero is used for the resulting $\boldsymbol{\mu}_0$ after transformation.

NIMPUTE=*number*

specifies the number of imputations. The default is NIMPUTE=5. You can specify NIMPUTE=0 to skip the imputation. In this case, only tables of model information, missing data patterns, descriptive statistics (SIMPLE option), and MLE from the EM algorithm (EM statement) are displayed.

NOPRINT

suppresses the display of all output. Note that this option temporarily disables the Output Delivery System (ODS); see Chapter 20, "Using the Output Delivery System," for more information.

OUT=*SAS-data-set*

creates an output SAS data set containing imputation results. The data set includes an index variable, _Imputation_, to identify the imputation number. For each imputation, the data set contains all variables in the input data set with missing values being replaced by the imputed values. See the section "Output Data Sets" on page 4416 for a description of this data set.

ROUND=*numbers*

specifies the units to round variables in the imputation. If only one number is specified, that number is used for all continuous variables. If more than one number is specified, you must use a VAR statement, and the specified numbers must correspond to variables in the VAR statement. When the classification variables are listed in the VAR statement, their corresponding roundoff units are not used. The default number is a missing value, which indicates no rounding for imputed variables.

When specifying a roundoff unit for the first variable only, you must also specify a missing value after the roundoff unit. Otherwise, the roundoff unit is used for all variables. For example, the option "ROUND= 10 ." sets a roundoff unit of 10 for the first analysis variable only and no rounding for the remaining variables. The option "ROUND= . 10" sets a roundoff unit of 10 for the second analysis variable only and no rounding for other variables.

The ROUND= option sets the precision of imputed values. For example, with a roundoff unit of 0.001, each value is rounded to the nearest multiple of 0.001. That is, each value has three significant digits after the decimal point. See Example 54.3 for an illustration of this option.

SEED=*number*

specifies a positive integer to start the pseudo-random number generator. The default is a value generated from reading the time of day from the computer's clock. However, in order to duplicate the results under identical situations, you must use the same value of the seed explicitly in subsequent runs of the MI procedure.

The seed information is displayed in the "Model Information" table so that the results can be reproduced by specifying this seed with the SEED= option. You need to specify the same seed number in the future to reproduce the results.

SIMPLE

displays simple descriptive univariate statistics and pairwise correlations from available cases. For a detailed description of these statistics, see the section "Descriptive Statistics" on page 4393.

SINGULAR=*p*

specifies the criterion for determining the singularity of a covariance matrix based on standardized variables, where $0 < p < 1$. The default is SINGULAR=1E−8.

Suppose that $\mathbf{S}$ is a covariance matrix and v is the number of variables in $\mathbf{S}$. Based on the spectral decomposition $\mathbf{S} = \mathbf{\Gamma \Lambda \Gamma}'$, where $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues λ_j, $j = 1, \ldots, v$, where $\lambda_i \geq \lambda_j$ when $i < j$, and $\mathbf{\Gamma}$ is a matrix with the corresponding orthonormal eigenvectors of $\mathbf{S}$ as columns, $\mathbf{S}$ is considered singular when an eigenvalue λ_j is less than $p\bar{\lambda}$, where the average $\bar{\lambda} = \sum_{k=1}^{v} \lambda_k / v$.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC MI to obtain separate analyses on observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the MI procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

You can specify a BY statement with PROC MI to obtain separate analyses on observations in groups defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the BY statement option NOTSORTED or DESCENDING in the BY statement for the MI procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.

- Create an index on the BY variables by using the DATASETS procedure.

For more information about the BY statement, see *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the *Base SAS Procedures Guide*.

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

CLASS Statement

CLASS *variables* **;**

The CLASS statement specifies the classification variables in the VAR statement. Classification variables can be either character or numeric. The CLASS statement must be used in conjunction with the MONOTONE statement.

Classification levels are determined from the formatted values of the classification variables. See "The FORMAT Procedure" in the *Base SAS Procedures Guide* for details.

EM Statement

EM < *options* > **;**

The expectation-maximization (EM) algorithm is a technique for maximum likelihood estimation in parametric models for incomplete data. The EM statement uses the EM algorithm to compute the MLE for $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, the means and covariance matrix, of a multivariate normal distribution from the input data set with missing values. Either the means and covariances from complete cases or the means and standard deviations from available cases can be used as the initial estimates for the EM algorithm. You can also specify the correlations for the estimates from available cases.

You can also use the EM statement with the NIMPUTE=0 option in the PROC MI statement to compute the EM estimates without multiple imputation, as shown in Example 54.1.

The following seven options are available with the EM statement:

CONVERGE=*p*

XCONV=*p*

sets the convergence criterion. The value must be between 0 and 1. The iterations are considered to have converged when the change in the parameter estimates between iteration steps is less than p for each parameter—that is, for each of the means and covariances. For each parameter, the change is a relative change if the parameter is greater than 0.01 in absolute value; otherwise, it is an absolute change. By default, CONVERGE=1E−4.

INITIAL=CC | AC | AC(R=*r*)

sets the initial estimates for the EM algorithm. The INITIAL=CC option uses the means and covariances from complete cases; the INITIAL=AC option uses the means and standard deviations from available cases and the correlations are set to zero; and the INITIAL=AC(R= r) option uses the means and standard deviations from available cases with correlation r, where $-1/(p-1) < r < 1$ and p is the number of variables to be analyzed. The default is INITIAL=AC.

ITPRINT

prints the iteration history in the EM algorithm.

MAXITER=*number*

specifies the maximum number of iterations used in the EM algorithm. The default is MAXITER=200.

OUT=*SAS-data-set*

creates an output SAS data set containing results from the EM algorithm. The data set contains all variables in the input data set, with missing values being replaced by the expected values from the EM algorithm. See the section "Output Data Sets" on page 4416 for a description of this data set.

OUTEM=*SAS-data-set*

creates an output SAS data set of TYPE=COV containing the MLE of the parameter vector $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. These estimates are computed with the EM algorithm. See the section "Output Data Sets" on page 4416 for a description of this output data set.

OUTITER < (*options* **) > =***SAS-data-set*

creates an output SAS data set of TYPE=COV containing parameters for each iteration. The data set includes a variable named _Iteration_ to identify the iteration number. The parameters in the output data set depend on the options specified. You can specify the MEAN and COV options to output the mean and covariance parameters. When no options are specified, the output data set contains the mean parameters for each iteration. See the section "Output Data Sets" on page 4416 for a description of this data set.

FREQ Statement

FREQ *variable* **;**

If one variable in your input data set represents the frequency of occurrence of other values in the observation, specify the variable name in a FREQ statement. PROC MI then treats the data set as if each observation appears n times, where n is the value of the FREQ variable for the observation. If the value of the FREQ variable is less than one, the observation is not used in the analysis. Only the integer portion of the value is used. The total number of observations is considered to be equal to the sum of the FREQ variable when PROC MI calculates significance probabilities.

MCMC Statement

MCMC < *options* > **;**

The MCMC statement specifies the details of the MCMC method for imputation.

Table 54.2 summarizes the options available for the MCMC statement.

Table 54.2 Summary of Options in MCMC

Option	Description
Data Sets	
INEST=	Inputs parameter estimates for imputations
OUTEST=	Outputs parameter estimates used in imputations
OUTITER=	Outputs parameter estimates used in iterations
Imputation Details	
IMPUTE=	Specifies monotone or full imputation
CHAIN=	Specifies single or multiple chain
NBITER=	Specifies the number of burn-in iterations for each chain
NITER=	Specifies the number of iterations between imputations in a chain
INITIAL=	Specifies initial parameter estimates for MCMC
PRIOR=	Specifies the prior parameter information
START=	Specifies starting parameters
ODS Output Graphics	
PLOTS=TRACE	Displays trace plots
PLOTS=ACF	Displays autocorrelation plots
Traditional Graphics	
TIMEPLOT	Displays trace plots
ACFPLOT	Displays autocorrelation plots
GOUT=	Specifies the graphics catalog name for saving graphics output
Printed Output	
WLF	Displays the worst linear function
DISPLAYINIT	Displays initial parameter values for MCMC

The following options are available for the MCMC statement (in alphabetical order).

ACFPLOT < *(options < / display-options >)* >

displays the traditional autocorrelation function plots of parameters from iterations. The ACFPLOT option is applicable only if the ODS GRAPHICS ON statement is not specified.

The available options are as follows.

COV < *(< variables > < variable1*variable2 > < . . . variable1*variable2 >)* >

displays plots of variances for variables in the list and covariances for pairs of variables in the list. When the option COV is specified without variables, variances for all variables and covariances for all pairs of variables are used.

MEAN *< (variables) >*

displays plots of means for variables in the list. When the option MEAN is specified without variables, all variables are used.

WLF

displays the plot for the worst linear function.

When the ACFPLOT is specified without the preceding options, the procedure displays plots of means for all variables that are used.

The display options provide additional information for the autocorrelation function plots. The available display options are as follows:

CCONF=*color*

specifies the color of the displayed confidence limits. The default is CCONF=BLACK.

CFRAME=*color*

specifies the color for filling the area enclosed by the axes and the frame. By default, this area is not filled.

CNEEDLES=*color*

specifies the color of the vertical line segments (needles) that connect autocorrelations to the reference line. The default is CNEEDLES=BLACK.

CREF=*color*

specifies the color of the displayed reference line. The default is CREF=BLACK.

CSYMBOL=*color*

specifies the color of the displayed data points. The default is CSYMBOL=BLACK.

HSYMBOL=*number*

specifies the height of data points in percentage screen units. The default is HSYMBOL=1.

LCONF=*linetype*

specifies the line type for the displayed confidence limits. The default is LCONF=1, a solid line.

LOG

requests that the logarithmic transformations of parameters be used to compute the autocorrelations; it is generally used for the variances of variables. When a parameter has values less than or equal to zero, the corresponding plot is not created.

LREF=*linetype*

specifies the line type for the displayed reference line. The default is LREF=3, a dashed line.

NAME=*'string'*

specifies a descriptive name, up to eight characters, that appears in the name field of the PROC GREPLAY master menu. The default is NAME='MI'.

NLAG=*number*

specifies the maximum lag of the series. The default is NLAG=20. The autocorrelations at each lag are displayed in the graph.

SYMBOL=*value*

specifies the symbol for data points in percentage screen units. The default is SYMBOL=STAR.

TITLE=*'string'*

specifies the title to be displayed in the autocorrelation function plots. The default is TITLE='Autocorrelation Plot'.

WCONF=*number*

specifies the width of the displayed confidence limits in percentage screen units. If you specify the WCONF=0 option, the confidence limits are not displayed. The default is WCONF=1.

WNEEDLES=*number*

specifies the width of the displayed needles that connect autocorrelations to the reference line, in percentage screen units. If you specify the WNEEDLES=0 option, the needles are not displayed. The default is WNEEDLES=1.

WREF=*number*

specifies the width of the displayed reference line in percentage screen units. If you specify the WREF=0 option, the reference line is not displayed. The default is WREF=1.

For example, the following statement requests autocorrelation function plots for the means and variances of the variable y1, respectively:

```
acfplot( mean( y1) cov(y1) /log);
```

Logarithmic transformations of both the means and variances are used in the plots. For a detailed description of the autocorrelation function plot, see the section "Autocorrelation Function Plot" on page 4414; see also Schafer (1997, pp. 120–126) and the *SAS/ETS User's Guide*.

CHAIN=SINGLE | MULTIPLE

specifies whether a single chain is used for all imputations or a separate chain is used for each imputation. The default is CHAIN=SINGLE.

DISPLAYINIT

displays initial parameter values in the MCMC method for each imputation.

GOUT=*graphics-catalog*

specifies the graphics catalog for saving graphics output from PROC MI. The default is WORK.GSEG. For more information, see "The GREPLAY Procedure" in *SAS/GRAPH Software: Reference*.

IMPUTE=FULL | MONOTONE

specifies whether a full-data imputation is used for all missing values or a monotone-data imputation is used for a subset of missing values to make the imputed data sets have a monotone missing pattern. The default is IMPUTE=FULL. When IMPUTE=MONOTONE is specified, the order in the VAR statement is used to complete the monotone pattern.

INEST=*SAS-data-set*

names a SAS data set of TYPE=EST containing parameter estimates for imputations. These estimates are used to impute values for observations in the DATA= data set. A detailed description of the data set is provided in the section "Input Data Sets" on page 4414.

INITIAL=EM < (*options*) >

INITIAL=INPUT=*SAS-data-set*

specifies the initial mean and covariance estimates for the MCMC method. The default is INITIAL=EM.

You can specify INITIAL=INPUT=*SAS-data-set* to read the initial estimates of the mean and covariance matrix for each imputation from a SAS data set. See the section "Input Data Sets" on page 4414 for a description of this data set.

With INITIAL=EM, PROC MI derives parameter estimates for a posterior mode, the highest observed-data posterior density, from the EM algorithm. The MLE from the EM algorithm is used to start the EM algorithm for the posterior mode, and the resulting EM estimates are used to begin the MCMC method. The prior information specified in the PRIOR= option is also used in the process to compute the posterior mode.

The following four options are available with INITIAL=EM:

BOOTSTRAP < =*number* >

requests bootstrap resampling, which uses a simple random sample with replacement from the input data set for the initial estimate. You can explicitly specify the number of observations in the random sample. Alternatively, you can implicitly specify the number of observations in the random sample by specifying the proportion $p, 0 < p <= 1$, to request $[np]$ observations in the random sample, where n is the number of observations in the data set and $[np]$ is the integer part of np. This produces an overdispersed initial estimate that provides different starting values for the MCMC method. If you specify the BOOTSTRAP option without the number, *p*=0.75 is used by default.

CONVERGE=*p*

XCONV=*p*

sets the convergence criterion. The value must be between 0 and 1. The iterations are considered to have converged when the change in the parameter estimates between iteration steps is less than p for each parameter—that is, for each of the means and covariances. For each parameter, the change is a relative change if the parameter is greater than 0.01 in absolute value; otherwise, it is an absolute change. By default, CONVERGE=1E–4.

ITPRINT

prints the iteration history in the EM algorithm for the posterior mode.

MAXITER=*number*

specifies the maximum number of iterations used in the EM algorithm. The default is MAXITER=200.

NBITER=*number*

specifies the number of burn-in iterations before the first imputation in each chain. The default is NBITER=200.

NITER=*number*

specifies the number of iterations between imputations in a single chain. The default is NITER=100.

OUTEST=*SAS-data-set*

creates an output SAS data set of TYPE=EST. The data set contains parameter estimates used in each imputation. The data set also includes a variable named _Imputation_ to identify the imputation number. See the section "Output Data Sets" on page 4416 for a description of this data set.

OUTITER < (*options* **) > =***SAS-data-set*

creates an output SAS data set of TYPE=COV containing parameters used in the imputation step for each iteration. The data set includes variables named _Imputation_ and _Iteration_ to identify the imputation number and iteration number.

The parameters in the output data set depend on the options specified. You can specify the options MEAN, STD, COV, LR, LR_POST, and WLF to output parameters of means, standard deviations, covariances, −2 log LR statistic, −2 log LR statistic of the posterior mode, and the worst linear function, respectively. When no options are specified, the output data set contains the mean parameters used in the imputation step for each iteration. See the section "Output Data Sets" on page 4416 for a description of this data set.

PLOTS < (LOG) > < = *plot-request* **>**

PLOTS < (LOG) > < = (*plot-request* **< . . .** *plot-request* **>) >**

requests statistical graphics via the Output Delivery System (ODS). To request these graphs, you must specify the ODS GRAPHICS ON statement in addition to the following options in the MCMC statement. For more information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS."

The global plot option LOG requests that the logarithmic transformations of parameters be used. The plot request options include the following:

ACF < (*acf-options* **) >**

displays plots of the autocorrelation function of parameters from iterations. The default is ACF(MEAN).

ALL

produces all appropriate plots.

NONE

suppresses all plots.

TRACE < (*trace-options* **) >**

displays trace plots of parameters from iterations. The default is TRACE(MEAN).

The available *acf-options* are as follows:

NLAG=*n*

specifies the maximum lag of the series. The default is NLAG=20. The autocorrelations at each lag are displayed in the graph.

COV < (< **variables** > < **variable1*variable2** > < **...variable1*variable2** >) >

displays plots of variances for variables in the list and covariances for pairs of variables in the list. When the option COV is specified without variables, variances for all variables and covariances for all pairs of variables are used.

MEAN < (**variables**) >

displays plots of means for variables in the list. When the option MEAN is specified without variables, all variables are used.

WLF

displays the plot for the worst linear function.

The available *trace-options* are as follows:

COV < (< **variables** > < **variable1*variable2** > < **...variable1*variable2** >) >

displays plots of variances for variables in the list and covariances for pairs of variables in the list. When the option COV is specified without variables, variances for all variables and covariances for all pairs of variables are used.

MEAN < (**variables**) >

displays plots of means for variables in the list. When the option MEAN is specified without variables, all variables are used.

WLF

displays the plot of the worst linear function.

PRIOR=*name*

specifies the prior information for the means and covariances. Valid values for *name* are as follows:

JEFFREYS	specifies a noninformative prior.
RIDGE=*number*	specifies a ridge prior.
INPUT=*SAS-data-set*	specifies a data set containing prior information.

For a detailed description of the prior information, see the section "Bayesian Estimation of the Mean Vector and Covariance Matrix" on page 4406 and the section "Posterior Step" on page 4407. If you do not specify the PRIOR= option, the default is PRIOR=JEFFREYS.

The PRIOR=INPUT= option specifies a TYPE=COV data set from which the prior information of the mean vector and the covariance matrix is read. See the section "Input Data Sets" on page 4414 for a description of this data set.

START=VALUE | DIST

specifies that the initial parameter estimates are used either as the starting value (START=VALUE) or as the starting distribution (START=DIST) in the first imputation step of each chain. If the IMPUTE=MONOTONE option is specified, then START=VALUE is used in the procedure. The default is START=VALUE.

TIMEPLOT *< (options < / display-options>) >*

displays the traditional trace (time series) plots of parameters from iterations. The TIMEPLOT option is applicable only if the ODS GRAPHICS ON statement is not specified.

The available options are as follows:

COV *< (< variables > < variable1*variable2 > < . . . variable1*variable2 >) >*

displays plots of variances for variables in the list and covariances for pairs of variables in the list. When the option COV is specified without variables, variances for all variables and covariances for all pairs of variables are used.

MEAN *< (variables) >*

displays plots of means for variables in the list. When the option MEAN is specified without variables, all variables are used.

WLF

displays the plot of the worst linear function.

When the TIMEPLOT is specified without the preceding options, the procedure displays plots of means for all variables that are used.

The display options provide additional information for the trace plots. The available display options are as follows:

CCONNECT=*color*

specifies the color of the line segments that connect data points in the trace plots. The default is CCONNECT=BLACK.

CFRAME=*color*

specifies the color for filling the area enclosed by the axes and the frame. By default, this area is not filled.

CSYMBOL=*color*

specifies the color of the data points to be displayed in the trace plots. The default is CSYMBOL=BLACK.

HSYMBOL=*number*

specifies the height of data points in percentage screen units. The default is HSYMBOL=1.

LCONNECT=*linetype*

specifies the line type for the line segments that connect data points in the trace plots. The default is LCONNECT=1, a solid line.

LOG

requests that the logarithmic transformations of parameters be used; it is generally used for the variances of variables. When a parameter value is less than or equal to zero, the value is not displayed in the corresponding plot.

NAME=*'string'*

specifies a descriptive name, up to eight characters, that appears in the name field of the PROC GREPLAY master menu. The default is NAME='MI'.

SYMBOL=*value*

specifies the symbol for data points in percentage screen units. The default is SYMBOL=PLUS.

TITLE=*'string'*

specifies the title to be displayed in the trace plots. The default is TITLE='Trace Plot'.

WCONNECT=*number*

specifies the width of the line segments that connect data points in the trace plots, in percentage screen units. If you specify the WCONNECT=0 option, the data points are not connected. The default is WCONNECT=1.

For a detailed description of the trace plot, see the section "Trace Plot" on page 4413 and Schafer (1997, pp. 120–126).

WLF

displays the worst linear function of parameters. This scalar function of parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ is "worst" in the sense that its values from iterations converge most slowly among parameters. For a detailed description of this statistic, see the section "Worst Linear Function of Parameters" on page 4412.

MONOTONE Statement

MONOTONE *< method < (< imputed < = effects> > < / options>) > > < . . . method < (< imputed < = effects> > < / options>) > >* **;**

The MONOTONE statement specifies imputation methods for data sets with monotone missingness. You must also specify a VAR statement, and the data set must have a monotone missing pattern with variables ordered in the VAR list. When both MONOTONE and MCMC statements are specified, the MONOTONE statement is not used.

For each method, you can specify the imputed variables and, optionally, a set of the effects to impute these variables. Each effect is a variable or a combination of variables preceding the imputed variable in the VAR statement. The syntax for specification of effects is the same as for the GLM procedure. See Chapter 39, "The GLM Procedure," for more information.

One general form of an effect involving several variables is

X1 ∗ X2 ∗ A ∗ B ∗ C (D E)

where A, B, C, D, and E are classification variables and X1 and X2 are continuous variables.

If no covariates are specified, then all preceding variables are used as the covariates. That is, each preceding continuous variable is used as a regressor effect, and each preceding classification variable is used as a main effect. For the discriminant function method, only the continuous variables can be used as covariate effects.

When a method for continuous variables is specified without imputed variables, the method is used for all continuous variables in the VAR statement that are not specified in other methods. Similarly, when a method for classification variables is specified without imputed variables, the method is used for all classification variables in the VAR statement that are not specified in other methods.

When a MONOTONE statement is used without specifying any methods, the regression method is used for all continuous variables and the discriminant function method is used for all classification variables. The preceding variables of each imputed variable in the VAR statement are used as the covariates.

With a MONOTONE statement, the variables are imputed sequentially in the order given by the VAR statement. For a continuous variable, you can use a regression method, a regression predicted mean matching method, or a propensity score method to impute missing values.

For a nominal classification variable, you can use a discriminant function method to impute missing values without using the ordering of the class levels. For an ordinal classification variable, you can use a logistic regression method to impute missing values by using the ordering of the class levels. For a binary classification variable, either a discriminant function method or a logistic regression method can be used.

Note that except for the regression method, all other methods impute values from the observed observation values. You can specify the following methods in a MONOTONE statement.

DISCRIM < (*imputed* < = *effects* > < / *options* >) >

specifies the discriminant function method of classification variables. Only the continuous variables are allowed as covariate effects. The available options are DETAILS, PCOV=, and PRIOR=. The DETAILS option displays the group means and pooled covariance matrix used in each imputation. The PCOV= option specifies the pooled covariance used in the discriminant method. Valid values for the PCOV= option are as follows:

FIXED	uses the observed-data pooled covariance matrix for each imputation.
POSTERIOR	draws a pooled covariance matrix from its posterior distribution.

The default is PCOV=POSTERIOR. See the section "Discriminant Function Method for Monotone Missing Data" on page 4401 for a detailed description of the method.

The PRIOR= option specifies the prior probabilities of group membership. Valid values for the PRIOR= option are as follows:

EQUAL	sets the prior probabilities equal for all groups.
PROPORTIONAL	sets the prior probabilities proportion to the group sample sizes.
JEFFREYS < =*c* >	specifies a noninformative prior, $0 < c < 1$. If the number c is not specified, JEFFREYS=0.5.
RIDGE < =*d* >	specifies a ridge prior, $d > 0$. If the number d is not specified, RIDGE=0.25.

The default is PRIOR=JEFFREYS. See the section "Discriminant Function Method for Monotone Missing Data" on page 4401 for a detailed description of the method.

LOGISTIC < (*imputed* < = *effects* > < / *options* >) >

specifies the logistic regression method of classification variables. The available options are DETAILS, ORDER=, and DESCENDING. The DETAILS option displays the regression coefficients in the logistic regression model used in each imputation.

When the imputed variable has more than two response levels, the ordinal logistic regression method is used. The ORDER= option specifies the sorting order for the levels of the response variable. Valid values for the ORDER= option are as follows:

DATA	sorts by the order of appearance in the input data set.
FORMATTED	sorts by their external formatted values.
FREQ	sorts by the descending frequency counts.
INTERNAL	sorts by the unformatted values.

By default, ORDER=FORMATTED.

The option DESCENDING reverses the sorting order for the levels of the response variables.

See the section "Logistic Regression Method for Monotone Missing Data" on page 4403 for a detailed description of the method.

PROPENSITY < (*imputed* < = *effects* > < / *options* >) >

specifies the propensity scores method of variables. Each variable is either a classification variable or a continuous variable. The available options are DETAILS and NGROUPS=. The DETAILS option displays the regression coefficients in the logistic regression model for propensity scores. The NGROUPS= option specifies the number of groups created based on propensity scores. The default is NGROUPS=5.

See the section "Propensity Score Method for Monotone Missing Data" on page 4400 for a detailed description of the method.

REG | REGRESSION < (*imputed* < = *effects* > < / **DETAILS** >) >

specifies the regression method of continuous variables. The DETAILS option displays the regression coefficients in the regression model used in each imputation.

With a regression method, the MAXIMUM=, MINIMUM=, and ROUND= options can be used to make the imputed values more consistent with the observed variable values.

See the section "Regression Method for Monotone Missing Data" on page 4398 for a detailed description of the method.

REGPMM < (*imputed* < = *effects* > < *options* >) >

REGPREDMEANMATCH < (*imputed* < = *effects* > < *options* >) >

specifies the predictive mean matching method for continuous variables. This method is similar to the regression method except that it imputes a value randomly from a set of observed values whose predicted values are closest to the predicted value for the missing value from the simulated regression model (Heitjan and Little 1991; Schenker and Taylor 1996).

The available options are DETAILS and K=. The DETAILS option displays the regression coefficients in the regression model used in each imputation. The K= option specifies the number of closest observations to be used in the selection. The default is K=5.

See the section "Predictive Mean Matching Method for Monotone Missing Data" on page 4399 for a detailed description of the method.

With a MONOTONE statement, the missing values of a variable are imputed when the variable is either explicitly specified in the method or implicitly specified when a method is specified without imputed variables. These variables are imputed sequentially in the order specified in the VAR statement. For example, the following MI procedure statements use the logistic regression method to impute variable c1 from effects y1, y2, and y1 $*$ y2 first, and then use the regression method to impute variable y3 from effects y1, y2, and c1:

```
proc mi;
   class c1;
   var y1 y2 c1 y3;
   monotone reg(y3= y1 y2 c1) logistic(c1= y1 y2 y1*y2);
run;
```

The variables y1 and y2 are not imputed since y1 is the leading variable in the VAR statement and y2 is not specified as an imputed variable in the MONOTONE statement.

TRANSFORM Statement

TRANSFORM *transform (variables < / options >) < . . . transform (variables < / options >) >* **;**

The TRANSFORM statement lists the transformations and their associated variables to be transformed. The options are transformation options that provide additional information for the transformation.

The MI procedure assumes that the data are from a multivariate normal distribution when either the regression method or the MCMC method is used. When some variables in a data set are clearly nonnormal, it is useful to transform these variables to conform to the multivariate normality assumption. With a TRANSFORM statement, variables are transformed before the imputation process, and these transformed variable values are displayed in all of the results. When you specify an OUT= option, the variable values are back-transformed to create the imputed data set.

The following transformations can be used in the TRANSFORM statement:

BOXCOX

specifies the Box-Cox transformation of variables. The variable Y is transformed to $\frac{(\mathsf{Y}+c)^{\lambda}-1}{\lambda}$, where c is a constant such that each value of $\mathsf{Y}+c$ must be positive. If the specified constant $\lambda = 0$, the logarithmic transformation is used.

EXP

specifies the exponential transformation of variables. The variable Y is transformed to $\mathrm{e}^{(\mathsf{Y}+c)}$, where c is a constant.

LOG

specifies the logarithmic transformation of variables. The variable Y is transformed to $\log(\mathsf{Y}+c)$, where c is a constant such that each value of $\mathsf{Y}+c$ must be positive.

LOGIT

specifies the logit transformation of variables. The variable Y is transformed to $\log(\frac{\mathsf{Y}/c}{1-\mathsf{Y}/c})$, where the constant $c > 0$ and the values of Y/c must be between 0 and 1.

POWER

specifies the power transformation of variables. The variable Y is transformed to $(\mathsf{Y} + c)^{\lambda}$, where c is a constant such that each value of $\mathsf{Y} + c$ must be positive and the constant $\lambda \neq 0$.

The following options provide the constant c and λ values in the transformations.

C=*number*

specifies the c value in the transformation. The default is $c = 1$ for logit transformation and $c = 0$ for other transformations.

LAMBDA=*number*

specifies the λ value in the power and Box-Cox transformations. You must specify the λ value for these two transformations.

For example, the following statement requests that variables $\log(\mathsf{y1})$, a logarithmic transformation for the variable y1, and $\sqrt{\mathsf{y2} + 1}$, a power transformation for the variable y2, be used in the imputation:

```
transform log(y1) power(y2/c=1 lambda=.5);
```

If the MU0= option is used to specify a parameter value $\boldsymbol{\mu}_0$ for a transformed variable, the same transformation for the variable is also applied to its corresponding MU0= value in the t test. Otherwise, $\boldsymbol{\mu}_0 = 0$ is used for the transformed variable. See Example 54.10 for a usage of the TRANSFORM statement.

VAR Statement

VAR *variables* **;**

The VAR statement lists the variables to be analyzed. The variables can be either character or numeric. If you omit the VAR statement, all continuous variables not mentioned in other statements are used. The VAR statement is required if you specify a MONOTONE statement, an IMPUTE=MONOTONE option in the MCMC statement, or more than one number in the MU0=, MAXIMUM=, MINIMUM=, or ROUND= option.

The character variables are allowed only when they are specified as CLASS variables and the MONOTONE statement is also specified.

Details: MI Procedure

Descriptive Statistics

Suppose $\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n)'$ is the $(n \times p)$ matrix of complete data, which might not be fully observed, n_0 is the number of observations fully observed, and n_j is the number of observations with observed values for variable Y_j.

With complete cases, the sample mean vector is

$$\overline{\mathbf{y}} = \frac{1}{n_0} \sum \mathbf{y}_i$$

and the CSSCP matrix is

$$\sum (\mathbf{y}_i - \overline{\mathbf{y}})(\mathbf{y}_i - \overline{\mathbf{y}})'$$

where each summation is over the fully observed observations.

The sample covariance matrix is

$$\mathbf{S} = \frac{1}{n_0 - 1} \sum (\mathbf{y}_i - \overline{\mathbf{y}})(\mathbf{y}_i - \overline{\mathbf{y}})'$$

and is an unbiased estimate of the covariance matrix.

The correlation matrix $\mathbf{R}$ containing the Pearson product-moment correlations of the variables is derived by scaling the corresponding covariance matrix:

$$\mathbf{R} = \mathbf{D}^{-1} \mathbf{S} \, \mathbf{D}^{-1}$$

where $\mathbf{D}$ is a diagonal matrix whose diagonal elements are the square roots of the diagonal elements of $\mathbf{S}$.

With available cases, the corrected sum of squares for variable Y_j is

$$\sum (y_{ji} - \overline{y}_j)^2$$

where $\overline{y}_j = \frac{1}{n_j} \sum y_{ji}$ is the sample mean and each summation is over observations with observed values for variable Y_j.

The variance is

$$s_{jj}^2 = \frac{1}{n_j - 1} \sum (y_{ji} - \overline{y}_j)^2$$

The correlations for available cases contain pairwise correlations for each pair of variables. Each correlation is computed from all observations that have nonmissing values for the corresponding pair of variables.

EM Algorithm for Data with Missing Values

The EM algorithm (Dempster, Laird, and Rubin 1977) is a technique that finds maximum likelihood estimates in parametric models for incomplete data. The books by Little and Rubin (2002), Schafer (1997), and McLachlan and Krishnan (1997) provide a detailed description and applications of the EM algorithm.

The EM algorithm is an iterative procedure that finds the MLE of the parameter vector by repeating the following steps:

1. The expectation E-step
Given a set of parameter estimates, such as a mean vector and covariance matrix for a multivariate normal distribution, the E-step calculates the conditional expectation of the complete-data log likelihood given the observed data and the parameter estimates.

2. The maximization M-step
Given a complete-data log likelihood, the M-step finds the parameter estimates to maximize the complete-data log likelihood from the E-step.

The two steps are iterated until the iterations converge.

In the EM process, the observed-data log likelihood is nondecreasing at each iteration. For multivariate normal data, suppose there are G groups with distinct missing patterns. Then the observed-data log likelihood being maximized can be expressed as

$$\log L(\theta | Y_{obs}) = \sum_{g=1}^{G} \log L_g(\boldsymbol{\theta} | Y_{obs})$$

where $\log L_g(\boldsymbol{\theta} | Y_{obs})$ is the observed-data log likelihood from the gth group, and

$$\log L_g(\boldsymbol{\theta} | Y_{obs}) = -\frac{n_g}{2} \log |\Sigma_g| - \frac{1}{2} \sum_{ig} (\mathbf{y}_{ig} - \boldsymbol{\mu}_g)' \boldsymbol{\Sigma}_g{}^{-1} (\mathbf{y}_{ig} - \boldsymbol{\mu}_g)$$

where n_g is the number of observations in the gth group, the summation is over observations in the gth group, $\mathbf{y}_{ig}$ is a vector of observed values corresponding to observed variables, $\boldsymbol{\mu}_g$ is the corresponding mean vector, and $\boldsymbol{\Sigma}_g$ is the associated covariance matrix.

A sample covariance matrix is computed at each step of the EM algorithm. If the covariance matrix is singular, the linearly dependent variables for the observed data are excluded from the likelihood function. That is, for each observation with linear dependency among its observed variables, the dependent variables are excluded from the likelihood function. Note that this can result in an unexpected change in the likelihood between iterations prior to the final convergence.

See Schafer (1997, pp. 163–181) for a detailed description of the EM algorithm for multivariate normal data.

PROC MI uses the means and standard deviations from available cases as the initial estimates for the EM algorithm. The correlations are set to zero. These initial estimates provide a good starting value with positive definite covariance matrix. For a discussion of suggested starting values for the algorithm, see Schafer (1997, p. 169).

You can specify the convergence criterion with the CONVERGE= option in the EM statement. The iterations are considered to have converged when the maximum change in the parameter estimates between iteration steps is less than the value specified. You can also specify the maximum number of iterations used in the EM algorithm with the MAXITER= option.

The MI procedure displays tables of the initial parameter estimates used to begin the EM process and the MLE parameter estimates derived from EM. You can also display the EM iteration history with the ITPRINT option. PROC MI lists the iteration number, the likelihood -2 log L, and the parameter values $\boldsymbol{\mu}$ at each iteration. You can also save the MLE derived from the EM algorithm in a SAS data set by specifying the OUTEM= option.

Statistical Assumptions for Multiple Imputation

The MI procedure assumes that the data are from a continuous multivariate distribution and contain missing values that can occur for any of the variables. It also assumes that the data are from a multivariate normal distribution when either the regression method or the MCMC method is used.

Suppose $\mathbf{Y}$ is the $n \times p$ matrix of complete data, which is not fully observed, and denote the observed part of $\mathbf{Y}$ by $\mathbf{Y}_{obs}$ and the missing part by $\mathbf{Y}_{mis}$. The MI and MIANALYZE procedures assume that the missing data are missing at random (MAR); that is, the probability that an observation is missing can depend on $\mathbf{Y}_{obs}$, but not on $\mathbf{Y}_{mis}$ (Rubin 1976; 1987, p. 53).

To be more precise, suppose that $\mathbf{R}$ is the $n \times p$ matrix of response indicators whose elements are zero or one depending on whether the corresponding elements of Y are missing or observed. Then the MAR assumption is that the distribution of $\mathbf{R}$ can depend on Y_{obs} but not on Y_{mis}:

$$\mathrm{pr}(\mathbf{R}|Y_{obs}, Y_{mis}) = \mathrm{pr}(\mathbf{R}|Y_{obs})$$

For example, consider a trivariate data set with variables Y_1 and Y_2 fully observed, and a variable Y_3 that has missing values. MAR assumes that the probability that Y_3 is missing for an individual can be related to the individual's values of variables Y_1 and Y_2, but not to its value of Y_3. On the other hand, if a complete case and an incomplete case for Y_3 with exactly the same values for variables Y_1 and Y_2 have systematically different values, then there exists a response bias for Y_3, and MAR is violated.

The MAR assumption is not the same as missing completely at random (MCAR), which is a special case of MAR. Under the MCAR assumption, the missing data values are a simple random sample of all data values; the missingness does not depend on the values of any variables in the data set.

Although the MAR assumption cannot be verified with the data and it can be questionable in some situations, the assumption becomes more plausible as more variables are included in the imputation model (Schafer 1997, pp. 27–28; van Buuren, Boshuizen, and Knook, 1999, p. 687).

Furthermore, the MI and MIANALYZE procedures assume that the parameters $\boldsymbol{\theta}$ of the data model and the parameters $\boldsymbol{\phi}$ of the model for the missing-data indicators are distinct. That is, knowing the values of $\boldsymbol{\theta}$ does not provide any additional information about $\boldsymbol{\phi}$, and vice versa. If both the MAR and distinctness assumptions are satisfied, the missing-data mechanism is said to be ignorable (Rubin 1987, pp. 50–54; Schafer 1997, pp. 10–11) .

Missing Data Patterns

The MI procedure sorts the data into groups based on whether the analysis variables are observed or missing. Note that the input data set does not need to be sorted in any order.

For example, with variables Y_1, Y_2, and Y_3 (in that order) in a data set, up to eight groups of observations can be formed from the data set. Figure 54.6 displays the eight groups of observations and an unique missing pattern for each group:

Figure 54.6 Missing Data Patterns

```
          Missing Data Patterns

         Group     Y1     Y2     Y3

           1       X      X      X
           2       X      X      .
           3       X      .      X
           4       X      .      .
           5       .      X      X
           6       .      X      .
           7       .      .      X
           8       .      .      .
```

Here, an "X" means that the variable is observed in the corresponding group and a "." means that the variable is missing.

The variable order is used to derive the order of the groups from the data set, and thus determines the order of missing values in the data to be imputed. If you specify a different order of variables in the VAR statement, then the results are different even if the other specifications remain the same.

A data set with variables Y_1, Y_2, ..., Y_p (in that order) is said to have a *monotone missing pattern* when the event that a variable Y_j is missing for a particular individual implies that all subsequent variables Y_k, $k > j$, are missing for that individual. Alternatively, when a variable Y_j is observed for a particular individual, it is assumed that all previous variables Y_k, $k < j$, are also observed for that individual.

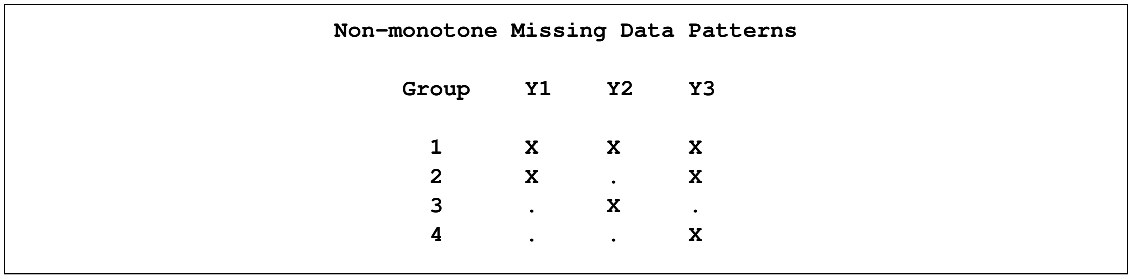
For example, Figure 54.7 displays a data set of three variables with a monotone missing pattern.

Figure 54.7 Monotone Missing Patterns

```
      Monotone Missing Data Patterns

         Group     Y1     Y2     Y3

           1       X      X      X
           2       X      X      .
           3       X      .      .
```

Figure 54.8 displays a data set of three variables with a non-monotone missing pattern.

Figure 54.8 Non-monotone Missing Patterns



```
              Non-monotone Missing Data Patterns

                    Group    Y1    Y2    Y3

                      1      X     X     X
                      2      X     .     X
                      3      .     X     .
                      4      .     .     X
```

A data set with an *arbitrary missing pattern* is a data set with either a monotone missing pattern or a non-monotone missing pattern.

Imputation Methods

This section describes the methods for multiple imputation that are available in the MI procedure. The method of choice depends on the pattern of missingness in the data and the type of the imputed variable, as summarized in Table 54.3.

Table 54.3 Imputation Methods in PROC MI

Pattern of Missingness	Type of Imputed Variable	Recommended Methods
Monotone	Continuous	• Regression • Predicted mean matching • Propensity score
Monotone	Classification (Ordinal)	• Logistic regression
Monotone	Classification (Nominal)	• Discriminant function method
Arbitrary	Continuous	• MCMC full-data imputation • MCMC monotone-data imputation

To impute missing values for a continuous variable in data sets with monotone missing patterns, you should use either a parametric method that assumes multivariate normality or a nonparametric method that uses propensity scores (Rubin 1987, pp. 124, 158; Lavori, Dawson, and Shera 1995). Parametric methods available include the regression method (Rubin 1987, pp. 166–167) and the predictive mean matching method (Heitjan and Little 1991; Schenker and Taylor 1996).

To impute missing values for a classification variable in data sets with monotone missing patterns, you should use the logistic regression method or the discriminant function method. Use the logistic regression method when the classification variable has a binary or ordinal response, and use the discriminant function method when the classification variable has a binary or nominal response.

For continuous variables in data sets with arbitrary missing patterns, you can use the Markov chain Monte Carlo (MCMC) method (Schafer 1997) to impute either all the missing values or just enough missing values to make the imputed data sets have monotone missing patterns.

With a monotone missing data pattern, you have greater flexibility in your choice of imputation models. In addition to the MCMC method, you can implement other methods, such as the regression method, that do not use Markov chains. You can also specify a different set of covariates for each imputed variable.

With an arbitrary missing data pattern, you can often use the MCMC method, which creates multiple imputations by drawing simulations from a Bayesian predictive distribution for normal data. Another way to handle a data set with an arbitrary missing data pattern is to use the MCMC approach to impute just enough values to make the missing data pattern monotone. Then, you can use a more flexible imputation method. This approach is described in the section "Producing Monotone Missingness with the MCMC Method" on page 4409.

Note that all continuous variables are standardized before the imputation process and then are transformed back to the original scale after the imputation process.

Although the regression and MCMC methods assume multivariate normality, inferences based on multiple imputation can be robust to departures from multivariate normality if the amount of missing information is not large, because the imputation model is effectively applied not to the entire data set but only to its missing part (Schafer 1997, pp. 147–148).

You can also use a TRANSFORM statement to transform variables to conform to the multivariate normality assumption. Variables are transformed before the imputation process and then are reverse-transformed to create the imputed data set.

Li (1988) presents a theoretical argument for convergence of the MCMC method in the continuous case and uses it to create imputations for incomplete multivariate continuous data. In practice, however, it is not easy to check the convergence of a Markov chain, especially for a large number of parameters. PROC MI generates statistics and plots that you can use to check for convergence of the MCMC method. The details are described in the section "Checking Convergence in MCMC" on page 4412.

Regression Method for Monotone Missing Data

The regression method is the default imputation method for continuous variables in a data set with a monotone missing pattern.

In the regression method, a regression model is fitted for a continuous variable with the covariates constructed from a set of effects. Based on the fitted regression model, a new regression model is simulated from the posterior predictive distribution of the parameters and is used to impute the missing values for each variable (Rubin 1987, pp. 166–167). That is, for a continuous variable Y_j with missing values, a model

$$Y_j = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

is fitted using observations with observed values for the variable Y_j and its covariates X_1, X_2, ..., X_k.

The fitted model includes the regression parameter estimates $\hat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k)$ and the associated covariance matrix $\hat{\sigma}_j^2 \mathbf{V}_j$, where $\mathbf{V}_j$ is the usual $\mathbf{X}'\mathbf{X}$ inverse matrix derived from the intercept and covariates X_1, X_2, ..., X_k.

The following steps are used to generate imputed values for each imputation:

1. New parameters $\boldsymbol{\beta}_* = (\beta_{*0}, \beta_{*1}, \ldots, \beta_{*(k)})$ and σ_{*j}^2 are drawn from the posterior predictive distribution of the parameters. That is, they are simulated from $(\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k)$, σ_j^2, and $\mathbf{V}_j$. The variance is drawn as

$$\sigma_{*j}^2 = \hat{\sigma}_j^2 (n_j - k - 1)/g$$

where g is a $\chi^2_{n_j-k-1}$ random variate and n_j is the number of nonmissing observations for Y_j. The regression coefficients are drawn as

$$\boldsymbol{\beta}_* = \hat{\boldsymbol{\beta}} + \sigma_{*j} \mathbf{V}'_{hj} \mathbf{Z}$$

where $\mathbf{V}'_{hj}$ is the upper triangular matrix in the Cholesky decomposition, $\mathbf{V}_j = \mathbf{V}'_{hj} \mathbf{V}_{hj}$, and $\mathbf{Z}$ is a vector of $k+1$ independent random normal variates.

2. The missing values are then replaced by

$$\beta_{*0} + \beta_{*1}\, x_1 + \beta_{*2}\, x_2 + \ldots + \beta_{*(k)}\, x_k + z_i\, \sigma_{*j}$$

where $x_1, x_2, \ldots, x_k$ are the values of the covariates and z_i is a simulated normal deviate.

Predictive Mean Matching Method for Monotone Missing Data

The predictive mean matching method is also an imputation method available for continuous variables. It is similar to the regression method except that for each missing value, it imputes a value randomly from a set of observed values whose predicted values are closest to the predicted value for the missing value from the simulated regression model (Heitjan and Little 1991; Schenker and Taylor 1996).

Following the description of the model in the section "Regression Method for Monotone Missing Data" on page 4398, the following steps are used to generate imputed values:

1. New parameters $\boldsymbol{\beta}_* = (\beta_{*0}, \beta_{*1}, \ldots, \beta_{*(k)})$ and σ_{*j}^2 are drawn from the posterior predictive distribution of the parameters. That is, they are simulated from $(\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k)$, σ_j^2, and $\mathbf{V}_j$. The variance is drawn as

$$\sigma_{*j}^2 = \hat{\sigma}_j^2 (n_j - k - 1)/g$$

where g is a $\chi^2_{n_j-k-1}$ random variate and n_j is the number of nonmissing observations for Y_j. The regression coefficients are drawn as

$$\boldsymbol{\beta}_* = \hat{\boldsymbol{\beta}} + \sigma_{*j} \mathbf{V}'_{hj} \mathbf{Z}$$

where $\mathbf{V}'_{hj}$ is the upper triangular matrix in the Cholesky decomposition, $\mathbf{V}_j = \mathbf{V}'_{hj}\mathbf{V}_{hj}$, and $\mathbf{Z}$ is a vector of $k + 1$ independent random normal variates.

2. For each missing value, a predicted value

$$y_{i*} = \beta_{*0} + \beta_{*1}\, x_1 + \beta_{*2}\, x_2 + \ldots + \beta_{*(k)}\, x_k$$

   is computed with the covariate values $x_1, x_2, \ldots, x_k$.

3. A set of k_0 observations whose corresponding predicted values are closest to y_{i*} is generated. You can specify k_0 with the K= option.

4. The missing value is then replaced by a value drawn randomly from these k_0 observed values.

The predictive mean matching method requires the number of closest observations to be specified. A smaller k_0 tends to increase the correlation among the multiple imputations for the missing observation and results in a higher variability of point estimators in repeated sampling. On the other hand, a larger k_0 tends to lessen the effect from the imputation model and results in biased estimators (Schenker and Taylor 1996, p. 430).

The predictive mean matching method ensures that imputed values are plausible and might be more appropriate than the regression method if the normality assumption is violated (Horton and Lipsitz 2001, p. 246).

Propensity Score Method for Monotone Missing Data

The propensity score method is another imputation method available for continuous variables when the data set has a monotone missing pattern.

A propensity score is generally defined as the conditional probability of assignment to a particular treatment given a vector of observed covariates (Rosenbaum and Rubin 1983). In the propensity score method, for a variable with missing values, a propensity score is generated for each observation to estimate the probability that the observation is missing. The observations are then grouped based on these propensity scores, and an approximate Bayesian bootstrap imputation (Rubin 1987, p. 124) is applied to each group (Lavori, Dawson, and Shera 1995).

The propensity score method uses the following steps to impute values for variable Y_j with missing values:

1. Create an indicator variable R_j with the value 0 for observations with missing Y_j and 1 otherwise.

2. Fit a logistic regression model

$$\text{logit}(p_j) = \beta_0 + \beta_1\, X_1 + \beta_2\, X_2 + \ldots + \beta_k\, X_k$$

   where $X_1, X_2, \ldots, X_k$ are covariates for Y_j, $p_j = Pr(R_j = 0 | X_1, X_2, \ldots, X_k)$, and $\text{logit}(p) = \log(p/(1-p))$.

3. Create a propensity score for each observation to estimate the probability that it is missing.
4. Divide the observations into a fixed number of groups (typically assumed to be five) based on these propensity scores.
5. Apply an approximate Bayesian bootstrap imputation to each group. In group k, suppose that Y_{obs} denotes the n_1 observations with nonmissing Y_j values and Y_{mis} denotes the n_0 observations with missing Y_j. The approximate Bayesian bootstrap imputation first draws n_1 observations randomly with replacement from Y_{obs} to create a new data set Y^*_{obs}. This is a nonparametric analog of drawing parameters from the posterior predictive distribution of the parameters. The process then draws the n_0 values for Y_{mis} randomly with replacement from Y^*_{obs}.

Steps 1 through 5 are repeated sequentially for each variable with missing values.

Note that the propensity score method was originally designed for a randomized experiment with repeated measures on the response variables. The goal was to impute the missing values on the response variables. The method uses only the covariate information that is associated with whether the imputed variable values are missing. It does not use correlations among variables. It is effective for inferences about the distributions of individual imputed variables, such as a univariate analysis, but it is not appropriate for analyses involving relationship among variables, such as a regression analysis (Schafer 1999, p. 11). It can also produce badly biased estimates of regression coefficients when data on predictor variables are missing (Allison 2000).

Discriminant Function Method for Monotone Missing Data

The discriminant function method is the default imputation method for classification variables in a data set with a monotone missing pattern.

For a nominal classification variable Y_j with responses 1, …, g and a set of effects from its preceding variables, if the covariates $X_1, X_2, \ldots, X_k$ associated with these effects within each group are approximately multivariate normal and the within-group covariance matrices are approximately equal, the discriminant function method (Brand 1999, pp. 95–96) can be used to impute missing values for the variable Y_j.

Denote the group-specific means for covariates $X_1, X_2, \ldots, X_k$ by

$$\overline{\mathbf{X}}_t = (\overline{X}_{t1}, \overline{X}_{t2}, \ldots, \overline{X}_{tk}),\ t = 1, 2, \ldots, g$$

then the pooled covariance matrix is computed as

$$\mathbf{S} = \frac{1}{n-g} \sum_{t=1}^{g} (n_t - 1)\mathbf{S}_t$$

where $\mathbf{S}_t$ is the within-group covariance matrix, n_t is the group-specific sample size, and $n = \sum_{t=1}^{g} n_t$ is the total sample size.

In each imputation, new parameters of the group-specific means ($\mathbf{m}_{*t}$), pooled covariance matrix ($\mathbf{S}_*$), and prior probabilities of group membership (q_{*t}) can be drawn from their corresponding posterior distributions (Schafer 1997, p. 356).

Pooled Covariance Matrix and Group-Specific Means

For each imputation, the MI procedure uses either the fixed observed pooled covariance matrix (PCOV=FIXED) or a drawn pooled covariance matrix (PCOV=POSTERIOR) from its posterior distribution with a noninformative prior. That is,

$$\mathbf{\Sigma} | \mathbf{X} \quad \sim \quad W^{-1} \left(n-g, \, (n-g)\mathbf{S} \right)$$

where W^{-1} is an inverted Wishart distribution.

The group-specific means are then drawn from their posterior distributions with a noninformative prior

$$\boldsymbol{\mu}_t | (\mathbf{\Sigma}, \overline{\mathbf{X}}_t) \quad \sim \quad N \left(\overline{\mathbf{X}}_t, \, \frac{1}{n_t} \mathbf{\Sigma} \right)$$

See the section "Bayesian Estimation of the Mean Vector and Covariance Matrix" on page 4406 for a complete description of the inverted Wishart distribution and posterior distributions that use a noninformative prior.

Prior Probabilities of Group Membership

The prior probabilities are computed through the drawing of new group sample sizes. When the total sample size n is considered fixed, the group sample sizes $(n_1, n_2, \ldots, n_g)$ have a multinomial distribution. New multinomial parameters (group sample sizes) can be drawn from their posterior distribution by using a Dirichlet prior with parameters $(\alpha_1, \alpha_2, \ldots, \alpha_g)$.

After the new sample sizes are drawn from the posterior distribution of $(n_1, n_2, \ldots, n_g)$, the prior probabilities q_{*t} are computed proportionally to the drawn sample sizes.

See Schafer (1997, pp. 247–255) for a complete description of the Dirichlet prior.

Imputation Steps

The discriminant function method uses the following steps in each imputation to impute values for a nominal classification variable Y_j with g responses:

1. Draw a pooled covariance matrix $\mathbf{S}_*$ from its posterior distribution if the PCOV=POSTERIOR option is used.
2. For each group, draw group means $\mathbf{m}_{*t}$ from the observed group mean $\overline{\mathbf{X}}_t$ and either the observed pooled covariance matrix (PCOV=FIXED) or the drawn pooled covariance matrix $\mathbf{S}_*$ (PCOV=POSTERIOR).

3. For each group, compute or draw q_{*t}, prior probabilities of group membership, based on the PRIOR= option:

   - PRIOR=EQUAL, $q_{*t} = 1/g$, prior probabilities of group membership are all equal.
   - PRIOR=PROPORTIONAL, $q_{*t} = n_t/n$, prior probabilities are proportional to their group sample sizes.
   - PRIOR=JEFFREYS=*c*, a noninformative Dirichlet prior with $\alpha_t = c$ is used.
   - PRIOR=RIDGE=*d*, a ridge prior is used with $\alpha_t = d * n_t/n$ for $d \geq 1$ and $\alpha_t = d * n_t$ for $d < 1$.

4. With the group means $\mathbf{m}_{*t}$, the pooled covariance matrix $\mathbf{S}_*$, and the prior probabilities of group membership q_{*t}, the discriminant function method derives linear discriminant function and computes the posterior probabilities of an observation belonging to each group

   $$p_t(\mathbf{x}) = \frac{\exp(-0.5 D_t^2(\mathbf{x}))}{\sum_{u=1}^{g} \exp(-0.5 D_u^2(\mathbf{x}))}$$

   where $D_t^2(\mathbf{x}) = (\mathbf{x} - \mathbf{m}_{*t})' \mathbf{S}_*^{-1} (\mathbf{x} - \mathbf{m}_{*t}) - 2\log(q_{*t})$ is the generalized squared distance from $\mathbf{x}$ to group t.

5. Draw a random uniform variate u, between 0 and 1, for each observation with missing group value. With the posterior probabilities, $p_1(\mathbf{x}) + p_2(\mathbf{x}) + \ldots, + p_g(\mathbf{x}) = 1$, the discriminant function method imputes $Y_j = 1$ if the value of u is less than $p_1(\mathbf{x})$, $Y_j = 2$ if the value is greater than or equal to $p_1(\mathbf{x})$ but less than $p_1(\mathbf{x}) + p_2(\mathbf{x})$, and so on.

Logistic Regression Method for Monotone Missing Data

The logistic regression method is another imputation method available for classification variables in a data set with a monotone missing pattern.

In the logistic regression method, a logistic regression model is fitted for a classification variable with a set of covariates constructed from the effects. For a binary classification variable, based on the fitted regression model, a new logistic regression model is simulated from the posterior predictive distribution of the parameters and is used to impute the missing values for each variable (Rubin 1987, pp. 169–170).

For a binary variable Y_j with responses 1 and 2, a logistic regression model is fitted using observations with observed values for the imputed variable Y_j and its covariates $X_1, X_2, \ldots, X_k$:

$$\text{logit}(p_j) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

where $X_1, X_2, \ldots, X_k$ are covariates for Y_j, $p_j = \Pr(R_j = 1 | X_1, X_2, \ldots, X_k)$, and $\text{logit}(p) = \log(p/(1-p))$.

The fitted model includes the regression parameter estimates $\hat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k)$ and the associated covariance matrix $\mathbf{V}_j$.

The following steps are used to generate imputed values for a binary variable Y_j with responses 1 and 2:

1. New parameters $\boldsymbol{\beta}_* = (\beta_{*0}, \beta_{*1}, \ldots, \beta_{*(k)})$ are drawn from the posterior predictive distribution of the parameters.

   $$\boldsymbol{\beta}_* = \hat{\boldsymbol{\beta}} + \mathbf{V}'_{hj}\mathbf{Z}$$

   where $\mathbf{V}'_{hj}$ is the upper triangular matrix in the Cholesky decomposition, $\mathbf{V}_j = \mathbf{V}'_{hj}\mathbf{V}_{hj}$, and $\mathbf{Z}$ is a vector of $k+1$ independent random normal variates.

2. For an observation with missing Y_j and covariates $x_1, x_2, \ldots, x_k$, compute the expected probability that $Y_j = 1$:

   $$p_j = \frac{\exp(\mu_j)}{1 + \exp(\mu_j)}$$

   where $\mu_j = \beta_{*0} + \beta_{*1}\, x_1 + \beta_{*2}\, x_2 + \ldots + \beta_{*(k)}\, x_k$.

3. Draw a random uniform variate, u, between 0 and 1. If the value of u is less than p_j, impute $Y_j = 1$; otherwise impute $Y_j = 2$.

The preceding logistic regression method can be extended to include the ordinal classification variables with more than two levels of responses. The options ORDER= and DESCENDING can be used to specify the sorting order for the levels of the imputed variables.

MCMC Method for Arbitrary Missing Data

The Markov chain Monte Carlo (MCMC) method originated in physics as a tool for exploring equilibrium distributions of interacting molecules. In statistical applications, it is used to generate pseudo-random draws from multidimensional and otherwise intractable probability distributions via Markov chains. A Markov chain is a sequence of random variables in which the distribution of each element depends only on the value of the previous element.

In MCMC simulation, one constructs a Markov chain long enough for the distribution of the elements to stabilize to a stationary distribution, which is the distribution of interest. By repeatedly simulating steps of the chain, the method simulates draws from the distribution of interest. See Schafer (1997) for a detailed discussion of this method.

In Bayesian inference, information about unknown parameters is expressed in the form of a posterior probability distribution. This posterior distribution is computed using Bayes' theorem,

$$p(\boldsymbol{\theta}|y) = \frac{p(y|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(y|\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}}$$

MCMC has been applied as a method for exploring posterior distributions in Bayesian inference. That is, through MCMC, you can simulate the entire joint posterior distribution of the unknown quantities and obtain simulation-based estimates of posterior parameters that are of interest.

In many incomplete-data problems, the observed-data posterior $p(\boldsymbol{\theta}|Y_{obs})$ is intractable and cannot easily be simulated. However, when Y_{obs} is augmented by an estimated/simulated value of the missing data Y_{mis}, the complete-data posterior $p(\boldsymbol{\theta}|Y_{obs}, Y_{mis})$ is much easier to simulate. Assuming that the data are from a multivariate normal distribution, data augmentation can be applied to Bayesian inference with missing data by repeating the following steps:

1. The imputation I-step
Given an estimated mean vector and covariance matrix, the I-step simulates the missing values for each observation independently. That is, if you denote the variables with missing values for observation i by $Y_{i(mis)}$ and the variables with observed values by $Y_{i(obs)}$, then the I-step draws values for $Y_{i(mis)}$ from a conditional distribution for $Y_{i(mis)}$ given $Y_{i(obs)}$.

2. The posterior P-step
Given a complete sample, the P-step simulates the posterior population mean vector and covariance matrix. These new estimates are then used in the next I-step. Without prior information about the parameters, a noninformative prior is used. You can also use other informative priors. For example, a prior information about the covariance matrix can be helpful to stabilize the inference about the mean vector for a near singular covariance matrix.

The two steps are iterated long enough for the results to be reliable for a multiply imputed data set (Schafer 1997, p. 72). That is, with a current parameter estimate $\boldsymbol{\theta}^{(t)}$ at the tth iteration, the I-step draws $Y_{mis}^{(t+1)}$ from $p(Y_{mis}|Y_{obs}, \boldsymbol{\theta}^{(t)})$ and the P-step draws $\boldsymbol{\theta}^{(t+1)}$ from $p(\boldsymbol{\theta}|Y_{obs}, Y_{mis}^{(t+1)})$.

This creates a Markov chain $(Y_{mis}^{(1)}, \boldsymbol{\theta}^{(1)})$, $(Y_{mis}^{(2)}, \boldsymbol{\theta}^{(2)})$, ..., which converges in distribution to $p(Y_{mis}, \boldsymbol{\theta}|Y_{obs})$. Assuming the iterates converge to a stationary distribution, the goal is to simulate an approximately independent draw of the missing values from this distribution.

To validate the imputation results, you should repeat the process with different random number generators and starting values based on different initial parameter estimates.

The next three sections provide details for the imputation step, Bayesian estimation of the mean vector and covariance matrix, and the posterior step.

Imputation Step

In each iteration, starting with a given mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, the imputation step draws values for the missing data from the conditional distribution Y_{mis} given Y_{obs}.

Suppose $\boldsymbol{\mu} = \left[\boldsymbol{\mu}_1', \boldsymbol{\mu}_2'\right]'$ is the partitioned mean vector of two sets of variables, Y_{obs} and Y_{mis}, where $\boldsymbol{\mu}_1$ is the mean vector for variables Y_{obs} and $\boldsymbol{\mu}_2$ is the mean vector for variables Y_{mis}.

Also suppose

$$\boldsymbol{\Sigma} = \left[\begin{array}{cc} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}' & \boldsymbol{\Sigma}_{22} \end{array} \right]$$

is the partitioned covariance matrix for these variables, where $\boldsymbol{\Sigma}_{11}$ is the covariance matrix for variables Y_{obs}, $\boldsymbol{\Sigma}_{22}$ is the covariance matrix for variables Y_{mis}, and $\boldsymbol{\Sigma}_{12}$ is the covariance matrix between variables Y_{obs} and variables Y_{mis}.

By using the sweep operator (Goodnight 1979) on the pivots of the $\mathbf{\Sigma}_{11}$ submatrix, the matrix becomes

$$\left[\begin{array}{cc} \mathbf{\Sigma}_{11}^{-1} & \mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12} \\ -\mathbf{\Sigma}_{12}'\mathbf{\Sigma}_{11}^{-1} & \mathbf{\Sigma}_{22.1} \end{array} \right]$$

where $\mathbf{\Sigma}_{22.1} = \mathbf{\Sigma}_{22} - \mathbf{\Sigma}_{12}'\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12}$ can be used to compute the conditional covariance matrix of $\mathbf{Y}_{mis}$ after controlling for $\mathbf{Y}_{obs}$.

For an observation with the preceding missing pattern, the conditional distribution of Y_{mis} given $Y_{obs} = \mathbf{y}_1$ is a multivariate normal distribution with the mean vector

$$\boldsymbol{\mu}_{2.1} = \boldsymbol{\mu}_2 + \mathbf{\Sigma}_{12}'\mathbf{\Sigma}_{11}^{-1}(\mathbf{y}_1 - \boldsymbol{\mu}_1)$$

and the conditional covariance matrix

$$\mathbf{\Sigma}_{22.1} = \mathbf{\Sigma}_{22} - \mathbf{\Sigma}_{12}'\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12}$$

Bayesian Estimation of the Mean Vector and Covariance Matrix

Suppose that $\mathbf{Y} = (\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_n')'$ is an $(n \times p)$ matrix made up of n $(p \times 1)$ independent vectors $\mathbf{y}_i$, each of which has a multivariate normal distribution with mean zero and covariance matrix $\mathbf{\Lambda}$. Then the SSCP matrix

$$\mathbf{A} = \mathbf{Y}'\mathbf{Y} = \sum_i \mathbf{y}_i \mathbf{y}_i'$$

has a Wishart distribution $W(n, \mathbf{\Lambda})$.

When each observation $\mathbf{y}_i$ is distributed with a multivariate normal distribution with an unknown mean $\boldsymbol{\mu}$, then the CSSCP matrix

$$\mathbf{A} = \sum_i (\mathbf{y}_i - \overline{\mathbf{y}})(\mathbf{y}_i - \overline{\mathbf{y}})'$$

has a Wishart distribution $W(n-1, \mathbf{\Lambda})$.

If $\mathbf{A}$ has a Wishart distribution $W(n, \mathbf{\Lambda})$, then $\mathbf{B} = \mathbf{A}^{-1}$ has an inverted Wishart distribution $W^{-1}(n, \mathbf{\Psi})$, where n is the degrees of freedom and $\mathbf{\Psi} = \mathbf{\Lambda}^{-1}$ is the precision matrix (Anderson 1984).

Note that, instead of using the parameter $\mathbf{\Psi} = \mathbf{\Lambda}^{-1}$ for the inverted Wishart distribution, Schafer (1997) uses the parameter $\mathbf{\Lambda}$.

Suppose that each observation in the data matrix $\mathbf{Y}$ has a multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Sigma}$. Then with a prior inverted Wishart distribution for $\mathbf{\Sigma}$ and a prior normal distribution for $\boldsymbol{\mu}$

$$\begin{array}{rcl} \mathbf{\Sigma} & \sim & W^{-1}\left(m, \mathbf{\Psi}\right) \\ \boldsymbol{\mu}|\mathbf{\Sigma} & \sim & N\left(\boldsymbol{\mu}_0, \frac{1}{\tau}\mathbf{\Sigma}\right) \end{array}$$

where $\tau > 0$ is a fixed number.

The posterior distribution (Anderson 1984, p. 270; Schafer 1997, p. 152) is

$$\begin{aligned} \boldsymbol{\Sigma}|\mathbf{Y} &\sim W^{-1}\left(n+m,\ (n-1)\mathbf{S} + \boldsymbol{\Psi} + \frac{n\tau}{n+\tau} (\bar{\mathbf{y}} - \boldsymbol{\mu}_0)(\bar{\mathbf{y}} - \boldsymbol{\mu}_0)' \right) \\ \boldsymbol{\mu}|(\boldsymbol{\Sigma},\mathbf{Y}) &\sim N\left(\frac{1}{n+\tau}(n\bar{\mathbf{y}} + \tau\boldsymbol{\mu}_0),\ \frac{1}{n+\tau}\boldsymbol{\Sigma} \right) \end{aligned}$$

where $(n-1)\mathbf{S}$ is the CSSCP matrix.

Posterior Step

In each iteration, the posterior step simulates the posterior population mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ from prior information for $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, and the complete sample estimates.

You can specify the prior parameter information by using one of the following methods:

- PRIOR=JEFFREYS, which uses a noninformative prior
- PRIOR=INPUT=, which provides a prior information for $\boldsymbol{\Sigma}$ in the data set. Optionally, it also provides a prior information for $\boldsymbol{\mu}$ in the data set.
- PRIOR=RIDGE=, which uses a ridge prior

The next four subsections provide details of the posterior step for different prior distributions.

1. A Noninformative Prior

Without prior information about the mean and covariance estimates, you can use a noninformative prior by specifying the PRIOR=JEFFREYS option. The posterior distributions (Schafer 1997, p. 154) are

$$\begin{aligned} \boldsymbol{\Sigma}^{(t+1)}|\mathbf{Y} &\sim W^{-1}\left(n-1,\ (n-1)\mathbf{S} \right) \\ \boldsymbol{\mu}^{(t+1)}|(\boldsymbol{\Sigma}^{(t+1)},\mathbf{Y}) &\sim N\left(\bar{\mathbf{y}},\ \frac{1}{n}\boldsymbol{\Sigma}^{(t+1)} \right) \end{aligned}$$

2. An Informative Prior for μ and Σ

When prior information is available for the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, you can provide it with a SAS data set that you specify with the PRIOR=INPUT= option:

$$\begin{aligned} \boldsymbol{\Sigma} &\sim W^{-1}\left(d^*,\ d^*\mathbf{S}^* \right) \\ \boldsymbol{\mu}|\boldsymbol{\Sigma} &\sim N\left(\boldsymbol{\mu}_0,\ \frac{1}{n_0}\boldsymbol{\Sigma} \right) \end{aligned}$$

To obtain the prior distribution for $\boldsymbol{\Sigma}$, PROC MI reads the matrix $\mathbf{S}^*$ from observations in the data set with _TYPE_='COV', and it reads $n^* = d^* + 1$ from observations with _TYPE_='N'.

To obtain the prior distribution for $\boldsymbol{\mu}$, PROC MI reads the mean vector $\boldsymbol{\mu}_0$ from observations with _TYPE_='MEAN', and it reads n_0 from observations with _TYPE_='N_MEAN'. When there are no observations with _TYPE_='N_MEAN', PROC MI reads n_0 from observations with _TYPE_='N'.

The resulting posterior distribution, as described in the section "Bayesian Estimation of the Mean Vector and Covariance Matrix" on page 4406, is given by

$$\begin{aligned} \boldsymbol{\Sigma}^{(t+1)} | \mathbf{Y} &\sim W^{-1}\left(n + d^*,\ (n-1)\mathbf{S} + d^*\mathbf{S}^* + \mathbf{S}_m\right) \\ \boldsymbol{\mu}^{(t+1)} \mid \left(\boldsymbol{\Sigma}^{(t+1)}, \mathbf{Y}\right) &\sim N\left(\frac{1}{n+n_0}(n\overline{\mathbf{y}} + n_0\boldsymbol{\mu}_0),\ \frac{1}{n+n_0}\boldsymbol{\Sigma}^{(t+1)} \right) \end{aligned}$$

where

$$\mathbf{S}_m = \frac{n n_0}{n+n_0}(\overline{\mathbf{y}} - \boldsymbol{\mu}_0)(\overline{\mathbf{y}} - \boldsymbol{\mu}_0)'$$

3. An Informative Prior for $\boldsymbol{\Sigma}$

When the sample covariance matrix $\mathbf{S}$ is singular or near singular, prior information about $\boldsymbol{\Sigma}$ can also be used without prior information about $\boldsymbol{\mu}$ to stabilize the inference about $\boldsymbol{\mu}$. You can provide it with a SAS data set that you specify with the PRIOR=INPUT= option.

To obtain the prior distribution for $\boldsymbol{\Sigma}$, PROC MI reads the matrix $\mathbf{S}^*$ from observations in the data set with _TYPE_='COV', and it reads n^* from observations with _TYPE_='N'.

The resulting posterior distribution for $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ (Schafer 1997, p. 156) is

$$\begin{aligned} \boldsymbol{\Sigma}^{(t+1)} | \mathbf{Y} &\sim W^{-1}\left(n + d^*,\ (n-1)\mathbf{S} + d^*\mathbf{S}^*\right) \\ \boldsymbol{\mu}^{(t+1)} \mid \left(\boldsymbol{\Sigma}^{(t+1)}, \mathbf{Y}\right) &\sim N\left(\overline{\mathbf{y}},\ \frac{1}{n}\boldsymbol{\Sigma}^{(t+1)} \right) \end{aligned}$$

Note that if the PRIOR=INPUT= data set also contains observations with _TYPE_='MEAN', then a complete informative prior for both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ will be used.

4. A Ridge Prior

A special case of the preceding adjustment is a ridge prior with $\mathbf{S}^*$ = Diag $(\mathbf{S})$ (Schafer 1997, p. 156). That is, $\mathbf{S}^*$ is a diagonal matrix with diagonal elements equal to the corresponding elements in $\mathbf{S}$.

You can request a ridge prior by using the PRIOR=RIDGE= option. You can explicitly specify the number $d^* \geq 1$ in the PRIOR=RIDGE=d^* option. Or you can implicitly specify the number by specifying the proportion p in the PRIOR=RIDGE=p option to request $d^* = (n-1)p$.

The posterior is then given by

$$\begin{aligned}\boldsymbol{\Sigma}^{(t+1)}|\mathbf{Y} \quad &\sim \quad W^{-1}\left(n+d^*,\ (n-1)\mathbf{S}+d^*\mathrm{Diag}(\mathbf{S})\right)\\ \boldsymbol{\mu}^{(t+1)}\left|\left(\boldsymbol{\Sigma}^{(t+1)},\mathbf{Y}\right)\right. \quad &\sim \quad N\left(\overline{\mathbf{y}},\ \frac{1}{n}\,\boldsymbol{\Sigma}^{(t+1)}\right)\end{aligned}$$

Producing Monotone Missingness with the MCMC Method

The monotone data MCMC method was first proposed by Li (1988), and Liu (1993) described the algorithm. The method is useful especially when a data set is close to having a monotone missing pattern. In this case, the method needs to impute only a few missing values to the data set to have a monotone missing pattern in the imputed data set. Compared to a full data imputation that imputes all missing values, the monotone data MCMC method imputes fewer missing values in each iteration and achieves approximate stationarity in fewer iterations (Schafer 1997, p. 227).

You can request the monotone MCMC method by specifying the option IMPUTE=MONOTONE in the MCMC statement. The “Missing Data Patterns” table now denotes the variables with missing values by “.” or “O”. The value “.” means that the variable is missing and will be imputed, and the value “O” means that the variable is missing and will not be imputed. The “Variance Information” and “Parameter Estimates” tables are not created.

You must specify the variables in the VAR statement. The variable order in the list determines the monotone missing pattern in the imputed data set. With a different order in the VAR list, the results will be different because the monotone missing pattern to be constructed will be different.

Assuming that the data are from a multivariate normal distribution, then like the MCMC method, the monotone MCMC method repeats the following steps:

1. The imputation I-step
Given an estimated mean vector and covariance matrix, the I-step simulates the missing values for each observation independently. Only a subset of missing values are simulated to achieve a monotone pattern of missingness.

2. The posterior P-step
Given a new sample with a monotone pattern of missingness, the P-step simulates the posterior population mean vector and covariance matrix with a noninformative Jeffreys prior. These new estimates are then used in the next I-step.

Imputation Step

The I-step is almost identical to the I-step described in the section “MCMC Method for Arbitrary Missing Data” on page 4404 except that only a subset of missing values need to be simulated. To state this precisely, denote the variables with observed values for observation i by $Y_{i(obs)}$ and the variables with missing values by $Y_{i(mis)} = (Y_{i(m1)}, Y_{i(m2)})$, where $Y_{i(m1)}$ is a subset of the missing variables that will cause a monotone missingness when their values are imputed. Then the I-step draws values for $Y_{i(m1)}$ from a conditional distribution for $Y_{i(m1)}$ given $Y_{i(obs)}$.

Posterior Step

The P-step is different from the P-step described in the section "MCMC Method for Arbitrary Missing Data" on page 4404. Instead of simulating the $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ parameters from the full imputed data set, this P-step simulates the $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ parameters through simulated regression coefficients from regression models based on the imputed data set with a monotone pattern of missingness. The step is similar to the process described in the section "Regression Method for Monotone Missing Data" on page 4398.

That is, for the variable Y_j, a model

$$Y_j = \beta_0 + \beta_1 Y_1 + \beta_2 Y_2 + \ldots + \beta_{j-1} Y_{j-1}$$

is fitted using n_j nonmissing observations for variable Y_j in the imputed data sets.

The fitted model consists of the regression parameter estimates $\hat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_{j-1})$ and the associated covariance matrix $\hat{\sigma}_j^2 \mathbf{V}_j$, where $\mathbf{V}_j$ is the usual $\mathbf{X}'\mathbf{X}$ inverse matrix from the intercept and variables $Y_1, Y_2, \ldots, Y_{j-1}$.

For each imputation, new parameters $\boldsymbol{\beta}_* = (\beta_{*0}, \beta_{*1}, \ldots, \beta_{*(j-1)})$ and σ_{*j}^2 are drawn from the posterior predictive distribution of the parameters. That is, they are simulated from $(\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_{j-1})$, σ_j^2, and $\mathbf{V}_j$. The variance is drawn as

$$\sigma_{*j}^2 = \hat{\sigma}_j^2 (n_j - j)/g$$

where g is a $\chi^2_{n_j - p + j - 1}$ random variate and n_j is the number of nonmissing observations for Y_j. The regression coefficients are drawn as

$$\boldsymbol{\beta}_* = \hat{\boldsymbol{\beta}} + \sigma_{*j} \mathbf{V}'_{hj} \mathbf{Z}$$

where $\mathbf{V}'_{hj}$ is the upper triangular matrix in the Cholesky decomposition, $\mathbf{V}_j = \mathbf{V}'_{hj} \mathbf{V}_{hj}$, and $\mathbf{Z}$ is a vector of j independent random normal variates.

These simulated values of $\boldsymbol{\beta}_*$ and σ_{*j}^2 are then used to re-create the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. For a detailed description of how to produce monotone missingness with the MCMC method for a multivariate normal data, see Schafer (1997, pp. 226–235).

MCMC Method Specifications

With the MCMC method, you can impute either all missing values (IMPUTE=FULL) or just enough missing values to make the imputed data set have a monotone missing pattern (IMPUTE=MONOTONE). In the process, either a single chain for all imputations (CHAIN=SINGLE) or a separate chain for each imputation (CHAIN=MULTIPLE) is used. The single chain might be somewhat more precise for estimating a single quantity such as a posterior mean (Schafer 1997, p. 138). See Schafer (1997, pp. 137–138) for a discussion of single versus multiple chains.

You can specify the number of initial burn-in iterations before the first imputation with the NBITER= option. This number is also used for subsequent chains for multiple chains. For a single chain, you can also specify the number of iterations between imputations with the NITER= option.

You can explicitly specify initial parameter values for the MCMC method with the INITIAL=INPUT= data set option. Alternatively, you can use the EM algorithm to derive a set of initial parameter values for MCMC with the option INITIAL=EM. These estimates are used as either the starting value (START=VALUE) or the starting distribution (START=DIST) for the MCMC method. For multiple chains, these estimates are used again as either the starting value (START=VALUE) or the starting distribution (START=DIST) for the subsequent chains.

You can specify the prior parameter information in the PRIOR= option. You can use a noninformative prior (PRIOR=JEFFREYS), a ridge prior (PRIOR=RIDGE), or an informative prior specified in a data set (PRIOR=INPUT).

The parameter estimates used to generate imputed values in each imputation can be saved in a data set with the OUTEST= option. Later, this data set can be read with the INEST= option to provide the reference distribution for imputing missing values for a new data set.

By default, the MCMC method uses a single chain to produce five imputations. It completes 200 burn-in iterations before the first imputation and 100 iterations between imputations. The posterior mode computed from the EM algorithm with a noninformative prior is used as the starting values for the MCMC method.

INITIAL=EM Specifications

The EM algorithm is used to find the maximum likelihood estimates for incomplete data in the EM statement. You can also use the EM algorithm to find a posterior mode, the parameter estimates that maximize the observed-data posterior density. The resulting posterior mode provides a good starting value for the MCMC method.

With the INITIAL=EM option, PROC MI uses the MLE of the parameter vector as the initial estimates in the EM algorithm for the posterior mode. You can use the ITPRINT option within the INITIAL=EM option to display the iteration history for the EM algorithm.

You can use the CONVERGE= option to specify the convergence criterion in deriving the EM posterior mode. The iterations are considered to have converged when the maximum change in the parameter estimates between iteration steps is less than the value specified. By default, CONVERGE=1E−4.

You can also use the MAXITER= option to specify the maximum number of iterations of the EM algorithm. By default, MAXITER=200.

With the BOOTSTRAP option, you can use overdispersed starting values for the MCMC method. In this case, PROC MI applies the EM algorithm to a bootstrap sample, a simple random sample with replacement from the input data set, to derive the initial estimates for each chain (Schafer 1997, p. 128).

Checking Convergence in MCMC

The theoretical convergence of the MCMC method has been explored under various conditions, as described in Schafer (1997, p. 70). However, in practice, verification of convergence is not a simple matter.

The parameters used in the imputation step for each iteration can be saved in an output data set with the OUTITER= option. These include the means, standard deviations, covariances, worst linear function, and observed-data LR statistics. You can then monitor the convergence in a single chain by displaying trace plots and autocorrelations for those parameter values (Schafer 1997, p. 120). The trace and autocorrelation function plots for parameters such as variable means, covariances, and the worst linear function can be displayed by specifying the TIMEPLOT and ACFPLOT option.

You can apply the EM algorithm to a bootstrap sample to obtain overdispersed starting values for multiple chains (Gelman and Rubin 1992). This provides a conservative estimate of the number of iterations needed before each imputation.

The next four subsections describe useful statistics and plots that can be used to check the convergence of the MCMC method.

LR Statistics

You can save the observed-data likelihood ratio (LR) statistic in each iteration with the LR option in the OUTITER= data set. The statistic is based on the observed-data likelihood with parameter values used in the iteration and the observed-data maximum likelihood derived from the EM algorithm.

In each iteration, the LR statistic is given by

$$-2 \log \left(\frac{f(\hat{\boldsymbol{\theta}}_i)}{f(\hat{\boldsymbol{\theta}})} \right)$$

where $f(\hat{\boldsymbol{\theta}})$ is the observed-data maximum likelihood derived from the EM algorithm and $f(\hat{\boldsymbol{\theta}}_i)$ is the observed-data likelihood for $\hat{\boldsymbol{\theta}}_i$ used in the iteration.

Similarly, you can also save the observed-data LR posterior mode statistic for each iteration with the LR_POST option. This statistic is based on the observed-data posterior density with parameter values used in each iteration and the observed-data posterior mode derived from the EM algorithm for posterior mode.

For large samples, these LR statistics tends to be approximately χ^2 distributed with degrees of freedom equal to the dimension of $\boldsymbol{\theta}$ (Schafer 1997, p. 131). For example, with a large number of iterations, if the values of the LR statistic do not behave like a random sample from the described χ^2 distribution, then there is evidence that the MCMC method has not converged.

Worst Linear Function of Parameters

The worst linear function (WLF) of parameters (Schafer 1997, pp. 129–131) is a scalar function of parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ that is “worst” in the sense that its function values converge most slowly among parameters in the MCMC method. The convergence of this function is evidence that other parameters are likely to converge as well.

For linear functions of parameters $\boldsymbol{\theta} = (\boldsymbol{\mu}, \boldsymbol{\Sigma})$, a worst linear function of $\boldsymbol{\theta}$ has the highest asymptotic rate of missing information. The function can be derived from the iterative values of $\boldsymbol{\theta}$ near the posterior mode in the EM algorithm. That is, an estimated worst linear function of $\boldsymbol{\theta}$ is

$$w(\boldsymbol{\theta}) = \mathbf{v}' \, (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})$$

where $\hat{\boldsymbol{\theta}}$ is the posterior mode and the coefficients $\mathbf{v} = \hat{\boldsymbol{\theta}}_{(-1)} - \hat{\boldsymbol{\theta}}$ are the difference between the estimated value of $\boldsymbol{\theta}$ one step prior to convergence and the converged value $\hat{\boldsymbol{\theta}}$.

You can display the coefficients of the worst linear function, $\mathbf{v}$, by specifying the WLF option in the MCMC statement. You can save the function value from each iteration in an OUTITER= data set by specifying the WLF option within the OUTITER option. You can also display the worst linear function values from iterations in an autocorrelation plot or a trace plot by specifying WLF as an ACFPLOT or TIMEPLOT option, respectively.

Note that when the observed-data posterior is nearly normal, the WLF is one of the slowest functions to approach stationarity. When the posterior is not close to normal, other functions might take much longer than the WLF to converge, as described in Schafer (1997, p. 130).

Trace Plot

A trace plot for a parameter ξ is a scatter plot of successive parameter estimates ξ_i against the iteration number i. The plot provides a simple way to examine the convergence behavior of the estimation algorithm for ξ. Long-term trends in the plot indicate that successive iterations are highly correlated and that the series of iterations has not converged.

You can display trace plots for worst linear function, variable means, variable variances, and covariances of variables. You can also request logarithmic transformations for positive parameters in the plots with the LOG option. When a parameter value is less than or equal to zero, the value is not displayed in the corresponding plot.

By default, the MI procedure uses solid line segments to connect data points in a trace plot. You can use the CCONNECT=, LCONNECT=, and WCONNECT= options to change the color, line type, and width of the line segments, respectively. When WCONNECT=0 is specified, the data points are not connected, and the procedure uses the plus sign (+) as the plot symbol to display the points with a height of one (percentage screen unit) in a trace plot. You can use the SYMBOL=, CSYMBOL=, and HSYMBOL= options to change the shape, color, and height of the plot symbol, respectively.

By default, the plot title "Trace Plot" is displayed in a trace plot. You can request another title by using the TITLE= option in the TIMEPLOT option. When another title is also specified in a TITLE statement, this title is displayed as the main title and the plot title is displayed as a subtitle in the plot.

You can use options in the GOPTIONS statement to change the color and height of the title. See the chapter "The SAS/GRAPH Statements" in *SAS/GRAPH Software: Reference* for an illustration of title options. See Example 54.8 for a usage of the trace plot.

Autocorrelation Function Plot

To examine relationships of successive parameter estimates ξ, the autocorrelation function (ACF) can be used. For a stationary series, $\xi_i, i \geq 1$, in trace data, the autocorrelation function at lag k is

$$\rho_k = \frac{\mathrm{Cov}(\xi_i, \xi_{i+k})}{\mathrm{Var}(\xi_i)}$$

The sample kth order autocorrelation is computed as

$$r_k = \frac{\sum_{i=1}^{n-k} (\xi_i - \bar{\xi})(\xi_{i+k} - \bar{\xi})}{\sum_{i=1}^{n} (\xi_i - \bar{\xi})^2}$$

You can display autocorrelation function plots for the worst linear function, variable means, variable variances, and covariances of variables. You can also request logarithmic transformations for parameters in the plots with the LOG option. When a parameter has values less than or equal to zero, the corresponding plot is not created.

You specify the maximum number of lags of the series with the NLAG= option. The autocorrelations at each lag less than or equal to the specified lag are displayed in the graph. In addition, the plot also displays approximate 95% confidence limits for the autocorrelations. At lag k, the confidence limits indicate a set of approximate 95% critical values for testing the hypothesis $\rho_j = 0, j \geq k$.

By default, the MI procedure uses the star (*) as the plot symbol to display the points with a height of one (percentage screen unit) in the plot, a solid line to display the reference line of zero autocorrelation, vertical line segments to connect autocorrelations to the reference line, and a pair of dashed lines to display approximately 95% confidence limits for the autocorrelations.

You can use the SYMBOL=, CSYMBOL=, and HSYMBOL= options to change the shape, color, and height of the plot symbol, respectively, and the CNEEDLES= and WNEEDLES= options to change the color and width of the needles, respectively. You can also use the LREF=, CREF=, and WREF= options to change the line type, color, and width of the reference line, respectively. Similarly, you can use the LCONF=, CCONF=, and WCONF= options to change the line type, color, and width of the confidence limits, respectively.

By default, the plot title "Autocorrelation Plot" is displayed in a autocorrelation function plot. You can request another title by using the TITLE= option within the ACFPLOT option. When another title is also specified in a TITLE statement, this title is displayed as the main title and the plot title is displayed as a subtitle in the plot.

You can use options in the GOPTIONS statement to change the color and height of the title. See the chapter "The SAS/GRAPH Statements" in *SAS/GRAPH Software: Reference* for a description of title options. See Example 54.8 for an illustration of the autocorrelation function plot.

Input Data Sets

You can specify the input data set with missing values by using the DATA= option in the PROC MI statement. When an MCMC method is used, you can specify the data set containing the reference distribution information for imputation with the INEST= option, the data set containing initial parameter estimates for the MCMC method with the INITIAL=INPUT= option, and the data set containing information for the prior distribution with the PRIOR=INPUT= option in the MCMC statement.

DATA=*SAS-data-set*

The input DATA= data set is an ordinary SAS data set containing multivariate data with missing values.

INEST=*SAS-data-set*

The input INEST= data set is a TYPE=EST data set and contains a variable _Imputation_ to identify the imputation number. For each imputation, PROC MI reads the point estimate from the observations with _TYPE_='PARM' or _TYPE_='PARMS' and the associated covariances from the observations with _TYPE_='COV' or _TYPE_='COVB'. These estimates are used as the reference distribution to impute values for observations in the DATA= data set. When the input INEST= data set also contains observations with _TYPE_='SEED', PROC MI reads the seed information for the random number generator from these observations. Otherwise, the SEED= option provides the seed information.

INITIAL=INPUT=*SAS-data-set*

The input INITIAL=INPUT= data set is a TYPE=COV or CORR data set and provides initial parameter estimates for the MCMC method. The covariances derived from the TYPE=COV/CORR data set are divided by the number of observations to get the correct covariance matrix for the point estimate (sample mean).

If TYPE=COV, PROC MI reads the number of observations from the observations with _TYPE_='N', the point estimate from the observations with _TYPE_='MEAN', and the covariances from the observations with _TYPE_='COV'.

If TYPE=CORR, PROC MI reads the number of observations from the observations with _TYPE_='N', the point estimate from the observations with _TYPE_='MEAN', the correlations from the observations with _TYPE_='CORR', and the standard deviations from the observations with _TYPE_='STD'.

PRIOR=INPUT=*SAS-data-set*

The input PRIOR=INPUT= data set is a TYPE=COV data set that provides information for the prior distribution. You can use the data set to specify a prior distribution for $\mathbf{\Sigma}$ of the form

$$\mathbf{\Sigma} \sim W^{-1}\left(d^*,\, d^*\mathbf{S}^*\right)$$

where $d^* = n^* - 1$ is the degrees of freedom. PROC MI reads the matrix $\mathbf{S}^*$ from observations with _TYPE_='COV' and reads n^* from observations with _TYPE_='N'.

You can also use this data set to specify a prior distribution for $\boldsymbol{\mu}$ of the form

$$\boldsymbol{\mu} \sim N\left(\boldsymbol{\mu}_0, \frac{1}{n_0}\boldsymbol{\Sigma}\right)$$

PROC MI reads the mean vector $\boldsymbol{\mu}_0$ from observations with _TYPE_='MEAN' and reads n_0 from observations with _TYPE_='N_MEAN'. When there are no observations with _TYPE_='N_MEAN', PROC MI reads n_0 from observations with _TYPE_='N'.

Output Data Sets

You can specify the output data set of imputed values with the OUT= option in the PROC MI statement. When an EM statement is used, you can specify the data set containing the original data set with missing values being replaced by the expected values from the EM algorithm by using the OUT= option in the EM statement. You can also specify the data set containing MLE computed with the EM algorithm by using the OUTEM= option.

When an MCMC method is used, you can specify the data set containing parameter estimates used in each imputation with the OUTEST= option in the MCMC statement, and you can specify the data set containing parameters used in the imputation step for each iteration with the OUTITER option in the MCMC statement.

OUT=*SAS-data-set* in the PROC MI statement

The OUT= data set contains all the variables in the original data set and a new variable named _Imputation_ that identifies the imputation. For each imputation, the data set contains all variables in the input DATA= data set with missing values being replaced by imputed values. Note that when the NIMPUTE=1 option is specified, the variable _Imputation_ is not created.

OUT=*SAS-data-set* in an EM statement

The OUT= data set contains the original data set with missing values being replaced by expected values from the EM algorithm.

OUTEM=*SAS-data-set*

The OUTEM= data set is a TYPE=COV data set and contains the MLE computed with the EM algorithm. The observations with _TYPE_='MEAN' contain the estimated mean and the observations with _TYPE_='COV' contain the estimated covariances.

OUTEST=*SAS-data-set*

The OUTEST= data set is a TYPE=EST data set and contains parameter estimates used in each imputation in the MCMC method. It also includes an index variable named _Imputation_, which identifies the imputation.

The observations with _TYPE_='SEED' contain the seed information for the random number generator. The observations with _TYPE_='PARM' or _TYPE_='PARMS' contain the point estimate, and the observations with _TYPE_='COV' or _TYPE_='COVB' contain the associated covariances. These estimates are used as the parameters of the reference distribution to impute values for observations in the DATA= dataset.

Note that these estimates are the values used in the I-step before each imputation. These are not the parameter values simulated from the P-step in the same iteration. See Example 54.9 for a usage of this option.

OUTITER < (*options*) > =*SAS-data-set* in an EM statement

The OUTITER= data set in an EM statement is a TYPE=COV data set and contains parameters for each iteration. It also includes a variable _Iteration_ that provides the iteration number.

The parameters in the output data set depend on the options specified. You can specify the MEAN and COV options for OUTITER. With the MEAN option, the output data set contains the mean parameters in observations with the variable _TYPE_='MEAN'. Similarly, with the MEAN option, the output data set contains the covariance parameters in observations with the variable _TYPE_='COV'. When no options are specified, the output data set contains the mean parameters for each iteration.

OUTITER < (*options*) > =*SAS-data-set* in an MCMC statement

The OUTITER= data set in an MCMC statement is a TYPE=COV data set and contains parameters used in the imputation step for each iteration. It also includes variables named _Imputation_ and _Iteration_, which provide the imputation number and iteration number.

The parameters in the output data set depend on the options specified. Table 54.4 summarizes the options available for OUTITER and the corresponding values for the output variable _TYPE_.

Table 54.4 Summary of Options for OUTITER in an MCMC statement

Option	**Output Parameters**	**_TYPE_**
MEAN	mean parameters	MEAN
STD	standard deviations	STD
COV	covariances	COV
LR	−2 log LR statistic	LOG_LR
LR_POST	−2 log LR statistic of the posterior mode	LOG_POST
WLF	worst linear function	WLF

When no options are specified, the output data set contains the mean parameters used in the imputation step for each iteration. For a detailed description of the worst linear function and LR statistics, see the section "Checking Convergence in MCMC" on page 4412.

Combining Inferences from Multiply Imputed Data Sets

With m imputations, m different sets of the point and variance estimates for a parameter Q can be computed. Suppose $\hat{Q}_i$ and $\hat{W}_i$ are the point and variance estimates from the ith imputed data set, i = 1, 2, …, m. Then the combined point estimate for Q from multiple imputation is the average of the m complete-data estimates:

$$\overline{Q} = \frac{1}{m} \sum_{i=1}^{m} \hat{Q}_i$$

Suppose $\overline{W}$ is the within-imputation variance, which is the average of the m complete-data estimates,

$$\overline{W} = \frac{1}{m} \sum_{i=1}^{m} \hat{W}_i$$

and B is the between-imputation variance

$$B = \frac{1}{m-1} \sum_{i=1}^{m} (\hat{Q}_i - \overline{Q})^2$$

Then the variance estimate associated with $\overline{Q}$ is the total variance (Rubin 1987)

$$T = \overline{W} + (1 + \frac{1}{m})B$$

The statistic $(Q - \overline{Q})T^{-(1/2)}$ is approximately distributed as t with v_m degrees of freedom (Rubin 1987), where

$$v_m = (m-1) \left[1 + \frac{\overline{W}}{(1+m^{-1})B} \right]^2$$

The degrees of freedom v_m depend on m and the ratio

$$r = \frac{(1+m^{-1})B}{\overline{W}}$$

The ratio r is called the relative increase in variance due to nonresponse (Rubin 1987). When there is no missing information about Q, the values of r and B are both zero. With a large value of m or a small value of r, the degrees of freedom v_m will be large and the distribution of $(Q - \overline{Q})T^{-(1/2)}$ will be approximately normal.

Another useful statistic is the fraction of missing information about Q:

$$\hat{\lambda} = \frac{r + 2/(v_m + 3)}{r + 1}$$

Both statistics r and λ are helpful diagnostics for assessing how the missing data contribute to the uncertainty about Q.

When the complete-data degrees of freedom v_0 are small, and there is only a modest proportion of missing data, the computed degrees of freedom, v_m, can be much larger than v_0, which is inappropriate. For example, with $m = 5$ and $r = 10\%$, the computed degrees of freedom $v_m = 484$, which is inappropriate for data sets with complete-data degrees of freedom less than 484.

Barnard and Rubin (1999) recommend the use of adjusted degrees of freedom

$$v_m^* = \left[\frac{1}{v_m} + \frac{1}{\hat{v}_{obs}} \right]^{-1}$$

where $\hat{v}_{obs} = (1 - \gamma)\, v_0 (v_0 + 1)/(v_0 + 3)$ and $\gamma = (1 + m^{-1}) B / T$.

Note that the MI procedure uses the adjusted degrees of freedom, v_m^*, for inference.

Multiple Imputation Efficiency

The relative efficiency (RE) of using the finite m imputation estimator, rather than using an infinite number for the fully efficient imputation, in units of variance, is approximately a function of m and λ (Rubin 1987, p. 114):

$$RE = \left(1 + \frac{\lambda}{m} \right)^{-1}$$

Table 54.5 shows relative efficiencies with different values of m and λ.

Table 54.5 Relative Efficiencies

	λ				
m	10%	20%	30%	50%	70%
3	0.9677	0.9375	0.9091	0.8571	0.8108
5	0.9804	0.9615	0.9434	0.9091	0.8772
10	0.9901	0.9804	0.9709	0.9524	0.9346
20	0.9950	0.9901	0.9852	0.9756	0.9662

The table shows that for situations with little missing information, only a small number of imputations are necessary. In practice, the number of imputations needed can be informally verified by replicating sets of m imputations and checking whether the estimates are stable between sets (Horton and Lipsitz 2001, p. 246).

Imputer's Model Versus Analyst's Model

Multiple imputation inference assumes that the model you used to analyze the multiply imputed data (the analyst's model) is the same as the model used to impute missing values in multiple imputation (the imputer's model). But in practice, the two models might not be the same (Schafer 1997, p. 139).

Schafer (1997, pp. 139–143) provides comprehensive coverage of this topic, and the following example is based on his work.

Consider a trivariate data set with variables Y_1 and Y_2 fully observed, and a variable Y_3 with missing values. An imputer creates multiple imputations with the model $Y_3 = Y_1 \; Y_2$. However, the analyst can later use the simpler model $Y_3 = Y_1$. In this case, the analyst assumes more than the imputer. That is, the analyst assumes there is no relationship between variables Y_3 and Y_2.

The effect of the discrepancy between the models depends on whether the analyst's additional assumption is true. If the assumption is true, the imputer's model still applies. The inferences derived from multiple imputations will still be valid, although they might be somewhat conservative because they reflect the additional uncertainty of estimating the relationship between Y_3 and Y_2.

On the other hand, suppose that the analyst models $Y_3 = Y_1$, and there is a relationship between variables Y_3 and Y_2. Then the model $Y_3 = Y_1$ will be biased and is inappropriate. Appropriate results can be generated only from appropriate analyst models.

Another type of discrepancy occurs when the imputer assumes more than the analyst. For example, suppose that an imputer creates multiple imputations with the model $Y_3 = Y_1$, but the analyst later fits a model $Y_3 = Y_1 \; Y_2$. When the assumption is true, the imputer's model is a correct model and the inferences still hold.

On the other hand, suppose there is a relationship between Y_3 and Y_2. Imputations created under the incorrect assumption that there is no relationship between Y_3 and Y_2 will make the analyst's estimate of the relationship biased toward zero. Multiple imputations created under an incorrect model can lead to incorrect conclusions.

Thus, generally you should include as many variables as you can when doing multiple imputation. The precision you lose with included unimportant predictors is usually a relatively small price to pay for the general validity of analyses of the resultant multiply imputed data set (Rubin 1996). But at the same time, you need to keep the model building and fitting feasible (Barnard and Meng, 1999, pp. 19–20).

To produce high-quality imputations for a particular variable, the imputation model should also include variables that are potentially related to the imputed variable and variables that are potentially related to the missingness of the imputed variable (Schafer 1997, p. 143).

Similar suggestions were also given by van Buuren, Boshuizen, and Knook (1999, p. 687). They recommend that the imputation model include three sets of covariates: variables in the analyst's model, variables associated with the missingness of the imputed variable, and variables correlated with the imputed variable. They also recommend the removal of the covariates not in the analyst's model if they have too many missing values for observations with missing imputed variables.

Note that it is good practice to include a description of the imputer's model with the multiply imputed data set (Rubin 1996, p. 479). That way, the analysts will have information about the variables involved in the imputation and which relationships among the variables have been implicitly set to zero.

Parameter Simulation versus Multiple Imputation

As an alternative to multiple imputation, parameter simulation can also be used to analyze the data for many incomplete-data problems. Although the MI procedure does not offer parameter simulation, the trade-offs between the two methods (Schafer 1997, pp. 89–90, 135–136) are examined in this section.

The parameter simulation method simulates random values of parameters from the observed-data posterior distribution and makes simple inferences about these parameters (Schafer 1997, p. 89). When a set of well-defined population parameters $\boldsymbol{\theta}$ are of interest, parameter simulation can be used to directly examine and summarize simulated values of $\boldsymbol{\theta}$. This usually requires a large number of iterations, and involves calculating appropriate summaries of the resulting dependent sample of the iterates of the $\boldsymbol{\theta}$. If only a small set of parameters are involved, parameter simulation is suitable (Schafer 1997).

Multiple imputation requires only a small number of imputations. Generating and storing a few imputations can be more efficient than generating and storing a large number of iterations for parameter simulation.

When fractions of missing information are low, methods that average over simulated values of the missing data, as in multiple imputation, can be much more efficient than methods that average over simulated values of $\boldsymbol{\theta}$ as in parameter simulation (Schafer 1997).

Summary of Issues in Multiple Imputation

This section summarizes issues that are encountered in applications of the MI procedure.

The MAR Assumption

The missing at random (MAR) assumption is needed for the imputation methods in the MI procedure. Although this assumption cannot be verified with the data, it becomes more plausible as more variables are included in the imputation model (Schafer 1997, pp. 27–28; van Buuren, Boshuizen, and Knook 1999, p. 687).

Number of Imputations

Based on the theory of multiple imputation, only a small number of imputations are needed for a data set with little missing information (Rubin 1987, p. 114). The number of imputations can be informally verified by replicating sets of m imputations and checking whether the estimates are stable (Horton and Lipsitz 2001, p. 246).

Imputation Model

Generally you should include as many variables as you can in the imputation model (Rubin 1996), At the same time, however, it is important to keep the number of variables in control, as discussed by Barnard and Meng (1999, pp. 19–20). For the imputation of a particular variable, the model should include variables in the complete-data model, variables that are correlated with the imputed variable, and variables that are associated with the missingness of the imputed variable (Schafer 1997, p. 143; van Buuren, Boshuizen, and Knook 1999, p. 687).

Multivariate Normality Assumption

Although the regression and MCMC methods assume multivariate normality, inferences based on multiple imputation can be robust to departures from the multivariate normality if the amount of missing information is not large (Schafer 1997, pp. 147–148).

You can use variable transformations to make the normality assumption more tenable. Variables are transformed before the imputation process and then back-transformed to create imputed values.

Monotone Regression Method

With the multivariate normality assumption, either the regression method or the predictive mean matching method can be used to impute continuous variables in data sets with monotone missing patterns.

The predictive mean matching method ensures that imputed values are plausible and might be more appropriate than the regression method if the normality assumption is violated (Horton and Lipsitz 2001, p. 246).

Monotone Propensity Score Method

The propensity score method can also be used to impute continuous variables in data sets with monotone missing patterns.

The propensity score method does not use correlations among variables and is not appropriate for analyses involving relationship among variables, such as a regression analysis (Schafer 1999, p. 11). It can also produce badly biased estimates of regression coefficients when data on predictor variables are missing (Allison 2000).

MCMC Monotone-Data Imputation

The MCMC method is used to impute continuous variables in data sets with arbitrary missing patterns, assuming a multivariate normal distribution for the data. It can also be used to impute just enough missing values to make the imputed data sets have a monotone missing pattern. Then, a more flexible monotone imputation method can be used for the remaining missing values.

Checking Convergence in MCMC

In an MCMC method, parameters are drawn after the MCMC is run long enough to converge to its stationary distribution. In practice, however, it is not simple to verify the convergence of the process, especially for a large number of parameters.

You can check for convergence by examining the observed-data likelihood ratio statistic and worst linear function of the parameters in each iteration. You can also check for convergence by examining a plot of autocorrelation function, as well as a trace plot of parameters (Schafer 1997, p. 120).

EM Estimates

The EM algorithm can be used to compute the MLE of the mean vector and covariance matrix of the data with missing values, assuming a multivariate normal distribution for the data. However, the covariance matrix associated with the estimate of the mean vector cannot be derived from the EM algorithm.

In the MI procedure, you can use the EM algorithm to compute the posterior mode, which provides a good starting value for the MCMC method (Schafer 1997, p. 169).

ODS Table Names

PROC MI assigns a name to each table it creates. You must use these names to reference tables when using the Output Delivery System (ODS). These names are listed in Table 54.6. For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 54.6 ODS Tables Produced by PROC MI

ODS Table Name	Description	Statement	Option
Corr	Pairwise correlations		SIMPLE
EMEstimates	EM (MLE) estimates	EM	
EMInitEstimates	EM initial estimates	EM	
EMIterHistory	EM (MLE) iteration history	EM	ITPRINT
EMPostEstimates	EM (posterior mode) estimates	MCMC	INITIAL=EM
EMPostIterHistory	EM (posterior mode)	MCMC	INITIAL=EM (ITPRINT)

Table 54.6 *continued*

ODS Table Name	Description	Statement	Option
	iteration history		
EMWLF	Worst linear function	MCMC	WLF
MCMCInitEstimates	MCMC initial estimates	MCMC	DISPLAYINIT
MissPattern	Missing data patterns		
ModelInfo	Model information		
MonoDiscrim	Discriminant model group means	MONOTONE	DISCRIM (/DETAILS)
MonoLogistic	Logistic model	MONOTONE	LOGISTIC (/DETAILS)
MonoModel	Multiple monotone models	MONOTONE	
MonoPropensity	Propensity score model logistic function	MONOTONE	PROPENSITY (/DETAILS)
MonoReg	Regression model	MONOTONE	REG (/DETAILS)
MonoRegPMM	Predicted mean matching model	MONOTONE	REGPMM (/DETAILS)
ParameterEstimates	Parameter estimates		
Transform	Variable transformations	TRANSFORM	
Univariate	Univariate statistics		SIMPLE
VarianceInfo	Between, within, and total variances		

ODS Graphics

PROC MI assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when using ODS. The names are listed in Table 54.7.

To request these graphs, you must specify the ODS GRAPHICS ON statement in addition to the options indicated in Table 54.7. For more information about the ODS GRAPHICS statement, see Chapter 21, “Statistical Graphics Using ODS.”

Table 54.7 ODS Graphics Produced by PROC MI

ODS Graph Name	Plot Description	Statement	Option
ACFPlot	ACF plot	MCMC	PLOTS=ACF
TracePlot	Trace plot	MCMC	PLOTS= TRACE

Examples: MI Procedure

The Fish data described in the STEPDISC procedure are measurements of 159 fish of seven species caught in Finland's lake Laengelmavesi. For each fish, the length, height, and width are measured. Three different length measurements are recorded: from the nose of the fish to the beginning of its tail (Length1), from the nose to the notch of its tail (Length2), and from the nose to the end of its tail (Length3). See Chapter 83, "The STEPDISC Procedure," for more information.

The Fish1 data set is constructed from the Fish data set and contains only one species of the fish and the three length measurements. Some values have been set to missing, and the resulting data set has a monotone missing pattern in the variables Length1, Length2, and Length3. The Fish1 data set is used in Example 54.2 with the propensity score method and in Example 54.3 with the regression method.

The Fish2 data set is also constructed from the Fish data set and contains two species of fish. Some values have been set to missing, and the resulting data set has a monotone missing pattern in the variables Length3, Height, Width, and Species. The Fish2 data set is used in Example 54.4 with the logistic regression method and in Example 54.5 with the discriminant function method. Note that some values of the variable Species have also been altered in the data set.

The FitMiss data set created in the section "Getting Started: MI Procedure" on page 4370 is used in other examples. The following statements create the Fish1 data set:

```
/*----------- Fish of Species Bream ----------*/
 data Fish1;
    title 'Fish Measurement Data';
    input Length1 Length2 Length3 @@;
    datalines;
 23.2 25.4 30.0    24.0 26.3 31.2    23.9 26.5 31.1
 26.3 29.0 33.5    26.5 29.0   .     26.8 29.7 34.7
 26.8   .    .     27.6 30.0 35.0    27.6 30.0 35.1
 28.5 30.7 36.2    28.4 31.0 36.2    28.7   .    .
 29.1 31.5   .     29.5 32.0 37.3    29.4 32.0 37.2
 29.4 32.0 37.2    30.4 33.0 38.3    30.4 33.0 38.5
 30.9 33.5 38.6    31.0 33.5 38.7    31.3 34.0 39.5
 31.4 34.0 39.2    31.5 34.5   .     31.8 35.0 40.6
 31.9 35.0 40.5    31.8 35.0 40.9    32.0 35.0 40.6
 32.7 36.0 41.5    32.8 36.0 41.6    33.5 37.0 42.6
 35.0 38.5 44.1    35.0 38.5 44.0    36.2 39.5 45.3
 37.4 41.0 45.9    38.0 41.0 46.5
 ;
```

The Fish2 data set contains two of the seven species in the Fish data set. For each of the two species (Bream and Roach), the length from the nose of the fish to the end of its tail, the height, and the width of each fish are measured. The height and width are recorded as percentages of the length variable.

The following statements create the Fish2 data set:

```
/*-------- Fish of Species Bream and Roach --------*/
 data Fish2 (drop=HtPct WidthPct);
 title 'Fish Measurement Data';
 input Species $ Length3 HtPct WidthPct @@;
```

```
   Height= HtPct*Length3/100;
   Width= WidthPct*Length3/100;
   datalines;
Gp1  30.0 38.4 13.4   Gp1  31.2 40.0 13.8   Gp1  31.1 39.8 15.1
  .  33.5 38.0   .      .  34.0 36.6 15.1   Gp1  34.7 39.2 14.2
Gp1  34.5 41.1 15.3   Gp1  35.0 36.2 13.4   Gp1  35.1 39.9 13.8
  .  36.2 39.3 13.7   Gp1  36.2 39.4 14.1     .  36.2 39.7 13.3
Gp1  36.4 37.8 12.0     .  37.3 37.3 13.6   Gp1  37.2 40.2 13.9
Gp1  37.2 41.5 15.0   Gp1  38.3 38.8 13.8   Gp1  38.5 38.8 13.5
Gp1  38.6 40.5 13.3   Gp1  38.7 37.4 14.8   Gp1  39.5 38.3 14.1
Gp1  39.2 40.8 13.7     .  39.7 39.1   .    Gp1  40.6 38.1 15.1
Gp1  40.5 40.1 13.8   Gp1  40.9 40.0 14.8   Gp1  40.6 40.3 15.0
Gp1  41.5 39.8 14.1   Gp2  41.6 40.6 14.9   Gp1  42.6 44.5 15.5
Gp1  44.1 40.9 14.3   Gp1  44.0 41.1 14.3   Gp1  45.3 41.4 14.9
Gp1  45.9 40.6 14.7   Gp1  46.5 37.9 13.7
Gp2  16.2 25.6 14.0   Gp2  20.3 26.1 13.9   Gp2  21.2 26.3 13.7
Gp2  22.2 25.3 14.3   Gp2  22.2 28.0 16.1   Gp2  22.8 28.4 14.7
Gp2  23.1 26.7 14.7     .  23.7 25.8 13.9   Gp2  24.7 23.5 15.2
Gp2  24.3 27.3 14.6   Gp2  25.3 27.8 15.1   Gp2  25.0 26.2 13.3
Gp2  25.0 25.6 15.2   Gp2  27.2 27.7 14.1   Gp2  26.7 25.9 13.6
  .  26.8 27.6 15.4   Gp2  27.9 25.4 14.0   Gp2  29.2 30.4 15.4
Gp2  30.6 28.0 15.6   Gp2  35.0 27.1 15.3
;
```

Example 54.1: EM Algorithm for MLE

This example uses the EM algorithm to compute the maximum likelihood estimates for parameters of multivariate normally distributed data with missing values. The following statements invoke the MI procedure and request the EM algorithm to compute the MLE for $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ of a multivariate normal distribution from the input data set FitMiss:

```
proc mi data=FitMiss seed=1518971 simple nimpute=0;
   em itprint outem=outem;
   var Oxygen RunTime RunPulse;
run;
```

Note that when you specify the NIMPUTE=0 option, the missing values are not imputed.

The “Model Information” table in Output 54.1.1 describes the method and options used in the procedure if a positive number is specified in the NIMPUTE= option.

Output 54.1.1 Model Information

```
                              The MI Procedure

                             Model Information

           Data Set                              WORK.FITMISS
           Method                                MCMC
           Multiple Imputation Chain             Single Chain
           Initial Estimates for MCMC            EM Posterior Mode
           Start                                 Starting Value
           Prior                                 Jeffreys
           Number of Imputations                 0
           Number of Burn-in Iterations          200
           Number of Iterations                  100
           Seed for random number generator      1518971
```

The “Missing Data Patterns” table in Output 54.1.2 lists distinct missing data patterns with corresponding frequencies and percents. Here, a value of “X” means that the variable is observed in the corresponding group and a value of “.” means that the variable is missing. The table also displays group-specific variable means.

Output 54.1.2 Missing Data Patterns

```
                            Missing Data Patterns

                                 Run     Run
            Group     Oxygen     Time    Pulse       Freq     Percent

                1     X          X       X             21       67.74
                2     X          X       .              4       12.90
                3     X          .       .              3        9.68
                4     .          X       X              1        3.23
                5     .          X       .              2        6.45

                           Missing Data Patterns

                   -----------------Group Means----------------
             Group           Oxygen          RunTime         RunPulse

                 1        46.353810        10.809524       171.666667
                 2        47.109500        10.137500                .
                 3        52.461667                .                .
                 4                .        11.950000       176.000000
                 5                .         9.885000                .
```

With the SIMPLE option, the procedure displays simple descriptive univariate statistics for available cases in the “Univariate Statistics” table in Output 54.1.3 and correlations from pairwise available cases in the “Pairwise Correlations” table in Output 54.1.4.

Output 54.1.3 Univariate Statistics

```
                         Univariate Statistics

Variable          N          Mean       Std Dev       Minimum       Maximum

Oxygen           28      47.11618       5.41305      37.38800      60.05500
RunTime          28      10.68821       1.37988       8.63000      14.03000
RunPulse         22     171.86364      10.14324     148.00000     186.00000

                         Univariate Statistics

                                ---Missing Values--
                      Variable        Count     Percent

                      Oxygen              3        9.68
                      RunTime             3        9.68
                      RunPulse            9       29.03
```

Output 54.1.4 Pairwise Correlations

```
                         Pairwise Correlations

                         Oxygen           RunTime          RunPulse

       Oxygen       1.000000000      -0.849118562      -0.343961742
       RunTime     -0.849118562       1.000000000       0.247258191
       RunPulse    -0.343961742       0.247258191       1.000000000
```

When you use the EM statement, the MI procedure displays the initial parameter estimates for the EM algorithm in the “Initial Parameter Estimates for EM” table in Output 54.1.5.

Output 54.1.5 Initial Parameter Estimates for EM

```
                   Initial Parameter Estimates for EM

        _TYPE_    _NAME_          Oxygen         RunTime        RunPulse

        MEAN                   47.116179       10.688214      171.863636
        COV       Oxygen       29.301078               0               0
        COV       RunTime              0        1.904067               0
        COV       RunPulse             0               0      102.885281
```

When you use the ITPRINT option in the EM statement, the “EM (MLE) Iteration History” table in Output 54.1.6 displays the iteration history for the EM algorithm.

Output 54.1.6 EM (MLE) Iteration History

```
                          EM (MLE) Iteration History

   _Iteration_        -2 Log L        Oxygen         RunTime       RunPulse

             0      289.544782     47.116179       10.688214     171.863636
             1      263.549489     47.116179       10.688214     171.863636
             2      255.851312     47.139089       10.603506     171.538203
             3      254.616428     47.122353       10.571685     171.426790
             4      254.494971     47.111080       10.560585     171.398296
             5      254.483973     47.106523       10.556768     171.389208
             6      254.482920     47.104899       10.555485     171.385257
             7      254.482813     47.104348       10.555062     171.383345
             8      254.482801     47.104165       10.554923     171.382424
             9      254.482800     47.104105       10.554878     171.381992
            10      254.482800     47.104086       10.554864     171.381796
            11      254.482800     47.104079       10.554859     171.381708
            12      254.482800     47.104077       10.554858     171.381669
```

The "EM (MLE) Parameter Estimates" table in Output 54.1.7 displays the maximum likelihood estimates for $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ of a multivariate normal distribution from the data set FitMiss.

Output 54.1.7 EM (MLE) Parameter Estimates

```
                         EM (MLE) Parameter Estimates

       _TYPE_    _NAME_            Oxygen         RunTime        RunPulse

       MEAN                     47.104077       10.554858      171.381669
       COV       Oxygen         27.797931       -6.457975      -18.031298
       COV       RunTime        -6.457975        2.015514        3.516287
       COV       RunPulse      -18.031298        3.516287       97.766857
```

You can also output the EM (MLE) parameter estimates to an output data set with the OUTEM= option. The following statements list the observations in the output data set outem:

```
proc print data=outem;
   title 'EM Estimates';
run;
```

The output data set outem in Output 54.1.8 is a TYPE=COV data set. The observation with _TYPE_='MEAN' contains the MLE for the parameter $\boldsymbol{\mu}$, and the observations with _TYPE_='COV' contain the MLE for the parameter $\boldsymbol{\Sigma}$ of a multivariate normal distribution from the data set FitMiss.

Output 54.1.8 EM Estimates

```
                                EM Estimates

        Obs     _TYPE_      _NAME_        Oxygen       RunTime     RunPulse

         1       MEAN                     47.1041      10.5549      171.382
         2       COV       Oxygen         27.7979      -6.4580      -18.031
         3       COV       RunTime        -6.4580       2.0155        3.516
         4       COV       RunPulse      -18.0313       3.5163       97.767
```

Example 54.2: Propensity Score Method

This example uses the propensity score method to impute missing values for variables in a data set with a monotone missing pattern. The following statements invoke the MI procedure and request the propensity score method. The resulting data set is named outex2.

```
proc mi data=Fish1 seed=899603 out=outex2;
   monotone propensity;
   var Length1 Length2 Length3;
run;
```

Note that the VAR statement is required and the data set must have a monotone missing pattern with variables as ordered in the VAR statement.

The "Model Information" table in Output 54.2.1 describes the method and options used in the multiple imputation process. By default, five imputations are created for the missing data.

Output 54.2.1 Model Information

```
                              The MI Procedure

                             Model Information

          Data Set                             WORK.FISH1
          Method                               Monotone
          Number of Imputations                5
          Seed for random number generator     899603
```

When monotone methods are used in the imputation, MONOTONE is displayed as the method. The "Monotone Model Specification" table in Output 54.2.2 displays the detailed model specification. By default, the observations are sorted into five groups based on their propensity scores.

Output 54.2.2 Monotone Model Specification

```
                     Monotone Model Specification

                                           Imputed
            Method                         Variables

            Propensity( Groups= 5)         Length2 Length3
```

Without covariates specified for imputed variables Length2 and Length3, the variable Length1 is used as the covariate for Length2, and the variables Length1 and Length2 are used as covariates for Length3.

The "Missing Data Patterns" table in Output 54.2.3 lists distinct missing data patterns with corresponding frequencies and percents. Here, values of "X" and "." indicate that the variable is observed or missing, respectively, in the corresponding group. The table confirms a monotone missing pattern for these three variables.

Output 54.2.3 Missing Data Patterns

```
                         Missing Data Patterns

          Group    Length1    Length2    Length3       Freq     Percent

              1    X          X          X               30       85.71
              2    X          X          .                3        8.57
              3    X          .          .                2        5.71

                        Missing Data Patterns

                  -----------------Group Means----------------
              Group         Length1         Length2         Length3

                  1       30.603333       33.436667       38.720000
                  2       29.033333       31.666667               .
                  3       27.750000               .               .
```

For the imputation process, first, missing values of Length2 in group 3 are imputed using observed values of Length1. Then the missing values of Length3 in group 2 are imputed using observed values of Length1 and Length2. And finally, the missing values of Length3 in group 3 are imputed using observed values of Length1 and imputed values of Length2.

After the completion of *m* imputations, the "Variance Information" table in Output 54.2.4 displays the between-imputation variance, within-imputation variance, and total variance for combining complete-data inferences. It also displays the degrees of freedom for the total variance. The relative increase in variance due to missingness, the fraction of missing information, and the relative efficiency for each variable are also displayed. A detailed description of these statistics is provided in the section "Combining Inferences from Multiply Imputed Data Sets" on page 4418.

Output 54.2.4 Variance Information

```
                              Variance Information

                   -----------------Variance-----------------
         Variable          Between          Within           Total        DF

         Length2          0.001500        0.465422        0.467223    32.034
         Length3          0.049725        0.547434        0.607104    27.103

                               Variance Information

                              Relative        Fraction
                              Increase         Missing        Relative
              Variable     in Variance     Information      Efficiency

              Length2         0.003869        0.003861        0.999228
              Length3         0.108999        0.102610        0.979891
```

The "Parameter Estimates" table in Output 54.2.5 displays the estimated mean and standard error of the mean for each variable. The inferences are based on the *t* distributions. For each variable, the table also displays a 95% mean confidence interval and a *t* statistic with the associated *p*-value for the hypothesis that the population mean is equal to the value specified in the MU0= option, which is zero by default.

Output 54.2.5 Parameter Estimates

```
                               Parameter Estimates

    Variable             Mean       Std Error    95% Confidence Limits         DF

    Length2         33.006857        0.683537     31.61460     34.39912    32.034
    Length3         38.361714        0.779169     36.76328     39.96015    27.103

                               Parameter Estimates

                                                                t for H0:
  Variable          Minimum          Maximum             Mu0     Mean=Mu0    Pr > |t|

  Length2         32.957143        33.060000               0        48.29      <.0001
  Length3         38.080000        38.545714               0        49.23      <.0001
```

The following statements list the first 10 observations of the data set outex2, as shown in Output 54.2.6. The missing values are imputed from observed values with similar propensity scores.

```
proc print data=outex2(obs=10);
   title 'First 10 Observations of the Imputed Data Set';
run;
```

Output 54.2.6 Imputed Data Set

```
                  First 10 Observations of the Imputed Data Set

                 Obs    _Imputation_    Length1    Length2    Length3

                   1          1           23.2       25.4       30.0
                   2          1           24.0       26.3       31.2
                   3          1           23.9       26.5       31.1
                   4          1           26.3       29.0       33.5
                   5          1           26.5       29.0       38.6
                   6          1           26.8       29.7       34.7
                   7          1           26.8       29.0       35.0
                   8          1           27.6       30.0       35.0
                   9          1           27.6       30.0       35.1
                  10          1           28.5       30.7       36.2
```

Example 54.3: Regression Method

This example uses the regression method to impute missing values for all variables in a data set with a monotone missing pattern. The following statements invoke the MI procedure and request the regression method for the variable Length2 and the predictive mean matching method for variable Length3. The resulting data set is named outex3.

```
proc mi data=Fish1 round=.1  mu0= 0 35 45
        seed=13951639 out=outex3;
   monotone reg(Length2/ details)
            regpmm(Length3= Length1 Length2 Length1*Length2/ details);
   var Length1 Length2 Length3;
run;
```

The ROUND= option is used to round the imputed values to the same precision as observed values. The values specified with the ROUND= option are matched with the variables Length1, Length2, and Length3 in the order listed in the VAR statement. The MU0= option requests *t* tests for the hypotheses that the population means corresponding to the variables in the VAR statement are Length2=35 and Length3=45.

The "Missing Data Patterns" table lists distinct missing data patterns with corresponding frequencies and percents. It is identical to the table in Output 54.2.3 in Example 54.2.

The “Monotone Model Specification” table in Output 54.3.1 displays the model specification.

Output 54.3.1 Monotone Model Specification

```
                               The MI Procedure

                        Monotone Model Specification

                                               Imputed
                     Method                    Variables

                     Regression                Length2
                     Regression-PMM( K= 5)     Length3
```

When you use the DETAILS option, the parameters estimated from the observed data and the parameters used in each imputation are displayed in Output 54.3.2 and Output 54.3.3.

Output 54.3.2 Regression Model

```
                    Regression Models for Monotone Method

Imputed                             ----------------Imputation----------------
Variable   Effect        Obs-Data              1              2              3

Length2    Intercept     -0.04249      -0.049184      -0.055470      -0.051346
Length2    Length1        0.98587       1.001934       0.995275       0.992294

                    Regression Models for Monotone Method

             Imputed                ---------Imputation---------
             Variable   Effect                 4               5

             Length2    Intercept      -0.064193       -0.030719
             Length2    Length1         0.983122        0.995883
```

Output 54.3.3 Regression Predicted Mean Matching Model

```
         Regression Models for Monotone Predicted Mean Matching Method

Imputed                                ----------------Imputation---------------
Variable  Effect              Obs Data            1             2             3

Length3   Intercept           -0.01304     0.004134     -0.011417     -0.034177
Length3   Length1             -0.01332     0.025320     -0.037494      0.308765
Length3   Length2              0.98918     0.955510      1.025741      0.673374
Length3   Length1*Length2     -0.02521    -0.034964     -0.022017     -0.017919

         Regression Models for Monotone Predicted Mean Matching Method

            Imputed                    ---------Imputation---------
            Variable  Effect                      4               5

            Length3   Intercept           -0.010532        0.004685
            Length3   Length1              0.156606       -0.147118
            Length3   Length2              0.828384        1.146440
            Length3   Length1*Length2     -0.029335       -0.034671
```

After the completion of five imputations by default, the "Variance Information" table in Output 54.3.4 displays the between-imputation variance, within-imputation variance, and total variance for combining complete-data inferences. The relative increase in variance due to missingness, the fraction of missing information, and the relative efficiency for each variable are also displayed. These statistics are described in the section "Combining Inferences from Multiply Imputed Data Sets" on page 4418.

Output 54.3.4 Variance Information

```
                              Variance Information

                     -----------------Variance-----------------
        Variable          Between          Within           Total       DF

        Length2          0.000133        0.439512        0.439672    32.15
        Length3          0.000386        0.486913        0.487376   32.131

                               Variance Information

                              Relative       Fraction
                              Increase        Missing        Relative
             Variable      in Variance    Information      Efficiency

             Length2          0.000363       0.000363        0.999927
             Length3          0.000952       0.000951        0.999810
```

The “Parameter Estimates” table in Output 54.3.5 displays a 95% mean confidence interval and a *t* statistic with its associated *p*-value for each of the hypotheses requested with the MU0= option.

Output 54.3.5 Parameter Estimates

```
                           Parameter Estimates

  Variable           Mean       Std Error    95% Confidence Limits        DF

  Length2       33.104571        0.663078     31.75417     34.45497    32.15
  Length3       38.424571        0.698123     37.00277     39.84637   32.131

                           Parameter Estimates

                                                            t for H0:
Variable          Minimum         Maximum           Mu0     Mean=Mu0    Pr > |t|

Length2         33.088571       33.117143     35.000000        -2.86      0.0074
Length3         38.397143       38.445714     45.000000        -9.42      <.0001
```

The following statements list the first 10 observations of the data set outex3 in Output 54.3.6. Note that the imputed values of Length2 are rounded to the same precision as the observed values.

```
proc print data=outex3(obs=10);
   title 'First 10 Observations of the Imputed Data Set';
run;
```

Output 54.3.6 Imputed Data Set

```
                 First 10 Observations of the Imputed Data Set

                Obs    _Imputation_    Length1    Length2    Length3

                  1          1           23.2       25.4       30.0
                  2          1           24.0       26.3       31.2
                  3          1           23.9       26.5       31.1
                  4          1           26.3       29.0       33.5
                  5          1           26.5       29.0       34.7
                  6          1           26.8       29.7       34.7
                  7          1           26.8       28.8       34.7
                  8          1           27.6       30.0       35.0
                  9          1           27.6       30.0       35.1
                 10          1           28.5       30.7       36.2
```

Example 54.4: Logistic Regression Method for CLASS Variables

This example uses logistic regression method to impute values for a binary variable in a data set with a monotone missing pattern.

In the following statements, the logistic regression method is used for the binary CLASS variable Species:

```
proc mi data=Fish2 seed=1305417 out=outex4;
   class Species;
   monotone logistic( Species= Height Width Height*Width/ details);
   var Height Width Species;
run;
```

The “Model Information” table in Output 54.4.1 describes the method and options used in the multiple imputation process.

Output 54.4.1 Model Information

```
                               The MI Procedure

                              Model Information

          Data Set                              WORK.FISH2
          Method                                Monotone
          Number of Imputations                 5
          Seed for random number generator      1305417
```

The “Monotone Model Specification” table in Output 54.4.2 describes methods and imputed variables in the imputation model. The procedure uses the logistic regression method to impute the variable Species in the model. Missing values in other variables are not imputed.

Output 54.4.2 Monotone Model Specification

```
                         Monotone Model Specification

                                            Imputed
                       Method               Variables

                       Logistic Regression  Species
```

The “Missing Data Patterns” table in Output 54.4.3 lists distinct missing data patterns with corresponding frequencies and percents. The table confirms a monotone missing pattern for these variables.

Output 54.4.3 Missing Data Patterns

```
                              Missing Data Patterns

                                                        --------Group Means-------
  Group  Height  Width  Species      Freq   Percent          Height          Width

      1  X       X      X              47     85.45       12.097645       4.808204
      2  X       X      .               6     10.91       11.411050       4.567050
      3  X       .      .               2      3.64       14.126350              .
```

When you use the DETAILS option, parameters estimated from the observed data and the parameters used in each imputation are displayed in the “Logistic Models for Monotone Method” table in Output 54.4.4.

Output 54.4.4 Logistic Regression Model

```
                       Logistic Models for Monotone Method

 Imputed                                ---------------Imputation---------------
 Variable   Effect          Obs-Data             1             2             3

 Species    Intercept        2.14183      1.240681      5.018482      5.509416
 Species    Height           9.08604      3.774512     11.322763     11.230355
 Species    Width           -5.02065      0.674528     -6.245428     -5.785890
 Species    Height*Width    -1.91634     -3.299450     -3.326538     -5.045058

                       Logistic Models for Monotone Method

             Imputed                    ---------Imputation---------
             Variable   Effect                   4               5

             Species    Intercept        -1.325099        6.069734
             Species    Height            5.711366       12.766614
             Species    Width             2.394018       -9.689260
             Species    Height*Width     -2.570333       -2.214031
```

The following statements list the first 10 observations of the data set outex4 in Output 54.4.5:

```
proc print data=outex4(obs=10);
   title 'First 10 Observations of the Imputed Data Set';
run;
```

Output 54.4.5 Imputed Data Set

```
                   First 10 Observations of the Imputed Data Set

          Obs     _Imputation_     Species     Length3      Height      Width

            1           1            Gp1         30.0      11.5200     4.0200
            2           1            Gp1         31.2      12.4800     4.3056
            3           1            Gp1         31.1      12.3778     4.6961
            4           1                        33.5      12.7300      .
            5           1            Gp1         34.0      12.4440     5.1340
            6           1            Gp1         34.7      13.6024     4.9274
            7           1            Gp1         34.5      14.1795     5.2785
            8           1            Gp1         35.0      12.6700     4.6900
            9           1            Gp1         35.1      14.0049     4.8438
           10           1            Gp1         36.2      14.2266     4.9594
```

Note that a missing value of the variable Species is not imputed if the corresponding covariates are missing and not imputed, as shown by observation 4 in the table.

Example 54.5: Discriminant Function Method for CLASS Variables

This example uses discriminant monotone methods to impute values of a CLASS variable from the observed observation values in a data set with a monotone missing pattern.

The following statements impute the continuous variables Height and Width with the regression method and the classification variable Species with the discriminant function method:

```
proc mi data=Fish2 seed=7545417 nimpute=3 out=outex5;
   class Species;
   monotone reg( Height Width)
            discrim( Species= Length3 Height Width/ details);
   var Length3 Height Width Species;
run;
```

The "Model Information" table in Output 54.5.1 describes the method and options used in the multiple imputation process.

Output 54.5.1 Model Information

```
                              The MI Procedure

                             Model Information

          Data Set                              WORK.FISH2
          Method                                Monotone
          Number of Imputations                 3
          Seed for random number generator      7545417
```

The "Monotone Model Specification" table in Output 54.5.2 describes methods and imputed variables in the imputation model. The procedure uses the regression method to impute the variables Height and Width, and uses the logistic regression method to impute the variable Species in the model.

Output 54.5.2 Monotone Model Specification

```
                        Monotone Model Specification

                                             Imputed
                   Method                    Variables

                   Regression                Height Width
                   Discriminant Function     Species
```

The "Missing Data Patterns" table in Output 54.5.3 lists distinct missing data patterns with corresponding frequencies and percents. The table confirms a monotone missing pattern for these variables.

Output 54.5.3 Missing Data Patterns

```
                           Missing Data Patterns

     Group     Length3     Height     Width     Species          Freq      Percent

         1     X           X          X         X                  47        85.45
         2     X           X          X         .                   6        10.91
         3     X           X          .         .                   2         3.64

                          Missing Data Patterns

                  -----------------Group Means----------------
             Group          Length3          Height          Width

                 1        33.497872       12.097645       4.808204
                 2        32.366667       11.411050       4.567050
                 3        36.600000       14.126350              .
```

When you use the DETAILS option, the parameters estimated from the observed data and the parameters used in each imputation are displayed in Output 54.5.4.

Output 54.5.4 Discriminant Model

```
                Group Means for Monotone Discriminant Method

                                     ----------------Imputation----------------
 Species     Variable     Obs-Data              1              2              3

 Gp1         Length3       0.68104       0.766779       0.724277       0.577304
 Gp1         Height        0.74011       0.809770       0.794103       0.671612
 Gp1         Width         0.63865       0.700122       0.725179       0.579870
 Gp2         Length3      -1.00022      -0.809466      -0.999101      -0.908734
 Gp2         Height       -1.09007      -0.965672      -1.089324      -1.024453
 Gp2         Width        -0.88135      -0.710969      -0.827099      -0.746598
```

The following statements list the first 10 observations of the data set outex5 in Output 54.5.5. Note that all missing values of the variables Width and Species are imputed.

```
proc print data=outex5(obs=10);
   title 'First 10 Observations of the Imputed Data Set';
run;
```

Output 54.5.5 Imputed Data Set

```
                  First 10 Observations of the Imputed Data Set

        Obs    _Imputation_    Species    Length3     Height      Width

          1          1           Gp1        30.0     11.5200    4.02000
          2          1           Gp1        31.2     12.4800    4.30560
          3          1           Gp1        31.1     12.3778    4.69610
          4          1           Gp1        33.5     12.7300    4.67966
          5          1           Gp2        34.0     12.4440    5.13400
          6          1           Gp1        34.7     13.6024    4.92740
          7          1           Gp1        34.5     14.1795    5.27850
          8          1           Gp1        35.0     12.6700    4.69000
          9          1           Gp1        35.1     14.0049    4.84380
         10          1           Gp1        36.2     14.2266    4.95940
```

Example 54.6: MCMC Method

This example uses the MCMC method to impute missing values for a data set with an arbitrary missing pattern. The following statements invoke the MI procedure and specify the MCMC method with six imputations:

```
proc mi data=FitMiss seed=21355417 nimpute=6 mu0=50 10 180 ;
   mcmc chain=multiple displayinit initial=em(itprint);
   var Oxygen RunTime RunPulse;
run;
```

The “Model Information” table in Output 54.6.1 describes the method used in the multiple imputation process. When you use the CHAIN=MULTIPLE option, the procedure uses multiple chains and completes the default 200 burn-in iterations before each imputation. The 200 burn-in iterations are used to make the iterations converge to the stationary distribution before the imputation.

Output 54.6.1 Model Information

```
                               The MI Procedure

                              Model Information

           Data Set                             WORK.FITMISS
           Method                               MCMC
           Multiple Imputation Chain            Multiple Chains
           Initial Estimates for MCMC           EM Posterior Mode
           Start                                Starting Value
           Prior                                Jeffreys
           Number of Imputations                6
           Number of Burn-in Iterations         200
           Seed for random number generator     21355417
```

By default, the procedure uses a noninformative Jeffreys prior to derive the posterior mode from the EM algorithm as the starting values for the MCMC method.

The “Missing Data Patterns” table in Output 54.6.2 lists distinct missing data patterns with corresponding statistics.

Output 54.6.2 Missing Data Patterns

```
                              Missing Data Patterns

                               Run     Run
            Group    Oxygen    Time    Pulse        Freq     Percent

                1    X         X       X              21       67.74
                2    X         X       .               4       12.90
                3    X         .       .               3        9.68
                4    .         X       X               1        3.23
                5    .         X       .               2        6.45

                             Missing Data Patterns

                      -----------------Group Means----------------
             Group          Oxygen         RunTime        RunPulse

                 1       46.353810       10.809524      171.666667
                 2       47.109500       10.137500               .
                 3       52.461667               .               .
                 4               .       11.950000      176.000000
                 5               .        9.885000               .
```

When you use the ITPRINT option within the INITIAL=EM option, the procedure displays the “EM (Posterior Mode) Iteration History” table in Output 54.6.3.

Output 54.6.3 EM (Posterior Mode) Iteration History

```
                    EM (Posterior Mode) Iteration History

 _Iteration_        -2 Log L   -2 Log Posterior         Oxygen         RunTime

           0      254.482800         282.909549      47.104077       10.554858
           1      255.081168         282.051584      47.104077       10.554857
           2      255.271408         282.017488      47.104077       10.554857
           3      255.318622         282.015372      47.104002       10.554523
           4      255.330259         282.015232      47.103861       10.554388
           5      255.333161         282.015222      47.103797       10.554341
           6      255.333896         282.015222      47.103774       10.554325
           7      255.334085         282.015222      47.103766       10.554320

                    EM (Posterior Mode) Iteration History

                         _Iteration_        RunPulse

                                   0      171.381669
                                   1      171.381652
                                   2      171.381644
                                   3      171.381842
                                   4      171.382053
                                   5      171.382150
                                   6      171.382185
                                   7      171.382196
```

When you use the DISPLAYINIT option in the MCMC statement, the "Initial Parameter Estimates for MCMC" table in Output 54.6.4 displays the starting mean and covariance estimates used in the MCMC method. The same starting estimates are used in the MCMC method for multiple chains because the EM algorithm is applied to the same data set in each chain. You can explicitly specify different initial estimates for different imputations, or you can use the bootstrap method to generate different parameter estimates from the EM algorithm for the MCMC method.

Output 54.6.4 Initial Parameter Estimates

```
                     Initial Parameter Estimates for MCMC

       _TYPE_    _NAME_           Oxygen         RunTime        RunPulse

       MEAN                    47.103766       10.554320      171.382196
       COV       Oxygen        24.549967       -5.726112      -15.926036
       COV       RunTime       -5.726112        1.781407        3.124798
       COV       RunPulse     -15.926036        3.124798       83.164045
```

Output 54.6.5 and Output 54.6.6 display variance information and parameter estimates, respectively, from the multiple imputation.

Output 54.6.5 Variance Information

```
                          Variance Information

                -----------------Variance-----------------
Variable         Between          Within           Total        DF

Oxygen          0.051560        0.928170        0.988323    25.958
RunTime         0.003979        0.070057        0.074699    25.902
RunPulse        4.118578        4.260631        9.065638    7.5938

                           Variance Information

                     Relative        Fraction
                     Increase         Missing        Relative
     Variable     in Variance     Information      Efficiency

     Oxygen          0.064809        0.062253        0.989731
     RunTime         0.066262        0.063589        0.989513
     RunPulse        1.127769        0.575218        0.912517
```

Output 54.6.6 Parameter Estimates

```
                             Parameter Estimates

  Variable              Mean       Std Error     95% Confidence Limits        DF

  Oxygen           47.164819        0.994145       45.1212       49.2085    25.958
  RunTime          10.549936        0.273312        9.9880       11.1118    25.902
  RunPulse        170.969836        3.010920      163.9615      177.9782    7.5938

                             Parameter Estimates

                                                               t for H0:
Variable            Minimum         Maximum             Mu0     Mean=Mu0    Pr > |t|

Oxygen            46.858020       47.363540       50.000000        -2.85      0.0084
RunTime           10.476886       10.659412       10.000000         2.01      0.0547
RunPulse         168.252615      172.894991      180.000000        -3.00      0.0182
```

Example 54.7: Producing Monotone Missingness with MCMC

This example uses the MCMC method to impute just enough missing values for a data set with an arbitrary missing pattern so that each imputed data set has a monotone missing pattern based on the order of variables in the VAR statement.

The following statements invoke the MI procedure and specify the IMPUTE=MONOTONE option to create the imputed data set with a monotone missing pattern. You must specify a VAR statement to provide the order of variables in order for the imputed data to achieve a monotone missing pattern.

```
proc mi data=FitMiss seed=17655417 out=outex7;
   mcmc impute=monotone;
   var Oxygen RunTime RunPulse;
run;
```

The “Model Information” table in Output 54.7.1 describes the method used in the multiple imputation process.

Output 54.7.1 Model Information

```
                            The MI Procedure

                           Model Information

          Data Set                             WORK.FITMISS
          Method                               Monotone-data MCMC
          Multiple Imputation Chain            Single Chain
          Initial Estimates for MCMC           EM Posterior Mode
          Start                                Starting Value
          Prior                                Jeffreys
          Number of Imputations                5
          Number of Burn-in Iterations         200
          Number of Iterations                 100
          Seed for random number generator     17655417
```

The “Missing Data Patterns” table in Output 54.7.2 lists distinct missing data patterns with corresponding statistics. Here, an “X” means that the variable is observed in the corresponding group, a “.” means that the variable is missing and will be imputed to achieve the monotone missingness for the imputed data set, and an “O” means that the variable is missing and will not be imputed. The table also displays group-specific variable means.

Output 54.7.2 Missing Data Patterns

```
                         Missing Data Patterns

                                  Run     Run
            Group    Oxygen      Time    Pulse        Freq     Percent

                1    X           X       X              21       67.74
                2    X           X       O               4       12.90
                3    X           O       O               3        9.68
                4    .           X       X               1        3.23
                5    .           X       O               2        6.45

                         Missing Data Patterns

                      -----------------Group Means----------------
             Group          Oxygen         RunTime        RunPulse

                 1       46.353810       10.809524      171.666667
                 2       47.109500       10.137500               .
                 3       52.461667               .               .
                 4               .       11.950000      176.000000
                 5               .        9.885000               .
```

As shown in the table in Output 54.7.2, the MI procedure needs to impute only three missing values from group 4 and group 5 to achieve a monotone missing pattern for the imputed data set.

When you use the MCMC method to produce an imputed data set with a monotone missing pattern, tables of variance information and parameter estimates are not created.

The following statements are used just to show the monotone missingness of the output data set outex7:

```
proc mi data=outex7  nimpute=0;
   var Oxygen RunTime RunPulse;
run;
```

The “Missing Data Patterns” table in Output 54.7.3 displays a monotone missing data pattern.

Output 54.7.3 Monotone Missing Data Patterns

```
                          The MI Procedure

                        Missing Data Patterns

                            Run      Run
         Group     Oxygen   Time     Pulse          Freq     Percent

             1     X        X        X               110       70.97
             2     X        X        .                30       19.35
             3     X        .        .                15        9.68

                       Missing Data Patterns

                   ---------------Group Means----------------
          Group            Oxygen         RunTime        RunPulse

              1         46.152428       10.861364      171.863636
              2         47.796038       10.053333               .
              3         52.461667               .               .
```

The following statements impute one value for each missing value in the monotone missingness data set outex7:

```
proc mi data=outex7 nimpute=1 seed=51343672 out=outds;
   monotone method=reg;
   var Oxygen RunTime RunPulse;
   by _Imputation_;
run;
```

You can then analyze these data sets by using other SAS procedures and combine these results by using the MIANALYZE procedure. Note that the VAR statement is required with a MONOTONE statement to provide the variable order for the monotone missing pattern.

Example 54.8: Checking Convergence in MCMC

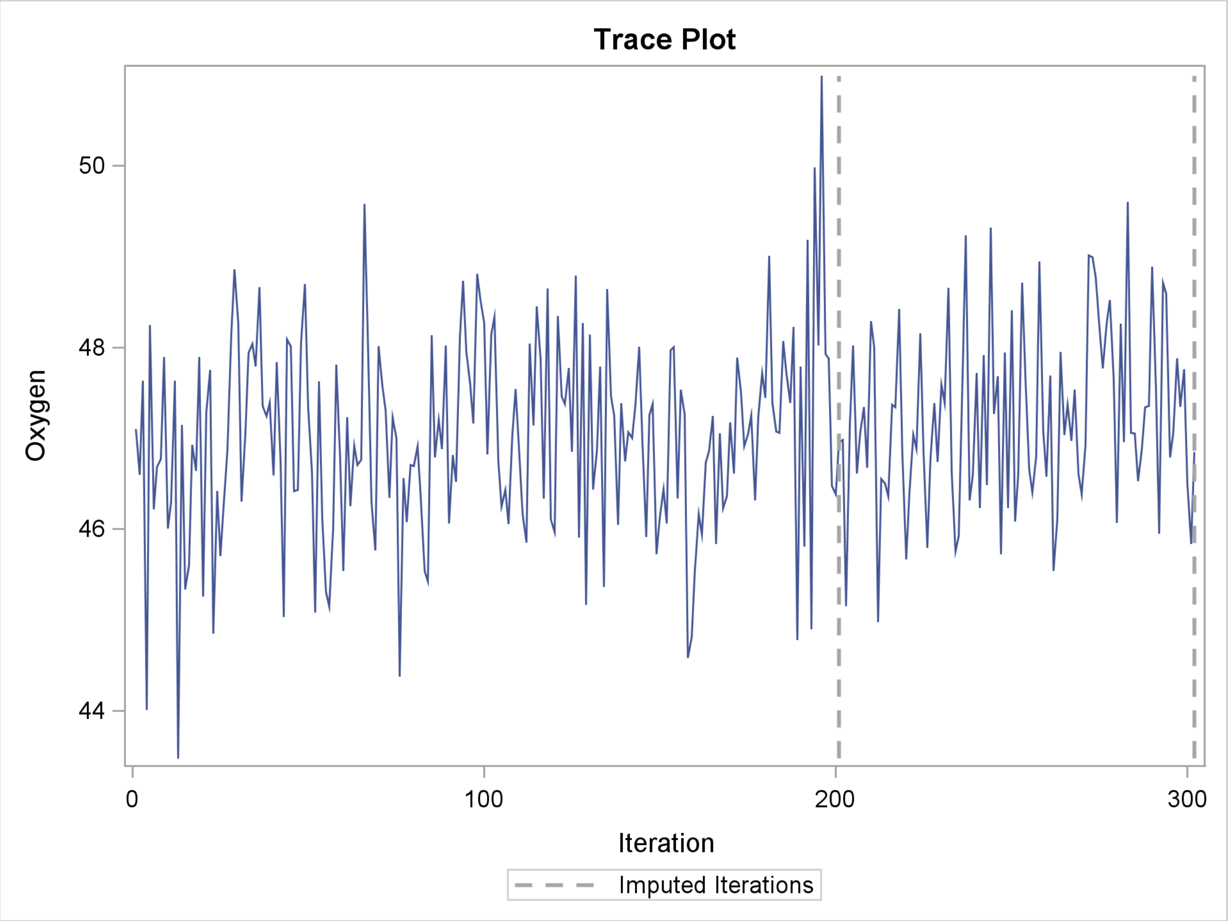
This example uses the MCMC method with a single chain. It also displays trace and autocorrelation plots to check convergence for the single chain.

The following statements use the MCMC method to create an iteration plot for the successive estimates of the mean of Oxygen. These statements also create an autocorrelation function plot for the variable Oxygen.

```
ods graphics on;
proc mi data=FitMiss seed=42037921 nimpute=2;
   mcmc plots=(trace(mean(Oxygen)) acf(mean(Oxygen)));
   var Oxygen RunTime RunPulse;
run;
ods graphics off;
```

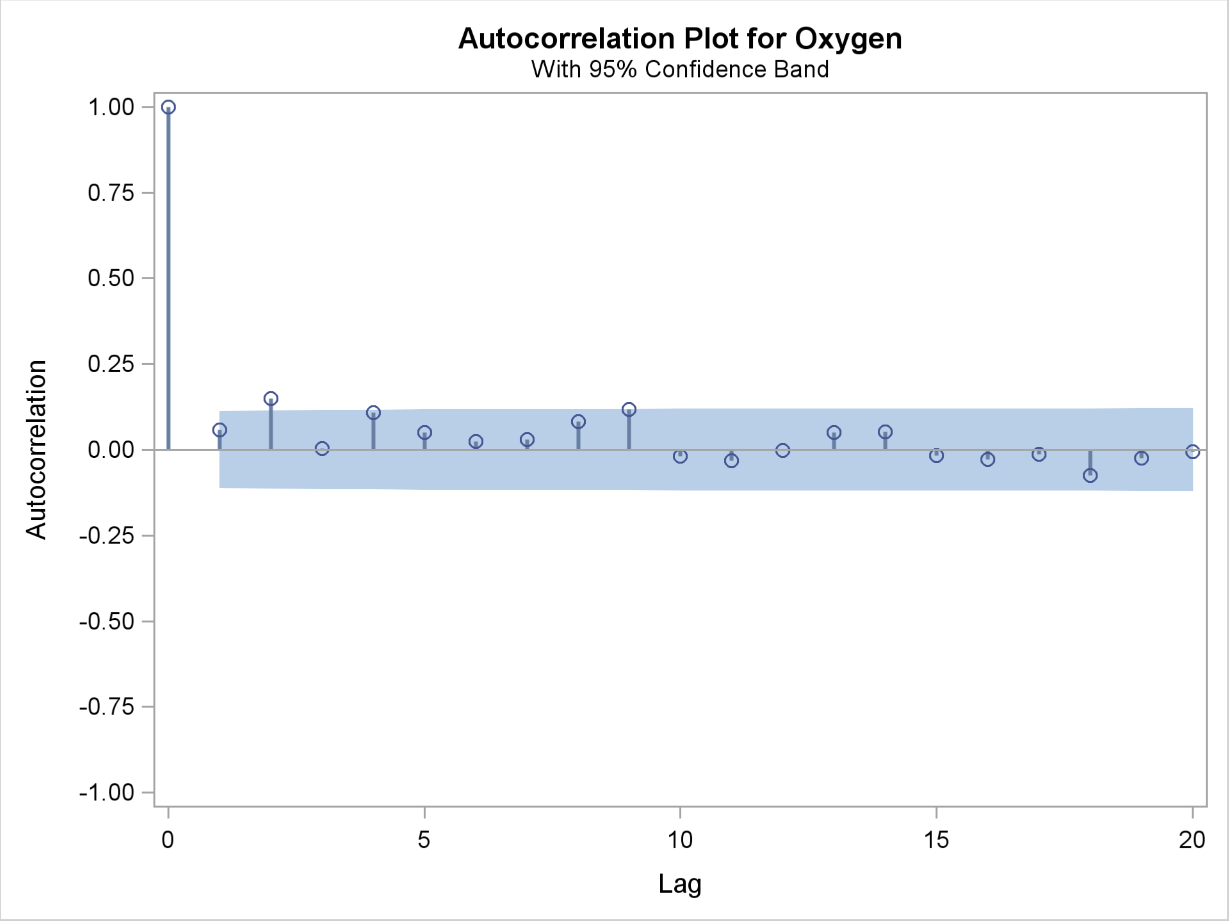
With the specified ODS GRAPHICS ON statement, the TRACE(MEAN(OXYGEN)) option in the PLOTS= option displays the trace plot of means for the variable Oxygen, as shown in Output 54.8.1. The dashed vertical lines indicate the imputed iterations—that is, the Oxygen values used in the imputations. The plot shows no apparent trends for the variable Oxygen.

Output 54.8.1 Trace Plot for Oxygen



The ACF(MEAN(OXYGEN)) option in the PLOTS= option displays the autocorrelation plot of means for the variable Oxygen, as shown in Output 54.8.2. The autocorrelation function plot shows no significant positive or negative autocorrelation.

Output 54.8.2 Autocorrelation Function Plot for Oxygen



You can also create plots for the worst linear function, the means of other variables, the variances of variables, and the covariances between variables. Alternatively, you can use the OUTITER option to save statistics such as the means, standard deviations, covariances, -2 log LR statistic, -2 log LR statistic of the posterior mode, and worst linear function from each iteration in an output data set. Then you can do a more in-depth trace (time series) analysis of the iterations with other procedures, such as PROC AUTOREG and PROC ARIMA in the *SAS/ETS User's Guide*.

For general information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS." For specific information about the graphics available in the MI procedure, see the section "ODS Graphics" on page 4424.

Example 54.9: Saving and Using Parameters for MCMC

This example uses the MCMC method with multiple chains as specified in Example 54.6. It saves the parameter values used for each imputation in an output data set of type EST called miest. This output data set can then be used to impute missing values in other similar input data sets. The following statements invoke the MI procedure and specify the MCMC method with multiple chains to create three imputations:

```
proc mi data=FitMiss seed=21355417 nimpute=6 mu0=50 10 180;
   mcmc chain=multiple initial=em outest=miest;
   var Oxygen RunTime RunPulse;
run;
```

The following statements list the parameters used for the imputations in Output 54.9.1. Note that the data set includes observations with _TYPE_='SEED' containing the seed to start the next random number generator.

```
proc print data=miest(obs=15);
   title 'Parameters for the Imputations';
run;
```

Output 54.9.1 OUTEST Data Set

```
                        Parameters for the Imputations

Obs _Imputation_ _TYPE_  _NAME_            Oxygen        RunTime       RunPulse

  1       1       SEED               825240167.00   825240167.00   825240167.00
  2       1       PARM                      46.77          10.47         169.41
  3       1       COV   Oxygen              30.59          -8.32         -50.99
  4       1       COV   RunTime             -8.32           2.90          17.03
  5       1       COV   RunPulse           -50.99          17.03         200.09
  6       2       SEED              1895925872.00  1895925872.00  1895925872.00
  7       2       PARM                      47.41          10.37         173.34
  8       2       COV   Oxygen              22.35          -4.44         -21.18
  9       2       COV   RunTime             -4.44           1.76           1.25
 10       2       COV   RunPulse           -21.18           1.25         125.67
 11       3       SEED               137653011.00   137653011.00   137653011.00
 12       3       PARM                      48.21          10.36         170.52
 13       3       COV   Oxygen              23.59          -5.25         -19.76
 14       3       COV   RunTime             -5.25           1.66           5.00
 15       3       COV   RunPulse           -19.76           5.00         110.99
```

The following statements invoke the MI procedure and use the INEST= option in the MCMC statement:

```
proc mi data=FitMiss mu0=50 10 180;
   mcmc inest=miest;
   var Oxygen RunTime RunPulse;
run;
```

The “Model Information” table in Output 54.9.2 describes the method used in the multiple imputation process. The remaining tables for the example are identical to the tables in Output 54.6.2, Output 54.6.4, Output 54.6.5, and Output 54.6.6 in Example 54.6.

Output 54.9.2 Model Information

```
                              The MI Procedure

                             Model Information

          Data Set                                  WORK.FITMISS
          Method                                    MCMC
          INEST Data Set                            WORK.MIEST
          Number of Imputations                     6
```

Example 54.10: Transforming to Normality

This example applies the MCMC method to the FitMiss data set in which the variable Oxygen is transformed. Assume that Oxygen is skewed and can be transformed to normality with a logarithmic transformation. The following statements invoke the MI procedure and specify the transformation. The TRANSFORM statement specifies the log transformation for Oxygen. Note that the values displayed for Oxygen in all of the results correspond to transformed values.

```
proc mi data=FitMiss seed=32937921 mu0=50 10 180 out=outex10;
   transform log(Oxygen);
   mcmc chain=multiple displayinit;
   var Oxygen RunTime RunPulse;
run;
```

The “Missing Data Patterns” table in Output 54.10.1 lists distinct missing data patterns with corresponding statistics for the FitMiss data. Note that the values of Oxygen shown in the tables are transformed values.

Output 54.10.1 Missing Data Patterns

```
                              The MI Procedure

                            Missing Data Patterns

                                 Run      Run
             Group     Oxygen    Time     Pulse        Freq     Percent

                 1     X         X        X              21       67.74
                 2     X         X        .               4       12.90
                 3     X         .        .               3        9.68
                 4     .         X        X               1        3.23
                 5     .         X        .               2        6.45

                        Transformed Variables: Oxygen

                           Missing Data Patterns

                       -----------------Group Means----------------
              Group             Oxygen         RunTime        RunPulse

                  1           3.829760       10.809524      171.666667
                  2           3.851813       10.137500               .
                  3           3.955298               .               .
                  4                  .       11.950000      176.000000
                  5                  .        9.885000               .

                       Transformed Variables: Oxygen
```

The “Variable Transformations” table in Output 54.10.2 lists the variables that have been transformed.

Output 54.10.2 Variable Transformations

```
                           Variable Transformations

                           Variable     _Transform_

                           Oxygen       LOG
```

The “Initial Parameter Estimates for MCMC” table in Output 54.10.3 displays the starting mean and covariance estimates used in the MCMC method.

Output 54.10.3 Initial Parameter Estimates

```
                       Initial Parameter Estimates for MCMC

       _TYPE_    _NAME_            Oxygen        RunTime       RunPulse

       MEAN                      3.846122      10.557605     171.382949
       COV       Oxygen          0.010827      -0.120891      -0.328772
       COV       RunTime        -0.120891       1.744580       3.011180
       COV       RunPulse       -0.328772       3.011180      82.747609

                          Transformed Variables: Oxygen
```

Output 54.10.4 displays variance information from the multiple imputation.

Output 54.10.4 Variance Information

```
                               Variance Information

                     -----------------Variance-----------------
         Variable         Between         Within          Total        DF

       * Oxygen       0.000016175       0.000401       0.000420    26.499
         RunTime         0.001762       0.065421       0.067536    27.118
         RunPulse        0.205979       3.116830       3.364004    25.222

                            * Transformed Variables

                               Variance Information

                              Relative       Fraction
                              Increase        Missing       Relative
              Variable     in Variance    Information     Efficiency

            * Oxygen          0.048454       0.047232       0.990642
              RunTime         0.032318       0.031780       0.993684
              RunPulse        0.079303       0.075967       0.985034

                             * Transformed Variables
```

Output 54.10.5 displays parameter estimates from the multiple imputation. Note that the parameter value of μ_0 has also been transformed using the logarithmic transformation.

Output 54.10.5 Parameter Estimates

```
                             Parameter Estimates

    Variable            Mean      Std Error    95% Confidence Limits        DF

  * Oxygen          3.845175       0.020494       3.8031       3.8873   26.499
    RunTime        10.560131       0.259876      10.0270      11.0932   27.118
    RunPulse      171.802181       1.834122     168.0264     175.5779   25.222

                          * Transformed Variables

                            Parameter Estimates

                                                            t for H0:
  Variable          Minimum         Maximum           Mu0    Mean=Mu0   Pr > |t|

* Oxygen           3.838599        3.848456      3.912023       -3.26     0.0030
  RunTime         10.493031       10.600498     10.000000        2.16     0.0402
  RunPulse       171.251777      172.498626    180.000000       -4.47     0.0001

                          * Transformed Variables
```

The following statements list the first 10 observations of the data set outmi in Output 54.10.6. Note that the values for Oxygen are in the original scale.

```
proc print data=outex10(obs=10);
   title 'First 10 Observations of the Imputed Data Set';
run;
```

Output 54.10.6 Imputed Data Set in Original Scale

```
                 First 10 Observations of the Imputed Data Set

                                                            Run
              Obs    _Imputation_     Oxygen    RunTime    Pulse

                1          1         44.6090    11.3700   178.000
                2          1         45.3130    10.0700   185.000
                3          1         54.2970     8.6500   156.000
                4          1         59.5710     7.1440   167.012
                5          1         49.8740     9.2200   170.092
                6          1         44.8110    11.6300   176.000
                7          1         38.5834    11.9500   176.000
                8          1         43.7376    10.8500   158.851
                9          1         39.4420    13.0800   174.000
               10          1         60.0550     8.6300   170.000
```

Note that the results in Output 54.10.6 can also be produced from the following statements without using a TRANSFORM statement. A transformed value of log(50)=3.91202 is used in the MU0= option.

```
data temp;
   set FitMiss;
   LogOxygen= log(Oxygen);
run;
proc mi data=temp seed=14337921 mu0=3.91202 10 180 out=outtemp;
   mcmc chain=multiple displayinit;
   var LogOxygen RunTime RunPulse;
run;
data outex10;
   set outtemp;
   Oxygen= exp(LogOxygen);
run;
```

Example 54.11: Multistage Imputation

This example uses two separate imputation procedures to complete the imputation process. In the first case, the MI procedure statements use the MCMC method to impute just enough missing values for a data set with an arbitrary missing pattern so that each imputed data set has a monotone missing pattern. In the second case, the MI procedure statements use a MONOTONE statement to impute missing values for data sets with monotone missing patterns.

The following statements are identical to those in Example 54.7. The statements invoke the MI procedure and specify the IMPUTE=MONOTONE option to create the imputed data set with a monotone missing pattern.

```
proc mi data=FitMiss seed=17655417 out=outex11;
   mcmc impute=monotone;
   var Oxygen RunTime RunPulse;
run;
```

The “Missing Data Patterns” table in Output 54.11.1 lists distinct missing data patterns with corresponding statistics. Here, an “X” means that the variable is observed in the corresponding group, a “.” means that the variable is missing and will be imputed to achieve the monotone missingness for the imputed data set, and an “O” means that the variable is missing and will not be imputed. The table also displays group-specific variable means.

Output 54.11.1 Missing Data Patterns

```
                              The MI Procedure

                            Missing Data Patterns

                                 Run      Run
            Group     Oxygen     Time     Pulse        Freq     Percent

                1     X          X        X              21       67.74
                2     X          X        O               4       12.90
                3     X          O        O               3        9.68
                4     .          X        X               1        3.23
                5     .          X        O               2        6.45

                           Missing Data Patterns

                   -----------------Group Means----------------
             Group           Oxygen         RunTime        RunPulse

                 1        46.353810       10.809524      171.666667
                 2        47.109500       10.137500               .
                 3        52.461667               .               .
                 4                .       11.950000      176.000000
                 5                .        9.885000               .
```

As shown in the table, the MI procedure needs to impute only three missing values from group 4 and group 5 to achieve a monotone missing pattern for the imputed data set. When the MCMC method is used to produce an imputed data set with a monotone missing pattern, tables of variance information and parameter estimates are not created.

The following statements impute one value for each missing value in the monotone missingness data set outex11:

```
proc mi data=outex11
        nimpute=1 seed=51343672
        out=outex11a;
   monotone reg;
   var Oxygen RunTime RunPulse;
   by _Imputation_;
run;
```

You can then analyze these data sets by using other SAS procedures and combine these results by using the MIANALYZE procedure. Note that the VAR statement is required with a MONOTONE statement to provide the variable order for the monotone missing pattern.

The “Model Information” table in Output 54.11.2 shows that a monotone method is used to generate imputed values in the first BY group.

Output 54.11.2 Model Information

```
----------------------------- Imputation Number=1 ------------------------------

                                The MI Procedure

                               Model Information

          Data Set                                    WORK.OUTEX11
          Method                                      Monotone
          Number of Imputations                       1
          Seed for random number generator            51343672
```

The “Monotone Model Specification” table in Output 54.11.3 describes methods and imputed variables in the imputation model. The MI procedure uses the regression method to impute the variables RunTime and RunPulse in the model.

Output 54.11.3 Monotone Model Specification

```
----------------------------- Imputation Number=1 ------------------------------

                          Monotone Model Specification

                                       Imputed
                         Method        Variables

                         Regression    RunTime RunPulse
```

The “Missing Data Patterns” table in Output 54.11.4 lists distinct missing data patterns with corresponding statistics. It shows a monotone missing pattern for the imputed data set.

Output 54.11.4 Missing Data Patterns

```
----------------------------- Imputation Number=1 ------------------------------

                              Missing Data Patterns

                               Run     Run
            Group    Oxygen    Time    Pulse        Freq     Percent

                1    X         X       X              22       70.97
                2    X         X       .               6       19.35
                3    X         .       .               3        9.68

                             Missing Data Patterns

                      -----------------Group Means----------------
             Group           Oxygen         RunTime        RunPulse

                 1        46.057479       10.861364      171.863636
                 2        46.745227       10.053333               .
                 3        52.461667               .               .
```

The following statements list the first 10 observations of the data set outex11a in Output 54.11.5:

```
proc print data=outex11a(obs=10);
   title 'First 10 Observations of the Imputed Data Set';
run;
```

Output 54.11.5 Imputed Data Set

```
                First 10 Observations of the Imputed Data Set

                                                            Run
             Obs    _Imputation_     Oxygen    RunTime     Pulse

               1          1          44.6090    11.3700    178.000
               2          1          45.3130    10.0700    185.000
               3          1          54.2970     8.6500    156.000
               4          1          59.5710     7.1569    169.914
               5          1          49.8740     9.2200    159.315
               6          1          44.8110    11.6300    176.000
               7          1          39.8345    11.9500    176.000
               8          1          45.3196    10.8500    151.252
               9          1          39.4420    13.0800    174.000
              10          1          60.0550     8.6300    170.000
```

This example presents an alternative to the full-data MCMC imputation, in which imputation of only a few missing values is needed to achieve a monotone missing pattern for the imputed data set. The example uses a monotone MCMC method that imputes fewer missing values in each iteration and achieves approximate stationarity in fewer iterations (Schafer 1997, p. 227). The example also demonstrates how to combine the monotone MCMC method with a method for monotone missing data, which does not rely on iterations of steps.

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Syntax Index

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