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SAS/STAT® 9.2 User's Guide

The GLMSELECT Procedure

(Book Excerpt)

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Chapter 42

The GLMSELECT Procedure

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Overview: GLMSELECT Procedure

The GLMSELECT procedure performs effect selection in the framework of general linear models. A variety of model selection methods are available, including the LASSO method of Tibshirani (1996) and the related LAR method of Efron et al. (2004). The procedure offers extensive capabilities for customizing the selection with a wide variety of selection and stopping criteria, from traditional and computationally efficient significance-level-based criteria to more computationally intensive validation-based criteria. The procedure also provides graphical summaries of the selection search.

The GLMSELECT procedure compares most closely to REG and GLM. The REG procedure supports a variety of model-selection methods but does not support a CLASS statement. The GLM procedure supports a CLASS statement but does not include effect selection methods. The GLMSELECT procedure fills this gap. GLMSELECT focuses on the standard independently and identically distributed general linear model for univariate responses and offers great flexibility for and insight into the model selection algorithm. GLMSELECT provides results (displayed tables, output data sets, and macro variables) that make it easy to take the selected model and explore it in more detail in a subsequent procedure such as REG or GLM.

Features

The main features of the GLMSELECT procedure are as follows:

- **Model Specification**
  - supports different parameterizations for classification effects
  - supports any degree of interaction (crossed effects) and nested effects
  - supports hierarchy among effects
  - supports partitioning of data into training, validation, and testing roles
  - supports constructed effects including spline and multimember effects

- **Selection Control**
  - provides multiple effect selection methods
  - enables selection from a very large number of effects (tens of thousands)
  - offers selection of individual levels of classification effects
  - provides effect selection based on a variety of selection criteria
  - provides stopping rules based on a variety of model evaluation criteria
  - provides leave-one-out and k-fold cross validation
- **Display and Output**
  - produces graphical representation of selection process
  - produces output data sets containing predicted values and residuals
  - produces an output data set containing the design matrix
  - produces macro variables containing selected models
  - supports parallel processing of BY groups
  - supports multiple SCORE statements

The GLMSELECT procedure supports the following effect selection methods. These methods are explained in detail in the section "Model-Selection Methods" on page 2717.

FORWARD	Forward selection. This method starts with no effects in the model and adds effects.
BACKWARD	Backward elimination. This method starts with all effects in the model and deletes effects.
STEPWISE	Stepwise regression. This is similar to the FORWARD method except that effects already in the model do not necessarily stay there.
LAR	Least angle regression. This method, like forward selection, starts with no effects in the model and adds effects. The parameter estimates at any step are "shrunk" when compared to the corresponding least squares estimates.
LASSO	This method adds and deletes parameters based on a version of ordinary least squares where the sum of the absolute regression coefficients is constrained.

Hybrid versions of LAR and LASSO are also supported. They use LAR or LASSO to select the model, but then estimate the regression coefficients by ordinary weighted least squares.

The GLMSELECT procedure is intended primarily as a model selection procedure and does not include regression diagnostics or other post-selection facilities such as hypothesis testing, testing of contrasts, and LS-means analyses. The intention is that you use PROC GLMSELECT to select a model or a set of candidate models. Further investigation of these models can be done by using these models in existing regression procedures.

Getting Started: GLMSELECT Procedure

The following data set contains salary and performance information for Major League Baseball players who played at least one game in both the 1986 and 1987 seasons, excluding pitchers. The salaries (*Sports Illustrated,* April 20, 1987) are for the 1987 season and the performance measures are from 1986 (Collier Books, *The 1987 Baseball Encyclopedia Update*).

```
data baseball;
  length name $ 18;
  length team $ 12;
  input name $ 1-18 nAtBat nHits nHome nRuns nRBI nBB
        yrMajor crAtBat crHits crHome crRuns crRbi crBB
        league $ division $ team $ position $ nOuts nAssts
        nError salary;
  label name="Player's Name"
     nAtBat="Times at Bat in 1986"
     nHits="Hits in 1986"
     nHome="Home Runs in 1986"
     nRuns="Runs in 1986"
     nRBI="RBIs in 1986"
     nBB="Walks in 1986"
     yrMajor="Years in the Major Leagues"
     crAtBat="Career times at bat"
     crHits="Career Hits"
     crHome="Career Home Runs"
     crRuns="Career Runs"
     crRbi="Career RBIs"
     crBB="Career Walks"
     league="League at the end of 1986"
     division="Division at the end of 1986"
     team="Team at the end of 1986"
     position="Position(s) in 1986"
     nOuts="Put Outs in 1986"
     nAssts="Assists in 1986"
     nError="Errors in 1986"
     salary="1987 Salary in $ Thousands";
     logSalary = log(Salary);
  datalines;
Allanson, Andy       293    66     1    30    29    14
                       1   293    66     1    30    29    14
                    American East Cleveland C 446 33 20 .
Ashby, Alan          315    81     7    24    38    39
                      14  3449   835    69   321   414   375
                    National West Houston C 632 43 10 475

... more lines ...

Wilson, Willie       631   170     9    77    44    31
                      11  4908  1457    30   775   357   249
                    American West KansasCity CF 408 4 3 1000
;
```

Suppose you want to investigate whether you can model the players' salaries for the 1987 season based on performance measures for the previous season. The aim is to obtain a parsimonious model that does not overfit this particular data, making it useful for prediction. This example shows how you can use PROC GLMSELECT as a starting point for such an analysis. Since the variation of salaries is much greater for the higher salaries, it is appropriate to apply a log transformation to the salaries before doing the model selection.

The following code selects a model with the default settings:

```
ods graphics on;
proc glmselect data=baseball plots=all;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor crAtBat crHits crHome crRuns crRbi
                    crBB league division nOuts nAssts nError
                  / details=all stats=all;
run;
ods graphics off;
```

PROC GLMSELECT performs effect selection where effects can contain classification variables that you specify in a CLASS statement. The "Class Level Information" table shown in Figure 42.1 lists the levels of the classification variables "division" and "league."

Figure 42.1 Class Level Information

The GLMSELECT Procedure

Class Level Information

Class	Levels	Values
league	2	American National
division	2	East West

When you specify effects that contain classification variables, the number of parameters is usually larger than the number of effects. The "Dimensions" table in Figure 42.2 shows the number of effects and the number of parameters considered.

Figure 42.2 Dimensions

Dimensions

Number of Effects	19
Number of Parameters	21

Figure 42.3 Model Information

The GLMSELECT Procedure

Data Set	WORK.BASEBALL
Dependent Variable	logSalary
Selection Method	Stepwise
Select Criterion	SBC
Stop Criterion	SBC
Effect Hierarchy Enforced	None

You find details of the default search settings in the "Model Information" table shown in Figure 42.3. The default selection method is a variant of the traditional stepwise selection where the decisions about what effects to add or drop at any step and when to terminate the selection are both based on the Schwarz Bayesian information criterion (SBC). The effect in the current model whose removal yields the maximal decrease in the SBC statistic is dropped provided this lowers the SBC value. Once no decrease in the SBC value can be obtained by dropping an effect in the model, the effect whose addition to the model yields the lowest SBC statistic is added and the whole process is repeated. The method terminates when dropping or adding any effect increases the SBC statistic.

Figure 42.4 Candidates for Entry at Step Two

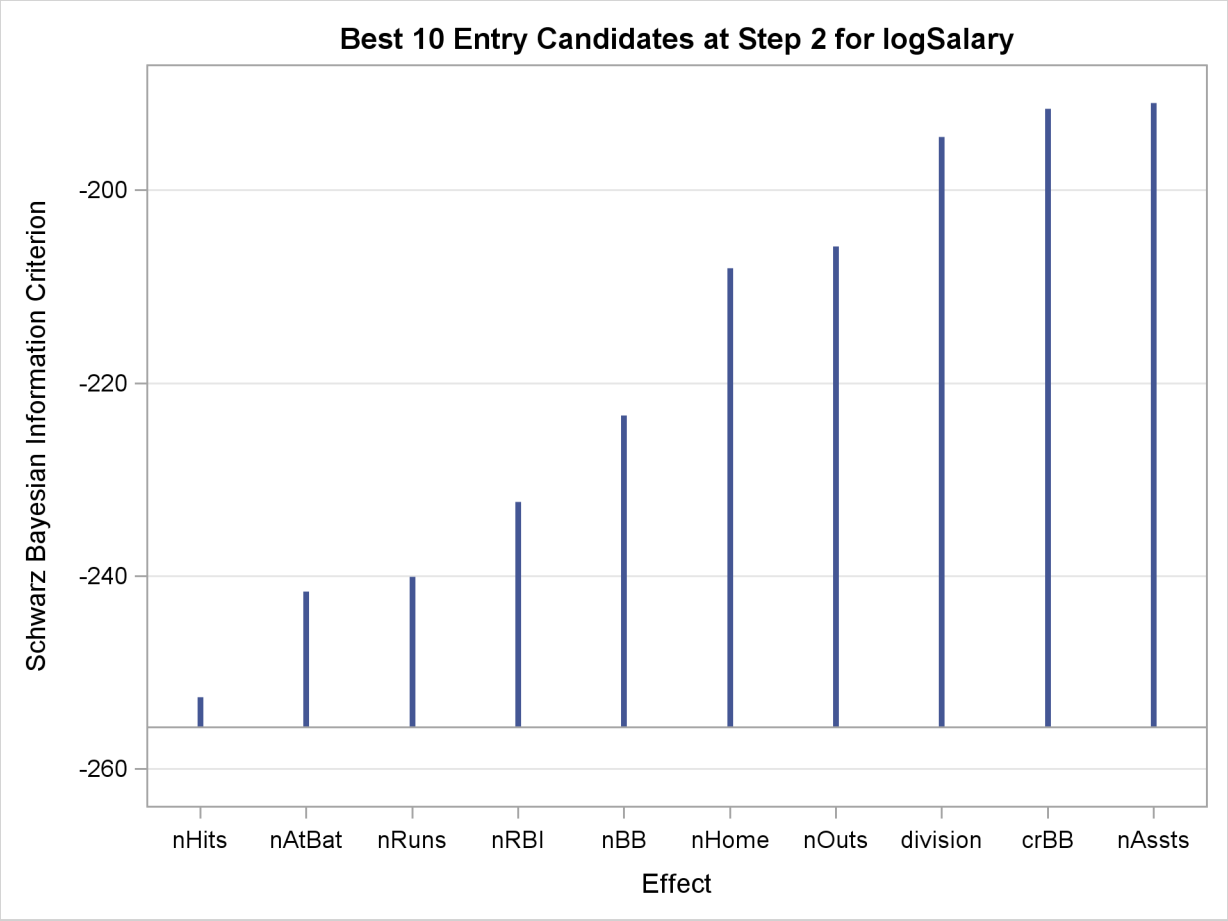
Best 10 Entry Candidates

Rank	Effect	SBC
1	nHits	-252.5794
2	nAtBat	-241.5789
3	nRuns	-240.1010
4	nRBI	-232.2880
5	nBB	-223.3741
6	nHome	-208.0565
7	nOuts	-205.8107
8	division	-194.4688
9	crBB	-191.5141
10	nAssts	-190.9425

The DETAILS=ALL option requests details of each step of the selection process. The "Best 10 Entry Candidates" table at each step shows the candidates for inclusion or removal at that step ranked from best to worst in terms of the selection criterion, which in this example is the SBC statistic. By default only the 10 best candidates are shown. Figure 42.4 shows the candidate table at step two.

To help in the interpretation of the selection process, you can use graphics supported by PROC GLMSELECT. You enable these graphical displays by specifying the ODS GRAPHICS statement. For general information about ODS Graphics, see Chapter 21, "Statistical Graphics Using ODS." With ODS Graphics enabled, the PLOTS=ALL option together with the DETAILS=STEPS option in the MODEL statement produces a needle plot view of the "Candidates" tables. The plot corresponding to the "Candidates" table at step two is shown in Figure 42.5. You can see that adding the effect "nHits" yields the smallest SBC value, and so this effect is added at step two.

Figure 42.5 Needle Plot of Entry Candidates at Step Two



The "Stepwise Selection Summary" table in Figure 42.6 shows the effect that was added or dropped at each step of the selection process together with fit statistics for the model at each step. The STATS=ALL option in the MODEL statement requests that all the available fit statistics are displayed. See the section "Criteria Used in Model Selection Methods" on page 2725 for descriptions and formulas. The criterion panel in Figure 42.7 provides a graphical view of the progression of these fit criteria as the selection process evolves. Note that none of these criteria has a local optimum before step five.

Figure 42.6 Selection Summary Table

The GLMSELECT Procedure

Stepwise Selection Summary

Step	Effect Entered	Effect Removed	Number Effects In	Number Parms In	Model R-Square	Adjusted R-Square
0	Intercept		1	1	0.0000	0.0000
1	crRuns		2	2	0.4187	0.4165
2	nHits		3	3	0.5440	0.5405
3	yrMajor		4	4	0.5705	0.5655
4		crRuns	3	3	0.5614	0.5581
5	nBB		4	4	0.5818	0.5770*

* Optimal Value Of Criterion

Stepwise Selection Summary

Step	Effect Entered	Effect Removed	AIC	AICC	BIC
0	Intercept		204.2238	204.2699	-60.6397
1	crRuns		63.5391	63.6318	-200.7872
2	nHits		1.7041	1.8592	-261.8807
3	yrMajor		-12.0208	-11.7873	-275.3333
4		crRuns	-8.5517	-8.3967	-271.9095
5	nBB		-19.0690*	-18.8356*	-282.1700*

* Optimal Value Of Criterion

Stepwise Selection Summary

Step	Effect Entered	Effect Removed	CP	SBC	PRESS
0	Intercept		375.9275	-57.2041	208.7381
1	crRuns		111.2315	-194.3166	123.9195
2	nHits		33.4438	-252.5794	97.6368
3	yrMajor		18.5870	-262.7322	92.2998
4		crRuns	22.3357	-262.8353	93.1482
5	nBB		11.3524*	-269.7804*	89.5434*

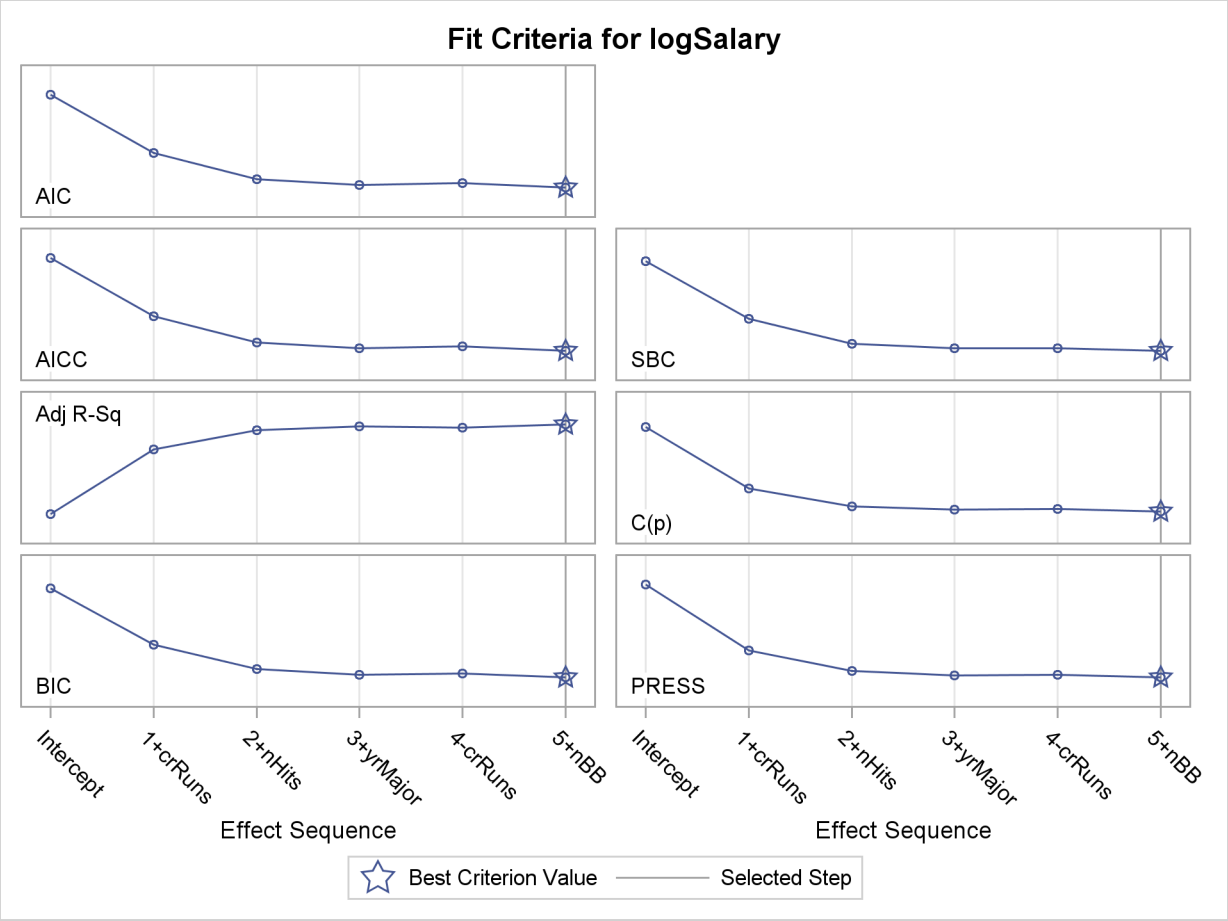
* Optimal Value Of Criterion

Stepwise Selection Summary

Step	Effect Entered	Effect Removed	ASE	F Value	Pr > F
0	Intercept		0.7877	0.00	1.0000
1	crRuns		0.4578	188.01	<.0001
2	nHits		0.3592	71.42	<.0001
3	yrMajor		0.3383	15.96	<.0001
4		crRuns	0.3454	5.44	0.0204
5	nBB		0.3294	12.62	0.0005

* Optimal Value Of Criterion

Figure 42.7 Criterion Panel



The stop reason and stop details tables in Figure 42.8 gives details of why the selection process terminated. This table shows that at step five the best add candidate, "division," and the best drop candidate, "nBB," yield models with SBC values of −268.6094 and −262.8353, respectively. Both of these values are larger than the current SBC value of −269.7804, and so the selection process stops at the model at step five.

Figure 42.8 Stopping Details

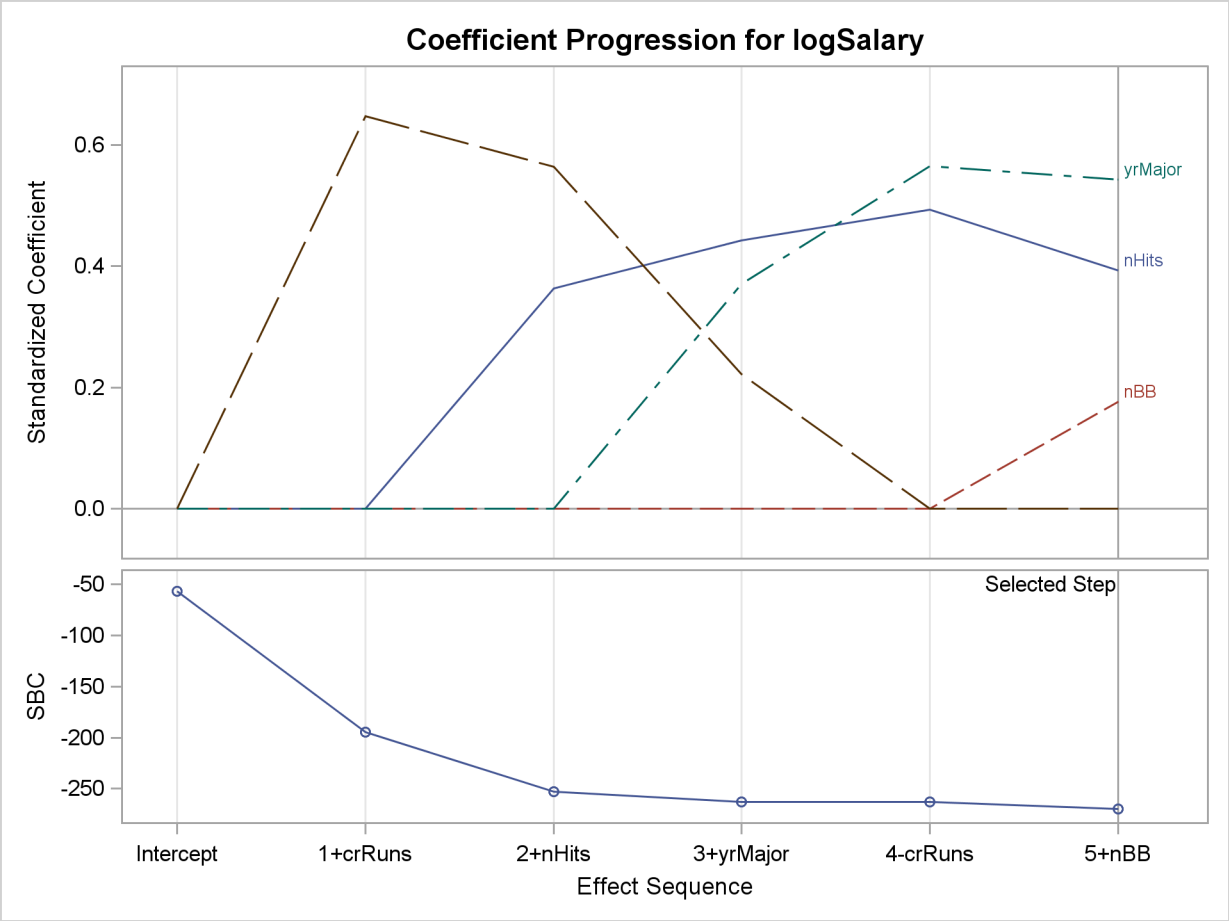
Selection stopped at a local minimum of the SBC criterion.

Stop Details

Candidate For	Effect	Candidate SBC		Compare SBC
Entry	division	−268.6094	>	−269.7804
Removal	nBB	−262.8353	>	−269.7804

The coefficient panel in Figure 42.9 enables you to visualize the selection process. In this plot, standardized coefficients of all the effects selected at some step of the stepwise method are plotted as a function of the step number. This enables you to assess the relative importance of the effects selected at any step of the selection process as well as providing information as to when effects entered the model. The lower plot in the panel shows how the criterion used to choose the selected model changes as effects enter or leave the model.

Figure 42.9 Coefficient Progression



The selected effects, analysis of variance, fit statistics, and parameter estimates tables shown in Figure 42.10 give details of the selected model.

Figure 42.10 Details of the Selected Model

```
                    The GLMSELECT Procedure
                        Selected Model

          The selected model is the model at the last step (Step 5).


                Effects: Intercept nHits nBB yrMajor
```

Figure 42.10 *continued*

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value
Model	3	120.52553	40.17518	120.12
Error	259	86.62820	0.33447	
Corrected Total	262	207.15373		

Root MSE	0.57834
Dependent Mean	5.92722
R-Square	0.5818
Adj R-Sq	0.5770
AIC	-19.06903
AICC	-18.83557
BIC	-282.17004
C(p)	11.35235
PRESS	89.54336
SBC	-269.78041
ASE	0.32938

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value
Intercept	1	4.013911	0.111290	36.07
nHits	1	0.007929	0.000994	7.98
nBB	1	0.007280	0.002049	3.55
yrMajor	1	0.100663	0.007551	13.33

PROC GLMSELECT provides you with the flexibility to use several selection methods and many fit criteria for selecting effects that enter or leave the model. You can also specify criteria to determine when to stop the selection process and to choose among the models at each step of the selection process. You can find continued exploration of the baseball data that uses a variety of these methods in Example 42.1.

Syntax: GLMSELECT Procedure

The following statements are available in PROC GLMSELECT:

PROC GLMSELECT < *options* > ;
BY *variables* ;
CLASS *variable* < *(v-options)* > < *variable* < *(v-options . . .)* > > < */ v-options* > < *options* > ;
EFFECT *name = effect-type (variables* < */ options* > *)* ;
FREQ *variable* ;
MODEL *variable* = < *effects* > < */ options* > ;
OUTPUT < *OUT=SAS-data-set* > < *keyword* < *=name* > > < *. . . keyword=name* > ;
PARTITION < *options* > ;
PERFORMANCE < *options* > ;
SCORE < *DATA=SAS-data-set* > < *OUT=SAS-data-set* > ;
WEIGHT *variable* ;

The PROC GLMSELECT statement invokes the procedure. All statements other than the MODEL statement are optional and multiple SCORE statements can be used. CLASS and EFFECT statements, if present, must precede the MODEL statement.

PROC GLMSELECT Statement

PROC GLMSELECT < *options* > ;

Table 42.1 lists the options available in the PROC GLMSELECT statement.

Table 42.1 PROC GLMSELECT Statement Options

Option	Description
Data Set Options	
DATA=	names a data set to use for the regression
MAXMACRO=	sets the maximum number of macro variables produced
TESTDATA=	names a data set containing test data
VALDATA=	names a data set containing validation data
ODS Graphics Options	
PLOTS=	produces ODS graphical displays
Other Options	
OUTDESIGN=	requests a data set containing the design matrix
NAMELEN=	sets the length of effect names in tables and output data sets
NOPRINT	suppresses displayed output including plots
SEED=	sets the seed used for pseudo-random number generation

Following are explanations of the options that you can specify in the PROC GLMSELECT statement (in alphabetical order).

DATA=*SAS-data-set*

names the SAS data set to be used by PROC GLMSELECT. If the DATA= option is not specified, PROC GLMSELECT uses the most recently created SAS data set. If the named data set contains a variable named _ROLE_, then this variable is used to assign observations for training, validation, and testing roles. See the section "Using Validation and Test Data" on page 2737 for details on using the _ROLE_ variable.

MAXMACRO=*n*

specifies the maximum number of macro variables with selected effects to create. By default, MAXMACRO=100. PROC GLMSELECT saves the list of selected effects in a macro variable, &_GLSIND. Say your input effect list consists of x1-x10. Then &_GLSIND would be set to x1 x3 x4 x10 if, for example, the first, third, fourth, and tenth effects were selected for the model. This list can be used, for example, in the model statement of a subsequent procedure. If you specify the OUTDESIGN= option in the PROC GLMSELECT statement, then PROC GLMSELECT saves the list of columns in the design matrix in a macro variable named &_GLSMOD.

With BY processing, one macro variable is created for each BY group, and the macro variables are indexed by the BY group number. The MAXMACRO= option can be used to either limit or increase the number of these macro variables when you are processing data sets with many BY groups.

With no BY processing, PROC GLMSELECT creates the following:

_GLSIND	selected effects
_GLSIND1	selected effects
_GLSMOD	design matrix columns
_GLSMOD1	design matrix columns
_GLSNUMBYS	number of BY groups
_GLSNUMMACROBYS	number of _GLSIND*i* macro variables actually made

With BY processing, PROC GLMSELECT creates the following:

_GLSIND	selected effects for BY group 1
_GLSIND1	selected effects for BY group 1
_GLSIND2	selected effects for BY group 2
.	
.	
.	
_GLSIND*m*	selected effects for BY group *m*, where a number is substituted for *m*
_GLSMOD	design matrix columns for BY group 1
_GLSMOD1	design matrix columns for BY group 1
_GLSMOD2	design matrix columns for BY group 2
.	
.	
.	
_GLSMOD*m*	design matrix columns for BY group *m*, where a number is substituted for *m*
_GLSNUMBYS	*n*, the number of BY groups
_GLSNUMMACROBYS	the number *m* of _GLSIND*i* macro variables actually made. This value can be less than _GLSNUMBYS = *n*, and it is less than or equal to the MAXMACRO= value.

See the section "Macro Variables Containing Selected Models" on page 2732 for further details.

NOPRINT

suppresses all displayed output including plots.

NAMELEN=*n*

specifies the length of effect names in tables and output data sets to be *n* characters long, where *n* is a value between 20 and 200 characters. The default length is 20 characters.

OUTDESIGN < (*options*) >< =SAS-data-set >

creates a data set that contains the design matrix. By default, the GLMSELECT procedure includes in the OUTDESIGN data set the **X** matrix corresponding to the parameters in the selected model. Two schemes for naming the columns of the design matrix are available. In the first scheme, names of the parameters are constructed from the parameter labels that appear in the "ParameterEstimates" table. This naming scheme is the default when you do not request BY processing and is not available when you do use BY processing. In the second scheme, the design matrix column names consist of a prefix followed by an index. The default naming prefix is "_X".

You can specify the following *options* in parentheses to control the contents of the OUTDESIGN data set:

ADDINPUTVARS

requests that all variables in the input data set be included in the OUTDESIGN= data set.

FULLMODEL
specifies that parameters corresponding to all the effects specified in the MODEL statement be included in the OUTDESIGN= data set. By default, only parameters corresponding to the selected model are included.

NAMES
produces a table associating columns in the OUTDESIGN data set with the labels of the parameters they represent.

PREFIX< =prefix >
requests that the design matrix column names consist of a prefix followed by an index. The default naming prefix is "_X". You can optionally specify a different prefix.

PARMLABELSTYLE=*options*
specifies how parameter names and labels are constructed for nested and crossed effects.

The following *options* are available:

INTERLACED < (SEPARATOR=*quoted string***) >**
forms parameter names and labels by positioning levels of classification variables and constructed effects adjacent to the associated variable or constructed effect name and using " * " as the delimiter for both crossed and nested effects. This style of naming parameters and labels is used in the TRANSREG procedure. You can request truncation of the classification variable names used in forming the parameter names and labels by using the CPREFIX= and LPREFIX= options in the CLASS statement. You can use the SEPARATOR= suboption to change the delimiter between the crossed variables in the effect. PARMLABELSTYLE=INTERLACED is not supported if you specify the SPLIT option in an EFFECT statement or a CLASS statement. The following are examples of the parameter labels in this style (Age is a continuous variable, Gender and City are classification variables):

```
Age
Gender male * City Beijing
City London * Age
```

SEPARATE
specifies that in forming parameter names and labels, the effect name appears before the levels associated with the classification variables and constructed effects in the effect. You can control the length of the effect name by using the NAMELEN= option in the PROC GLMSELECT statement. In forming parameter labels, the first level that is displayed is positioned so that it starts at the same offset in every parameter label—this enables you to easily distinguish the effect name from the levels when the parameter labels are displayed in a column in the "Parameter Estimates" table. This style of labeling is used in the GLM procedure and is the default if you do not specify the PARMLABELSTYLE option. The following are examples of the parameter labels in this style (Age is a continuous variable, Gender and City are classification variables):

```
Age
Gender*City male Beijing
Age*City    London
```

SEPARATECOMPACT
requests the same parameter naming and labeling scheme as PARMLABELSTYLE=SEPARATE except that the first level in the parameter label is separated from the effect name by a single blank. This style of labeling is used in the PLS procedure. The following are examples of the parameter labels in this style (Age is a continuous variable, Gender and City are classification variables):

```
Age
Gender*City male Beijing
Age*City London
```

PLOTS <(*global-plot-options***)> <= ***plot-request* *< (options) >* **>>**

PLOTS <(*global-plot-options***)> <= (***plot-request* *< (options) >* **<...** *plot-request* *< (options) >* **>)>**
controls the plots produced through ODS Graphics. When you specify only one plot request, you can omit the parentheses around the plot request. Here are some examples:

```
plots=all
plots=coefficients(unpack)
plots(unpack)=(criteria candidates)
```

You must enable ODS Graphics before requesting plots as shown in the following example. For general information about ODS Graphics, see Chapter 21, "Statistical Graphics Using ODS."

```
ods graphics on;

proc glmselect plots=all;
   model y = x1-x100;
run;

ods graphics off;
```

Global Plot Options

The *global-options* apply to all plots generated by the GLMSELECT procedure, unless it is altered by a *specific-plot-option*.

ENDSTEP=*n*
specifies that the step ranges shown on the horizontal axes of plots terminates at specified step. By default, the step range shown terminates at the final step of the selection process. If you specify the ENDSTEP= option as both a global plot option and a specific plot option, then the ENDSTEP= value on the specific plot is used.

LOGP | LOGPVALUE
requests that the natural logarithm of the entry and removal significance levels be displayed. This option is ignored if the select criterion is not significance level.

MAXSTEPLABEL=*n*
specifies the maximum number of characters beyond which labels of effects on plots are truncated.

MAXPARMLABEL= *n*
specifies the maximum number of characters beyond which parameter labels on plots are truncated.

STARTSTEP=*n*
specifies that the step ranges shown on the horizontal axes of plots start at the specified step. By default, the step range shown starts at the initial step of the selection process. If you specify the STARTSTEP= option both as a global plot option and a specific plot option, then the STARTSTEP= value on the specific plot is used.

STEPAXIS=EFFECT | NORMB | NUMBER
specifies the horizontal axis to be used on the plots, where this axis represents the sequence of entering or departing effects.

STEPAXIS=EFFECT
requests that each step be labeled by a prefix followed by the name of the effect that enters or leaves at that step. The prefix consists of the step number followed by a "+" sign or a "-" sign depending on whether the effect enters or leaves at that step.

STEPAXIS=NORMB
is valid only with LAR and LASSO selection methods and requests that the horizontal axis value at step i be the L1 norm of the parameters at step i, normalized by the L1 norm of the parameters at the final step.

STEPAXIS=NUMBER
requests that each step be labeled by the step number.

UNPACK
suppresses paneling. By default, multiple plots can appear in some output panels. Specify UNPACK to get each plot individually. You can also specify UNPACK as a suboption with CRITERIA and COEFFICIENTS.

Specific Plot Options

The following listing describes the specific plots and their options.

ALL
requests that all default plots be produced. Note that candidate plots are produced only if you specify DETAILS=STEPS or DETAILS=ALL in the MODEL statement.

ASE | ASEPLOT <(*aseplot-option***)>**
plots the progression of the average square error on the training data, and the test and validation data whenever these data are provided with the TESTDATA= and VALDATA= options or are produced by using a PARTITION statement. The following *aseplot-option* option is available:

STEPAXIS=EFFECT | NORMB | NUMBER
specifies the horizontal axis to be used.

CANDIDATES | CANDIDATESPLOT <(*candidatesplot-options*)>
produces a needle plot of the select criterion values for the candidates for entry or removal at each step of the selection process, ordered from best to worst. Candidates plots are not available if you specify SELECTION=NONE, SELECTION=LAR, or SELECTION=LASSO in the MODEL statement, or if you have not specified DETAILS=ALL or DETAILS=STEPS in the MODEL statement. The following *candidatesplot-options* are available:

LOGP | LOGPVALUE
requests that the natural logarithm of the entry and removal significance levels be displayed. This option is ignored if the select criterion is not significance level.

SHOW=*number*
specifies the maximum number of candidates displayed at each step. The default is SHOW=10.

COEFFICIENTS | COEFFICIENTPANEL <(*coefficientPanel-options*)>
plots a panel of two plots. The upper plot shows the progression of the parameter values as the selection process proceeds. The lower plot shows the progression of the CHOOSE= criterion. If no choose criterion is in effect, then the AICC criterion is displayed. The following *coefficientPanel-options* are available:

LABELGAP=*percentage*
specifies the percentage of the vertical axis range that forms the minimum gap between successive parameter labels at the final step of the coefficient progression plot. If the values of more than one parameter at the final step are closer than this gap, then the labels on all but one of these parameters is suppressed. The default value is LABELGAP=5. Planned enhancements to the automatic label collision avoidance algorithm will obviate the need for this option in future releases of the GLMSELECT procedure.

LOGP | LOGPVALUE
requests that the natural logarithm of the entry and removal significance levels be displayed if the choose criterion is significance level.

STEPAXIS=EFFECT | NORMB | NUMBER
specifies the horizontal axis to be used.

UNPACK | UNPACKPANEL
displays the coefficient progression and the choose criterion progression in separate plots.

CRITERIA | CRITERIONPANEL <(*criterionPanel-options*)>
plots a panel of model fit criteria. The criteria that are displayed are ADJRSQ, AIC, AICC, and SBC, as well as any other criteria that are named in the CHOOSE=, SELECT=, STOP=, or STATS= option in the MODEL statement. The following *criterionPanel-options* are available:

STEPAXIS=EFFECT | NORMB | NUMBER
specifies the horizontal axis to be used.

UNPACK | UNPACKPANEL
displays each criterion progression on a separate plot.

NONE
suppresses all plots.

SEED=*number*
specifies an integer used to start the pseudo-random number generator for random cross validation and random partitioning of data for training, testing, and validation. If you do not specify a seed, or if you specify a value less than or equal to zero, the seed is generated from reading the time of day from the computer's clock.

TESTDATA=*SAS-data-set*
names a SAS data set containing test data. This data set must contain all the variables specified in the MODEL statement. Furthermore, when a BY statement is used and the TESTDATA=data set contains any of the BY variables, then the TESTDATA= data set must also contain all the BY variables sorted in the order of the BY variables. In this case, only the test data for a specific BY group is used with the corresponding BY group in the analysis data. If the TESTDATA= data set contains none of the BY variables, then the entire TESTDATA = data set is used with each BY group of the analysis data.

If you specify a TESTDATA=data set, then you cannot also reserve observations for testing by using a PARTITION statement.

VALDATA=*SAS-data-set*
names a SAS data set containing validation data. This data set must contain all the variables specified in the MODEL statement. Furthermore, when a BY statement is used and the VALDATA=data set contains any of the BY variables, then the VALDATA= data set must also contain all the BY variables sorted in the order of the BY variables. In this case, only the validation data for a specific BY group are used with the corresponding BY group in the analysis data. If the VALDATA= data set contains none of the BY variables, then the entire VALDATA = data set is used with each BY group of the analysis data.

If you specify a VALDATA=data set, then you cannot also reserve observations for validation by using a PARTITION statement.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC GLMSELECT to obtain separate analyses on observations in groups defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. The *variables* are one or more variables in the input data set.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the BY statement option NOTSORTED or DESCENDING in the BY statement for the GLMSELECT procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in base SAS software).

For more information about the BY statement, see *SAS Language Reference: Contents*. For more information about the DATASETS procedure, see the *Base SAS Procedures Guide*.

CLASS Statement

The CLASS statement names the classification variables to be used in the analysis. The CLASS statement must precede the MODEL statement.

The following options can be specified after a slash (/):

DELIMITER=*quoted character*
specifies the delimiter that is used between levels of classification variables in building parameter names and lists of class level values. The default if you do not specify DELIMITER= is a space. This option is useful if the levels of a classification variable contain embedded blanks.

SHOW | SHOWCODING
requests a table for each classification variable that shows the coding used for that variable.

You can specify various *v-options* for each variable by enclosing them in parentheses after the variable name. You can also specify global *v-options* for the CLASS statement by placing them after a slash (/). Global *v-options* are applied to all the variables specified in the CLASS statement. If you specify more than one CLASS statement, the global *v-options* specified in any one CLASS statement apply to all CLASS statements. However, individual CLASS variable *v-options* override the global *v-options*.

The following *v-options* are available:

CPREFIX=*n*

specifies that, at most, the first n characters of a CLASS variable name be used in creating names for the corresponding design variables. The default is $32 - \min(32, \max(2, f))$, where f is the formatted length of the CLASS variable. The CPREFIX= applies only when you specify the PARMLABELSTYLE=INTERLACED option in the PROC GLMSELECT statement.

DESCENDING

DESC

reverses the sorting order of the classification variable.

LPREFIX=*n*

specifies that, at most, the first n characters of a CLASS variable label be used in creating labels for the corresponding design variables. The default is $256 - \min(256, \max(2, f))$, where f is the formatted length of the CLASS variable. The LPREFIX= applies only when you specify the PARMLABELSTYLE=INTERLACED option in the PROC GLMSELECT statement.

MISSING

allows missing value ('.' for a numeric variable and blanks for a character variables) as a valid value for the CLASS variable.

ORDER=DATA | FORMATTED | FREQ | INTERNAL

specifies the sorting order for the levels of classification variables. This ordering determines which parameters in the model correspond to each level in the data, so the ORDER= option might be useful when you use the CONTRAST or ESTIMATE statement. If ORDER=FORMATTED for numeric variables for which you have supplied no explicit format, the levels are ordered by their internal values. Note that this represents a change from previous releases for how class levels are ordered. Before SAS 8, numeric class levels with no explicit format were ordered by their BEST12. formatted values, and in order to revert to the previous ordering you can specify this format explicitly for the affected classification variables. The change was implemented because the former default behavior for ORDER=FORMATTED often resulted in levels not being ordered numerically and usually required the user to intervene with an explicit format or ORDER=INTERNAL to get the more natural ordering. The following table shows how PROC GLMSELECT interprets values of the ORDER= option.

Value of ORDER=	Levels Sorted By
DATA	order of appearance in the input data set
FORMATTED	external formatted value, except for numeric variables with no explicit format, which are sorted by their unformatted (internal) value
FREQ	descending frequency count; levels with the most observations come first in the order
INTERNAL	unformatted value

By default, ORDER=FORMATTED. For FORMATTED and INTERNAL, the sort order is machine dependent.

For more information about sorting order, refer to the chapter on the SORT procedure in the *Base SAS Procedures Guide* and the discussion of BY-group processing in *SAS Language Reference: Concepts*.

PARAM=*keyword*

specifies the parameterization method for the classification variable or variables. Design matrix columns are created from CLASS variables according to the following coding schemes. The default is PARAM=GLM. If PARAM=ORTHPOLY or PARAM=POLY, and the CLASS levels are numeric, then the ORDER= option in the CLASS statement is ignored, and the internal, unformatted values are used. See the section "CLASS Variable Parameterization" on page 2728 for further details.

EFFECT	specifies effect coding.
GLM	specifies less-than-full-rank, reference-cell coding; this option can be used only as a global option.
ORDINAL THERMOMETER	specifies the cumulative parameterization for an ordinal CLASS variable.
POLYNOMIAL POLY	specifies polynomial coding.
REFERENCE REF	specifies reference-cell coding.
ORTHEFFECT	orthogonalizes PARAM=EFFECT.
ORTHORDINAL	
ORTHOTHERM	orthogonalizes PARAM=ORDINAL.
ORTHPOLY	orthogonalizes PARAM=POLYNOMIAL.
ORTHREF	orthogonalizes PARAM=REFERENCE.

The EFFECT, POLYNOMIAL, REFERENCE, and ORDINAL schemes and their orthogonal parameterizations are full rank. The REF= option in the CLASS statement determines the reference level for the EFFECT and REFERENCE schemes and their orthogonal parameterizations.

REF=*'level'* | *keyword*

specifies the reference level for PARAM=EFFECT, PARAM=REFERENCE, and their orthogonalizations. For an individual (but not a global) variable REF= *option*, you can specify the *level* of the variable to use as the reference level. For a global or individual variable REF= *option*, you can use one of the following *keywords*. The default is REF=LAST.

FIRST	designates the first-ordered level as reference.
LAST	designates the last-ordered level as reference.

SPLIT

requests that the columns of the design matrix corresponding to any effect containing a split classification variable can be selected to enter or leave a model independently of the other design columns of that effect. For example, suppose a variable named temp has three levels with values "hot," "warm," and "cold," and a variable named sex has two levels with values "M" and "F" are used in a PROC GLMSELECT job as follows:

```
proc glmselect;
   class temp sex/split;
   model depVar = sex sex*temp;
run;
```

As both the classification variables are split, the two effects named in the MODEL statement are split into eight independent effects. The effect "sex" is split into two effects labeled "sex_M" and "sex_F". The effect "sex*temp" is split into six effects labeled "sex_M*temp_hot", "sex_F*temp_hot", "sex_M*temp_warm", "sex_F*temp_warm", "sex_M*temp_cold", and "sex_F*temp_cold", and the previous PROC GLMSELECT step is equivalent to the following:

```
proc glmselect;
   model depVar =  sex_M sex_F sex_M*temp_hot  sex_F*temp_hot
                               sex_M*temp_warm sex_F*temp_warm
                               sex_M*temp_cold sex_F*temp_cold;
run;
```

The split option can be used on individual classification variables. For example, consider the following PROC GLMSELECT step:

```
proc glmselect;
   class temp(split) sex;
   model depVar = sex sex*temp;
run;
```

In this case the effect "sex" is not split and the effect "sex*temp" is split into three effects labeled "sex*temp_hot", "sex*temp_warm", and "sex*temp_cold". Furthermore each of these three split effects now has two parameters corresponding to the two levels of "sex," and the PROC GLMSELECT step is equivalent to the following:

```
proc glmselect;
   class sex;
   model depVar = sex sex*temp_hot sex*temp_warm sex*temp_cold;
run;
```

EFFECT Statement

EFFECT *effect-specification* **;**

The experimental EFFECT statement enables you to construct special collections of columns for the **X** matrix in your model. These collections are referred to as *constructed effects* to distinguish them from the usual model effects formed from continuous or classification variables.

For details about the syntax of the EFFECT statement and how columns of constructed effects are computed, see the section "Constructed Effects and the EFFECT Statement (Experimental)" on page 377 of Chapter 18, "Shared Concepts and Topics."

FREQ Statement

FREQ *variable* **;**

The variable specified in the FREQ statement identifies a variable in the input data set containing the frequency of occurrence of each observation. PROC GLMSELECT treats each observation as if it appears n times, where n is the value of the FREQ variable for the observation. If it is not an integer, the frequency value is truncated to an integer. If it is less than 1 or if it is missing, the observation is not used.

MODEL Statement

MODEL *dependent*=< *effects* > / < *options* > **;**

The MODEL statement names the dependent variable and the explanatory effects, including covariates, main effects, constructed effects, interactions, and nested effects; see the section "Specification of Effects" on page 2486 in Chapter 39, "The GLM Procedure," for more information. If you omit the explanatory effects, the procedure fits an intercept-only model.

After the keyword MODEL, the dependent (response) variable is specified, followed by an equal sign. The explanatory effects follow the equal sign.

Table 42.2 lists the options available in the MODEL statement.

Table 42.2 MODEL Statement Options

Option	Description
CVDETAILS=	requests details when cross validation is used
CVMETHOD=	specifies how subsets for cross validation are formed
DETAILS=	specifies details to be displayed
HIERARCHY=	specifies hierarchy of effects to impose
NOINT	specifies models without an explicit intercept
ORDERSELECT	requests that parameter estimates be displayed in the order in which the parameters first entered the model

Table 42.2 *continued*

Option	Description
SELECTION=	specifies model selection method
STATS=	specifies additional statistics to be displayed
SHOWPVALUES	requests p-values in "ANOVA" and "Parameter Estimates" tables
STB	adds standardized coefficients to "Parameter Estimates" tables

You can specify the following options in the MODEL statement after a slash (/):

CVDETAILS=ALL

CVDETAILS=COEFFS

CVDETAILS=CVPRESS

specifies the details produced when cross validation is requested as the CHOOSE=, SELECT=, or STOP= criterion in the MODEL statement. If n-fold cross validation is being used, then the training data are subdivided into n parts, and at each step of the selection process, models are obtained on each of the n subsets of the data obtained by omitting one of these parts. CVDETAILS=COEFFS requests that the parameter estimates obtained for each of these n subsets be included in the parameter estimates table. CVDETAILS=CVPRESS requests a table containing the predicted residual sum of squares of each of these models scored on the omitted subset. CVDETAILS=ALL requests both CVDETAILS=COEFFS and CVDETAILS=CVPRESS. If DETAILS=STEPS or DETAILS=ALL has been specified in the MODEL statement, then the requested CVDETAILS are produced for every step of the selection process.

CVMETHOD=BLOCK < (*n*) >

CVMETHOD=RANDOM < (*n*) >

CVMETHOD=SPLIT < (*n*) >

CVMETHOD=INDEX (*variable*)

specifies how the training data are subdivided into n parts when you request n-fold cross validation by using any of the CHOOSE=CV, SELECT=CV, and STOP=CV suboptions of the SELECTION= option in the MODEL statement.

- CVMETHOD=BLOCK requests that parts be formed of n blocks of consecutive training observations.
- CVMETHOD=SPLIT requests that the ith part consist of training observations $i, i + n, i + 2n, \ldots$.
- CVMETHOD=RANDOM assigns each training observation randomly to one of the n parts.
- CVMETHOD=INDEX(*variable*) assigns observations to parts based on the formatted value of the named variable. This input data set variable is treated as a classification variable and the number of parts n is the number of distinct levels of this variable. By optionally naming this variable in a CLASS statement you can use the CLASS statement options ORDER= and MISSING to control how the levelization of this variable is done.

n defaults to 5 with CVMETHOD=BLOCK, CVMETHOD=SPLIT, or CVMETHOD=RANDOM. If you do not specify the CVMETHOD= option, then the CVMETHOD defaults to CVMETHOD=RANDOM(5).

DETAILS=*level*

DETAILS=STEPS <(*step options***)>**

specifies the level of detail produced, where *level* can be ALL, STEPS, or SUMMARY. The default if the DETAILS= option is omitted is DETAILS=SUMMARY. The DETAILS=ALL option produces the following:

- entry and removal statistics for each variable selected in the model building process
- ANOVA, fit statistics, and parameter estimates
- entry and removal statistics for the top 10 candidates for inclusion or exclusion at each step
- a selection summary table

The DETAILS=SUMMARY option produces only the selection summary table.

The option DETAILS=STEPS <(*step options*)> provides the step information and the selection summary table. The following options can be specified within parentheses after the DETAILS=STEPS option:

ALL

requests ANOVA, fit statistics, parameter estimates, and entry or removal statistics for the top 10 candidates for inclusion or exclusion at each selection step.

ANOVA

requests ANOVA at each selection step.

FITSTATISTICS | FITSTATS | FIT

requests fit statistics at each selection step. The default set of statistics includes all of the statistics named in the CHOOSE=, SELECT=, and STOP= suboptions specified in the MODEL statement SELECTION= option, but additional statistics can be requested with the STATS= option in the MODEL statement.

PARAMETERESTIMATES | PARMEST

requests parameter estimates at each selection step.

CANDIDATES <(SHOW= ALL | *n*)>

requests entry or removal statistics for the best *n* candidate effects for inclusion or exclusion at each step. If you specify SHOW=ALL, then all candidates are shown. If SHOW= is not specified. then the best 10 candidates are shown. The entry or removal statistic is the statistic named in the SELECT= option that is specified in the MODEL statement SELECTION= option.

HIERARCHY=*keyword*

HIER=*keyword*

specifies whether and how the model hierarchy requirement is applied. This option also controls whether a single effect or multiple effects are allowed to enter or leave the model in

one step. You can specify that only classification effects, or both classification and continuous effects, be subject to the hierarchy requirement. The HIERARCHY= option is ignored unless you also specify one of the following options: SELECTION=FORWARD, SELECTION=BACKWARD, or SELECTION=STEPWISE.

Model hierarchy refers to the requirement that for any term to be in the model, all model effects contained in the term must be present in the model. For example, in order for the interaction A*B to enter the model, the main effects A and B must be in the model. Likewise, neither effect A nor effect B can leave the model while the interaction A*B is in the model.

The keywords you can specify in the HIERARCHY= option are as follows:

NONE
specifies that model hierarchy not be maintained. Any single effect can enter or leave the model at any given step of the selection process.

SINGLE
specifies that only one effect enter or leave the model at one time, subject to the model hierarchy requirement. For example, suppose that the model contains the main effects A and B and the interaction A*B. In the first step of the selection process, either A or B can enter the model. In the second step, the other main effect can enter the model. The interaction effect can enter the model only when both main effects have already entered. Also, before A or B can be removed from the model, the A*B interaction must first be removed. All effects (CLASS and interval) are subject to the hierarchy requirement.

SINGLECLASS
is the same as HIERARCHY=SINGLE except that only CLASS effects are subject to the hierarchy requirement.

The default value is HIERARCHY=NONE.

NOINT
suppresses the intercept term that is otherwise included in the model.

ORDERSELECT
specifies that for the selected model, effects be displayed in the order in which they first entered the model. If you do not specify the ORDERSELECT option, then effects in the selected model are displayed in the order in which they appeared in the MODEL statement.

SELECTION=*method* < (method options) >
specifies the method used to select the model, optionally followed by parentheses enclosing options applicable to the specified method. The default if the SELECTION= option is omitted is SELECTION=STEPWISE.

The following methods are available and are explained in detail in the section "Model-Selection Methods" on page 2717.

NONE	no model selection
FORWARD	forward selection. This method starts with no effects in the model and adds effects.

BACKWARD — backward elimination. This method starts with all effects in the model and deletes effects.

STEPWISE — stepwise regression. This is similar to the FORWARD method except that effects already in the model do not necessarily stay there.

LAR — least angle regression. This method, like forward selection, starts with no effects in the model and adds effects. The parameter estimates at any step are "shrunk" when compared to the corresponding least squares estimates. If the model contains classification variables, then these classification variables are split. See the SPLIT option in the CLASS statement for details.

LASSO — This method adds and deletes parameters based on a version of ordinary least squares where the sum of the absolute regression coefficients is constrained. If the model contains classification variables, then these classification variables are split. See the SPLIT option in the CLASS statement for details.

Table 42.3 lists the applicable suboptions for each of these methods.

Table 42.3 Applicable SELECTION= Options by Method

Option	FORWARD	BACKWARD	STEPWISE	LAR	LASSO
STOP =	x	x	x	x	
CHOOSE =	x	x	x	x	
STEPS =	x	x	x	x	
MAXSTEPS =	x	x	x	x	
SELECT =	x	x	x		
INCLUDE =	x	x	x		
SLENTRY =	x		x		
SLSTAY =		x	x		
DROP =			x		
LSCOEFFS					x

The syntax of the suboptions that you can specify in parentheses after the SELECTION= option method follows. Note that, as described in Table 42.3, not all selection suboptions are applicable to every SELECTION= method.

CHOOSE=*criterion*

chooses from the list of models at the steps of the selection process the model that yields the best value of the specified criterion. If the optimal value of the specified criterion occurs for models at more than one step, then the model with the smallest number of parameters is chosen. If you do not specify the CHOOSE= option, then the model selected is the model at the final step in the selection process.

The criteria that you can specify in the CHOOSE= option are shown in Table 42.4. See the section "Criteria Used in Model Selection Methods" on page 2725 for more detailed descriptions of these criteria.

Table 42.4 Criteria for the CHOOSE= Option

Option	Criteria
ADJRSQ	Adjusted R-square statistic
AIC	Akaike information criterion
AICC	Corrected Akaike information criterion
BIC	Sawa Bayesian information criterion
CP	Mallows C(p) statistic
CV	Predicted residual sum of square with k-fold cross validation
PRESS	Predicted residual sum of squares
SBC	Schwarz Bayesian information criterion
VALIDATE	Average square error for the validation data

For ADJRSQ the chosen value is the largest one; for all other criteria, the smallest value is chosen. You can use the VALIDATE option only if you have specified a VALDATA= data set in the PROC GLMSELECT statement or if you have reserved part of the input data for validation by using either a PARTITION statement or a _ROLE_ variable in the input data.

DROP=*policy*

specifies when effects are eligible to be dropped in the STEPWISE method. Valid values for policy are BEFOREADD and COMPETITIVE.

If you specify DROP=BEFOREADD, then effects currently in the model are examined to see if any meet the requirements to be removed from the model. If so, the effect that gives the best value of the removal criterion is dropped from the model and the stepwise method proceeds to the next step. Only when no effect currently in the model meets the requirement to be removed from the model are any effects added to the model.

DROP=COMPETITIVE can be specified only if the SELECT= criterion is not SL. If you specify DROP=COMPETITIVE, then the SELECT= criterion is evaluated for all models where an effect currently in the model is dropped or an effect not yet in the model is added. The effect whose removal or addition to the model yields the maximum improvement to the SELECT= criterion is dropped or added.

The default if you do not specify DROP= suboption with the STEPWISE method is DROP=BEFOREADD. If SELECT=SL, then this yields the traditional stepwise method as implemented in PROC REG.

INCLUDE=*n*

forces the first n effects listed in the MODEL statement to be included in all models. The selection methods are performed on the other effects in the MODEL statement. The INCLUDE= option is available only with SELECTION=FORWARD, SELECTION=STEPWISE, and SELECTION=BACKWARD.

LSCOEFFS

requests a hybrid version of the LAR and LASSO methods, where the sequence of models is determined by the LAR or LASSO algorithm but the coefficients of the parameters for the model at any step are determined by using ordinary least squares.

MAXSTEP=*n*

specifies the maximum number of selection steps that are done. The default value of *n* is the number of effects in the model statement for the FORWARD, BACKWARD, and LAR methods and is three times the number of effects for the STEPWISE and LASSO methods.

SELECT=*criterion*

specifies the criterion that PROC GLMSELECT uses to determine the order in which effects enter and/or leave at each step of the specified selection method. The SELECT option is not valid with the LAR and LASSO methods. The criteria that you can specify with the SELECT= option are ADJRSQ, AIC, AICC, BIC, CP, CV, PRESS, RSQUARE, SBC, SL, and VALIDATE. See the section "Criteria Used in Model Selection Methods" on page 2725 for a description of these criteria. The default value of the SELECT= criterion is SELECT=SBC. You can use SELECT=SL to request the traditional approach where effects enter and leave the model based on the significance level. With other SELECT= criteria, the effect that is selected to enter or leave at a step of the selection process is the effect whose addition to or removal from the current model gives the maximum improvement in the specified criterion.

SLENTRY=*value*

SLE=*value*

specifies the significance level for entry, used when the STOP=SL or SELECT=SL option is in effect. The default is 0.15.

SLSTAY=*value*

SLS=*value*

specifies the significance level for staying in the model, used when the STOP=SL or SELECT=SL option is in effect. The default is 0.15.

STEPS=*n*

specifies the number of selection steps to be done. If the STEPS= option is specified, the STOP= and MAXSTEP= options are ignored.

STOP=*n*

STOP=*criterion*

specifies when PROC GLMSELECT stops the selection process. If the STEPS= option is specified, then the STOP= option is ignored. If the STOP=option does not cause the selection process to stop before the maximum number of steps for the selection method, then the selection process terminates at the maximum number of steps.

If you do not specify the STOP= option but do specify the SELECT= option, then the criterion named in the SELECT=option is also used as the STOP= criterion. If you do not specify either the STOP= or SELECT= option, then the default is STOP=SBC.

If STOP=*n* is specified, then PROC GLMSELECT stops selection at the first step for which the selected model has *n* effects.

The nonnumeric arguments that you can specify in the STOP= option are shown in Table 42.5. See the section "Criteria Used in Model Selection Methods" on page 2725 for more detailed descriptions of these criteria.

Table 42.5 Nonnumeric Criteria for the STOP= Option

Option	Criteria
NONE	
ADJRSQ	Adjusted R-square statistic
AIC	Akaike information criterion
AICC	Corrected Akaike information criterion
BIC	Sawa Bayesian information criterion
CP	Mallows C(p) statistic
CV	Predicted residual sum of square with k-fold cross validation
PRESS	Predicted residual sum of squares
SBC	Schwarz Bayesian information criterion
SL	Significance level
VALIDATE	Average square error for the validation data

With the SL criterion, selection stops at the step where the significance level for entry of all the effects not yet in the model is greater than the SLE= value for addition steps in the FORWARDS and STEPWISE methods and where the significance level for removal of any effect in the current model is greater than the SLS= value in the BACKWARD and STEPWISE methods. With the ADJRSQ criterion, selection stops at the step where the next step would yield a model with a smaller value of the Adjusted R-square statistic; for all other criteria, selection stops at the step where the next step would yield a model with a larger value of the criteria. You can use the VALIDATE option only if you have specified a VALDATA= data set in the PROC GLMSELECT statement or if you have reserved part of the input data for validation by using either a PARTITION statement or a _ROLE_ variable in the input data.

STAT|STATS=*name*

STATS=(*names*)

specifies which model fit statistics are displayed in the fit summary table and fit statistics tables. If you omit the STATS= option, the default set of statistics that are displayed in these tables includes all the criteria specified in any of the CHOOSE=, SELECT=, and STOP= options specified in the MODEL statement SELECTION= option.

The statistics that you can specify follow:

ADJRSQ — the adjusted R-square statistic

AIC — the Akaike information criterion

AICC — the corrected Akaike information criterion

ASE — the average square errors for the training, test, and validation data. The ASE statistics for the test and validation data are reported only if you have specified TESTDATA= and/or VALDATA= in the PROC GLMSELECT statement or if you have reserved part of the input data for testing and/or validation by using either a PARTITION statement or a _ROLE_ variable in the input data.

BIC — the Sawa Bayesian information criterion

CP	the Mallows C(p) statistic
FVALUE	the F statistic for entering or departing effects
PRESS	the predicted residual sum of squares statistic
RSQUARE	the R-square statistic
SBC	the Schwarz Bayesian information criterion
SL	the significance level of the F statistic for entering or departing effects

The statistics ADJRSQ, AIC, AICC, FVALUE, RSQUARE, SBC, and SL can be computed with little computation cost. However, computing BIC, CP, CVPRESS, PRESS, and ASE for test and validation data when these are not used in any of the CHOOSE=, SELECT=, and STOP= options specified in the MODEL statement SELECTION= option can hurt performance.

SHOWPVALUES
SHOWPVALS

displays p-values in the "ANOVA" and "Parameter Estimates" tables. These p-values are generally liberal because they are not adjusted for the fact that the terms in the model have been selected.

STB

produces standardized regression coefficients. A standardized regression coefficient is computed by dividing a parameter estimate by the ratio of the sample standard deviation of the dependent variable to the sample standard deviation of the regressor.

OUTPUT Statement

OUTPUT < **OUT=***SAS-data-set* > < *keyword* < *=name* > > . . . < *keyword* < *=name* > > **;**

The OUTPUT statement creates a new SAS data set that saves diagnostic measures calculated for the selected model. If you do not specify a *keyword*, then the only diagnostic included is the predicted response.

All the variables in the original data set are included in the new data set, along with variables created in the OUTPUT statement. These new variables contain the values of a variety of statistics and diagnostic measures that are calculated for each observation in the data set. If you specify a BY statement, then a variable _BY_ that indexes the BY groups is included. For each observation, the value of _BY_ is the index of the BY group to which this observation belongs. This variable is useful for matching BY groups with macro variables that PROC GLMSELECT creates. See the section "Macro Variables Containing Selected Models" on page 2732 for details.

If you have requested n-fold cross validation by requesting CHOOSE=CV, SELECT=CV, or STOP=CV in the MODEL statement, then a variable _CVINDEX_ is included in the output data set. For each observation used for model training the value of _CVINDEX_ is i if that observation is omitted in forming the ith subset of the training data. See the CVMETHOD= for additional details. The value of _CVINDEX_ is 0 for all observations in the input data set that are not used for model training.

If you have partitioned the input data with a PARTITION statement, then a character variable _ROLE_ is included in the output data set. For each observation the value of _ROLE_ is as follows:

ROLE	Observation Role
TEST	testing
TRAIN	training
VALIDATE	validation

If you want to create a permanent SAS data set, you must specify a two-level name (for example, *libref.data-set-name*).

For more information on permanent SAS data sets, refer to the section "SAS Files" in *SAS Language Reference: Concepts*.

Details on the specifications in the OUTPUT statement follow.

keyword < =name >
: specifies the statistics to include in the output data set and optionally names the new variables that contain the statistics. Specify a keyword for each desired statistic (see the following list of keywords), followed optionally by an equal sign, and a variable to contain the statistic.

  If you specify *keyword=name*, the new variable that contains the requested statistic has the specified name. If you omit the optional *=name* after a *keyword*, then the new variable name is formed by using a prefix of one or more characters that identify the statistic, followed by an underscore (_), followed by the dependent variable name.

  The keywords allowed and the statistics they represent are as follows:

  PREDICTED | PRED | P predicted values. The prefix for the default name is *p*.

  RESIDUAL | RESID | R residual, calculated as ACTUAL − PREDICTED. The prefix for the default name is *r*.

OUT=*SAS data set*
: gives the name of the new data set. By default, the procedure uses the DATA*n* convention to name the new data set.

PARTITION Statement

The PARTITION statement specifies how observations in the input data set are logically partitioned into disjoint subsets for model training, validation, and testing. Either you can designate a variable in the input data set and a set of formatted values of that variable to determine the role of each observation, or you can specify proportions to use for random assignment of observations for each role.

An alternative to using a PARTITION statement is to provide a variable named _ROLE_ in the input data set to define roles of observations in the input data. If you specify a PARTITION statement then the _ROLE_ variable if present in the input data set is ignored. If you do not use a PARTITION

statement and the input data do not contain a variable named _ROLE_, then all observations in the input data set are assigned to model training.

The following mutually exclusive options are available:

ROLEVAR | ROLE=variable(< TEST='value' > < TRAIN='value' > < VALIDATE='value' >)
names the variable in the input data set whose values are used to assign roles to each observation. The formatted values of this variable that are used to assign observations roles are specified in the TEST=, TRAIN=, and VALIDATE= suboptions. If you do not specify the TRAIN= suboption, then all observations whose role is not determined by the TEST= or VALIDATE= suboptions are assigned to training. If you specify a TESTDATA= data set in the PROC GLMSELECT statement, then you cannot also specify the TEST= suboption in the PARTITION statement. If you specify a VALDATA= data set in the PROC GLMSELECT statement, then you cannot also specify the VALIDATE= suboption in the PARTITION statement.

FRACTION(< TEST=fraction > < VALIDATE=fraction >)
requests that specified proportions of the observations in the input data set be randomly assigned training and validation roles. You specify the proportions for testing and validation by using the TEST= and VALIDATE= suboptions. If you specify both the TEST= and the VALIDATE= suboptions, then the sum of the specified fractions must be less than one and the remaining fraction of the observations are assigned to the training role. If you specify a TESTDATA= data set in the PROC GLMSELECT statement, then you cannot also specify the TEST= suboption in the PARTITION statement. If you specify a VALDATA= data set in the PROC GLMSELECT statement, then you cannot also specify the VALIDATE= suboption in the PARTITION statement.

PERFORMANCE Statement

PERFORMANCE < *options* > **;**

The PERFORMANCE statement is used to change default options that affect the performance of PROC GLMSELECT and to request tables that show the performance options in effect and timing details.

The following options are available:

CPUCOUNT= *1-1024*

CPUCOUNT=*ACTUAL*
specifies the number of processors that PROC GLMSELECT assumes are available for multithreaded BY-group processing. CPUCOUNT=ACTUAL sets CPUCOUNT to be the number of physical processors available. Note that this can be less than the physical number of CPUs if the SAS process has been restricted by system administration tools. Setting CPUCOUNT= to a number greater than the actual number of available CPUs might result in reduced performance. This option overrides the SAS system option CPUCOUNT=. If CPUCOUNT=1, then NOTHREADS is in effect, or if no BY processing is being used, then PROC GLMSELECT uses singly threaded code.

DETAILS

requests the "PerfSettings" table that shows the performance settings in effect and the "Timing" table that provides a broad timing breakdown of the PROC GLMSELECT step.

BUILDSSCP=FULL

BUILDSSCP=INCREMENTAL

specifies whether the SSCP matrix is built incrementally as the selection process progresses or whether the SCCP matrix for the full model is built at the outset. Building the SSCP matrix incrementally can significantly reduce the memory required and the time taken to perform model selection in cases where the number of parameters in the selected model is much smaller than the number of parameters in the full model, but it can hurt performance in other cases since it requires at least one pass through the model training data at each step. If you use backward selection or no selection, or if the BIC or CP statistics are required in the selection process, then the BUILDSSCP=INCREMENTAL option is ignored. In other cases, BUILDSSCP=INCREMENTAL is used by default if the number of effects is greater than 100. See the section "Building the SSCP Matrix" on page 2735 for further details.

THREADS

enables multithreaded BY-group computation. This option overrides the SAS system option THREADS | NOTHREADS. If no BY processing is being used, then PROC GLMSELECT ignores this option and uses singly threaded code.

NOTHREADS

disables multithreaded BY-group computation. This option overrides the SAS system option THREADS | NOTHREADS.

SCORE Statement

SCORE < *DATA=SAS-data-set* > < *OUT=SAS-data-set* > < *keyword* <=*name*> > . . . < *keyword* <=*name*> > ;

The SCORE statement creates a new SAS data set containing predicted values and optionally residuals for data in a new data set that you name. If you do not specify a DATA= data set, then the input data are scored. If you have multiple data sets to predict, you can specify multiple SCORE statements. If you want to create a permanent SAS data set, you must specify a two-level name (for example, *libref.data-set-name*) in the OUT= option. For more information on permanent SAS data sets, refer to the section "SAS Files" in *SAS Language Reference: Concepts*.

When a BY statement is used, the score data set must either contain all the BY variables sorted in the order of the BY variables or contain none of the BY variables. If the score data set contains all of the BY variables, then the model selected for a given BY group is used to score just the matching observations in the score data set. If the score data set contains none of the BY variables, then the entire score data set is scored for each BY group.

All observations in the score data set are retained in the output data set. However, only those observations that contain nonmissing values for all the continuous regressors in the selected model and whose levels of the class variables appearing in effects of the selected model are represented in

the corresponding class variable in the procedure's input data set are scored. All the variables in the input data set are included in the output data set, along with variables containing predicted values and optionally residuals.

Details on the specifications in the SCORE statement follow:

DATA=*SAS data set*
names the data set to be scored. If you omit this option, then the input data set named in the DATA= option in the PROC GLMSELECT statement is scored.

keyword < =*name* >
specifies the statistics to include in the output data set and optionally names the new variables that contain the statistics. Specify a keyword for each desired statistic (see the following list of keywords), followed optionally by an equal sign, and a variable to contain the statistic.

If you specify *keyword*=*name*, the new variable that contains the requested statistic has the specified name. If you omit the optional =*name* after a *keyword*, then the new variable name is formed by using a prefix of one or more characters that identify the statistic, followed by an underscore (_), followed by the dependent variable name.

The keywords allowed and the statistics they represent are as follows:

PREDICTED | PRED | P predicted values. The prefix for the default name is *p*.

RESIDUAL | RESID | R residual, calculated as ACTUAL − PREDICTED. The prefix for the default name is *r*.

OUT=*SAS data set*
gives the name of the new output data set. By default, the procedure uses the DATA*n* convention to name the new data set.

WEIGHT Statement

WEIGHT *variable* **;**

A WEIGHT statement names a variable in the input data set with values that are relative weights for a weighted least squares fit. If the weight value is proportional to the reciprocal of the variance for each observation, then the weighted estimates are the best linear unbiased estimates (BLUE).

Values of the weight variable must be nonnegative. If an observation's weight is zero, the observation is deleted from the analysis. If a weight is negative or missing, it is set to zero, and the observation is excluded from the analysis. A more complete description of the WEIGHT statement can be found in Chapter 39, "The GLM Procedure."

Details: GLMSELECT Procedure

Model-Selection Methods

The model selection methods implemented in PROC GLMSELECT are specified with the SELECTION= option in the MODEL statement.

Full Model Fitted (NONE)

The complete model specified in the MODEL statement is used to fit the model and no effect selection is done. You request this by specifying SELECTION=NONE in the MODEL statement.

Forward Selection (FORWARD)

The forward selection technique begins with just the intercept and then sequentially adds the effect that most improves the fit. The process terminates when no significant improvement can be obtained by adding any effect.

In the traditional implementation of forward selection, the statistic used to gauge improvement in fit is an F statistic that reflects an effect's contribution to the model if it is included. At each step, the effect that yields the most significant F statistic is added. Note that because effects can contribute different degrees of freedom to the model, it is necessary to compare the p-values corresponding to these F statistics.

More precisely, if the current model has p parameters excluding the intercept, and if you denote its residual sum of squares by RSS_p and you add an effect with k degrees of freedom and denote the residual sum of squares of the resulting model by RSS_{p+k}, then the F statistic for entry with k numerator degrees of freedom and $n-(p+k)-1$ denominator degrees of freedom is given by

$$F = \frac{(RSS_p - RSS_{p+k})/k}{RSS_{p+k}/(n-(p+k)-1)}$$

where n is number of observations used in the analysis.

The process stops when the significance level for adding any effect is greater than some specified entry significance level. A well-known problem with this methodology is that these F statistics do not follow an F distribution (Draper, Guttman, and Kanemasu 1971). Hence these p-values cannot reliably be interpreted as probabilities. Various ways to approximate this distribution are described by Miller (2002). Another issue when you use significance levels of entering effects as a stopping criterion arises because the entry significance level is an a priori specification that does not depend on the data. Thus, the same entry significance level can result in overfitting for some data and underfitting for other data.

One approach to address the critical problem of when to stop the selection process is to assess the quality of the models produced by the forward selection method and choose the model from this sequence that "best" balances goodness of fit against model complexity. PROC GLMSELECT supports several criteria that you can use for this purpose. These criteria fall into two groups—information criteria and criteria based on out-of-sample prediction performance.

You use the CHOOSE= option of forward selection to specify the criterion for selecting one model from the sequence of models produced. If you do not specify a CHOOSE= criterion, then the model at the final step is the selected model.

For example, if you specify

```
selection=forward(select=SL choose=AIC SLE=0.2)
```

then forward selection terminates at the step where no effect can be added at the 0.2 significance level. However, the selected model is the first one with the minimal value of the Akaike information criterion. Note that in some cases this minimal value might occur at a step much earlier that the final step, while in other cases the AIC criterion might start increasing only if more steps are done (that is, a larger value of SLE is used). If what you are interested in is minimizing AIC, then too many steps are done in the former case and too few in the latter case. To address this issue, PROC GLMSELECT enables you to specify a stopping criterion with the STOP= option. With a stopping criterion specified, forward selection continues until a local extremum of the stopping criterion in the sequence of models generated is reached. You can also specify STOP= number, which causes forward selection to continue until there are the specified number of effects in the model.

For example, if you specify

```
selection=forward(select=SL stop=AIC)
```

then forward selection terminates at the step where the effect to be added at the next step would produce a model with an AIC statistic larger than the AIC statistic of the current model. Note that in most cases, provided that the entry significance level is large enough that the local extremum of the named criterion occurs before the final step, specifying

```
selection=forward(select=SL choose=CRITERION)
```

or

```
selection=forward(select=SL stop=CRITERION)
```

selects the same model, but more steps are done in the former case. In some cases there might be a better local extremum that cannot be reached if you specify the STOP= option but can be found if you use the CHOOSE= option. Also, you can use the CHOOSE= option in preference to the STOP= option if you want examine how the named criterion behaves as you move beyond the step where the first local minimum of this criterion occurs.

Note that you can specify both the CHOOSE= and STOP= options. You might want to consider models generated by forward selection that have at most some fixed number of effects but select from within this set based on a criterion you specify. For example, specifying

```
selection=forward(stop=20 choose=ADJRSQ)
```

requests that forward selection continue until there are 20 effects in the final model and chooses among the sequence of models the one that has the largest value of the adjusted R-square statistic.

You can also combine these options to select a model where one of two conditions is met. For example,

```
selection=forward(stop=AICC choose=PRESS)
```

chooses whatever occurs first between a local minimum of the predicted residual sum of squares (PRESS) and a local minimum of the corrected Akaike information criterion (AICC).

It is important to keep in mind that forward selection bases the decision about what effect to add at any step by considering models that differ by one effect from the current model. This search paradigm cannot guarantee reaching a "best" subset model. Furthermore, the add decision is greedy in the sense that the effect deemed most significant is the effect that is added. However, if your goal is to find a model that is best in terms of some selection criterion other than the significance level of the entering effect, then even this one step choice might not be optimal. For example, the effect you would add to get a model with the smallest value of the PRESS statistic at the next step is not necessarily the same effect that has the most significant entry F statistic. PROC GLMSELECT enables you to specify the criterion to optimize at each step by using the SELECT= option. For example,

```
selection=forward(select=CP)
```

requests that at each step the effect that is added be the one that gives a model with the smallest value of the Mallows' $C(p)$ statistic. Note that in the case where all effects are variables (that is, effects with one degree of freedom and no hierarchy), using ADJRSQ, AIC, AICC, BIC, CP, RSQUARE, or SBC as the selection criterion for forward selection produces the same sequence of additions. However, if the degrees of freedom contributed by different effects are not constant, or if an out-of-sample prediction-based criterion is used, then different sequences of additions might be obtained.

You can use SELECT= together with CHOOSE= and STOP=. If you specify only the SELECT= criterion, then this criterion is also used as the stopping criterion. In the previous example where only the selection criterion is specified, not only do effects enter based on the Mallows' $C(p)$ statistic, but the selection terminates when the $C(p)$ statistic first increases.

You can find discussion and references to studies about criteria for variable selection in Burnham and Anderson (2002), along with some cautions and recommendations.

Examples of Forward Selection Specifications

```
selection=forward
```

adds effects that at each step give the lowest value of the SBC statistic and stops at the step where adding any effect would increase the SBC statistic.

```
selection=forward(select=SL)
```

adds effects based on significance level and stops when all candidate effects for entry at a step have a significance level greater than the default entry significance level of 0.15.

```
selection=forward(select=SL stop=validation)
```

adds effects based on significance level and stops at a step where adding any effect increases the error sum of squares computed on the validation data.

```
selection=forward(select=AIC)
```

adds effects that at each step give the lowest value of the AIC statistic and stops at the step where adding any effect would increase the AIC statistic.

```
selection=forward(select=ADJRSQ stop=SL SLE=0.2)
```

adds effects that at each step give the largest value of the adjusted R-square statistic and stops at the step where the significance level corresponding to the addition of this effect is greater than 0.2.

Backward Elimination (BACKWARD)

The backward elimination technique starts from the full model including all independent effects. Then effects are deleted one by one until a stopping condition is satisfied. At each step, the effect showing the smallest contribution to the model is deleted. In traditional implementations of backward elimination, the contribution of an effect to the model is assessed by using an F statistic. At any step, the predictor producing the least significant F statistic is dropped and the process continues until all effects remaining in the model have F statistics significant at a stay significance level (SLS).

More precisely, if the current model has p parameters excluding the intercept, and if you denote its residual sum of squares by RSS_p and you drop an effect with k degrees of freedom and denote the residual sum of squares of the resulting model by RSS_{p-k}, then the F statistic for removal with k numerator degrees of freedom and $n - p - k$ denominator degrees of freedom is given by

$$F = \frac{(RSS_{p-k} - RSS_p)/k}{RSS_p/(n-p-k)}$$

where n is number of observations used in the analysis.

Just as with forward selection, you can change the criterion used to assess effect contributions with the SELECT= option. You can also specify a stopping criterion with the STOP= option and use a CHOOSE= option to provide a criterion used to select among the sequence of models produced. See the discussion in the section "Forward Selection (FORWARD)" on page 2717 for additional details.

Examples of Backward Selection Specifications

```
selection=backward
```

removes effects that at each step produce the largest value of the Schwarz Bayesian information criterion (SBC) statistic and stops at the step where removing any effect increases the SBC statistic.

```
selection=backward(stop=press)
```

removes effects based on the SBC statistic and stops at the step where removing any effect increases the predicted residual sum of squares (PRESS).

```
selection=backward(select=SL)
```

removes effects based on significance level and stops when all candidate effects for removal at a step have a significance level less than the default stay significance level of 0.15.

```
selection=backward(select=SL choose=validate SLS=0.1)
```

removes effects based on significance level and stops when all effects in the model are significant at the 0.1 level. Finally, from the sequence of models generated, choose the one that gives the smallest average square error when scored on the validation data.

Stepwise Selection(STEPWISE)

The stepwise method is a modification of the forward selection technique that differs in that effects already in the model do not necessarily stay there.

In the traditional implementation of stepwise selection method, the same entry and removal F statistics for the forward selection and backward elimination methods are used to assess contributions of effects as they are added to or removed from a model. If at a step of the stepwise method, any effect in the model is not significant at the SLSTAY= level, then the least significant of these effects is removed from the model and the algorithm proceeds to the next step. This ensures that no effect can be added to a model while some effect currently in the model is not deemed significant. Only after all necessary deletions have been accomplished can another effect be added to the model. In this case the effect whose addition yields the most significant F value is added to the model and the algorithm proceeds to the next step. The stepwise process ends when none of the effects outside the model has an F statistic significant at the SLENTRY= level and every effect in the model is significant at the SLSTAY= level. In some cases, neither of these two conditions for stopping is met and the sequence of models cycles. In this case, the stepwise method terminates at the end of the second cycle.

Just as with forward selection and backward elimination, you can change the criterion used to assess effect contributions, with the SELECT= option. You can also specify a stopping criterion with the STOP= option and use a CHOOSE= option to provide a criterion used to select among the sequence of models produced. See the discussion in the section "Forward Selection (FORWARD)" on page 2717 for additional details.

For selection criteria other than significance level, PROC GLMSELECT optionally supports a further modification in the stepwise method. In the standard stepwise method, no effect can enter the model if removing any effect currently in the model would yield an improved value of the selection criterion. In the modification, you can use the DROP=COMPETITIVE option to specify that addition and deletion of effects should be treated competitively. The selection criterion is evaluated for all models obtained by deleting an effect from the current model or by adding an effect to this model. The action that most improves the selection criterion is the action taken.

Examples of Stepwise Selection Specifications

```
selection=stepwise
```

requests stepwise selection based on the SBC criterion. First, if removing any effect yields a model with a lower SBC statistic than the current model, then the effect producing the smallest SBC statistic is removed. When removing any effect increases the SBC statistic, then provided that adding some effect lowers the SBC statistic, the effect producing the model with the lowest SBC is added.

```
selection=stepwise(select=SL)
```

requests the traditional stepwise method. First, if the removal of any effect yields an F statistic that is not significant at the default stay level of 0.15, then the effect whose removal produces the least significant F statistic is removed and the algorithm proceeds to the next step. Otherwise the effect whose addition yields the most significant F statistic is added, provided that it is significant at the default entry level of 0.15.

```
selection=stepwise(select=SL stop=SBC)
```

is the traditional stepwise method, where effects enter and leave based on significance levels, but with the following extra check: If any effect to be added or removed yields a model whose SBC statistic is greater than the SBC statistic of the current model, then the stepwise method terminates at the current model. Note that in this case, the entry and stay significance levels still play a role as they determine whether an effect is deleted from or added to the model. This might result in the selection terminating before a local minimum of the SBC criterion is found.

```
selection=stepwise(select=SL SLE=0.1 SLS=0.08 choose=AIC)
```

selects effects to enter or drop as in the previous example except that the significance level for entry is now 0.1 and the significance level to stay is 0.08. From the sequence of models produced, the selected model is chosen to yield the minimum AIC statistic.

```
selection=stepwise(select=AICC drop=COMPETITIVE)
```

requests stepwise selection based on the AICC criterion with steps treated competitively. At any step, evaluate the AICC statistics corresponding to the removal of any effect in the current model or the addition of any effect to the current model. Choose the addition or removal that produced this minimum value, provided that this minimum is lower than the AICC statistic of the current model.

```
selection=stepwise(select=SBC drop=COMPETITIVE stop=VALIDATE)
```

requests stepwise selection based on the SBC criterion with steps treated competitively and where stopping is based on the average square error over the validation data. At any step, SBC statistics corresponding to the removal of any effect from the current model or the addition of any effect to the current model are evaluated. The addition or removal that produces the minimum SBC value is made. The average square error on the validation data for the model with this addition or removal is evaluated. If this average square error is greater than the average square error on the validation data prior to this addition or deletion, then the algorithm terminates at this prior model.

Least Angle Regression (LAR)

Least angle regression was introduced by Efron et al. (2004). Not only does this algorithm provide a selection method in its own right, but with one additional modification it can be used to efficiently produce LASSO solutions. Just like the forward selection method, the LAR algorithm produces a sequence of regression models where one parameter is added at each step, terminating at the full least squares solution when all parameters have entered the model.

The algorithm starts by centering the covariates and response, and scaling the covariates so that they all have the same corrected sum of squares. Initially all coefficients are zero, as is the predicted response. The predictor that is most correlated with the current residual is determined and a step is taken in the direction of this predictor. The length of this step determines the coefficient of this predictor and is chosen so that some other predictor and the current predicted response have the same correlation with the current residual. At this point, the predicted response moves in the direction that is equiangular between these two predictors. Moving in this direction ensures that these two predictors continue to have a common correlation with the current residual. The predicted response moves in this direction until a third predictor has the same correlation with the current residual as the two predictors already in the model. A new direction is determined that is equiangular between these three predictors and the predicted response moves in this direction until a fourth predictor joins the set having the same correlation with the current residual. This process continues until all predictors are in the model.

As with other selection methods, the issue of when to stop the selection process is crucial. You can specify a criterion to use to choose among the models at each step with the CHOOSE= option. You can also specify a stopping criterion with the STOP= option. See the section "Criteria Used in Model Selection Methods" on page 2725 for details and Table 42.6 for the formulas for evaluating these criteria. These formulas use the approximation that at step k of the LAR algorithm, the model has k degrees of freedom. See Efron et al. (2004) for a detailed discussion of this so-called simple approximation.

A modification of LAR selection suggested in Efron et al. (2004) uses the LAR algorithm to select the set of covariates in the model at any step, but uses ordinary least squares regression with just these covariates to obtain the regression coefficients. You can request this hybrid method by specifying the LSCOEFFS suboption of SELECTION=LAR.

Lasso Selection (LASSO)

LASSO (least absolute shrinkage and selection operator) selection arises from a constrained form of ordinary least squares regression where the sum of the absolute values of the regression coefficients is constrained to be smaller than a specified parameter. More precisely let $X = (x_1, x_2, \ldots, x_m)$ denote the matrix of covariates and let y denote the response, where the x_is have been centered and scaled to have unit standard deviation and mean zero, and y has mean zero. Then for a given parameter t, the LASSO regression coefficients $\beta = (\beta_1, \beta_2, \ldots, \beta_m)$ are the solution to the constrained optimization problem

$$\text{minimize} ||y - X\beta||^2 \qquad \text{subject to} \quad \sum_{j=1}^{m} |\beta_j| \leq t$$

Provided that the LASSO parameter t is small enough, some of the regression coefficients will be exactly zero. Hence, you can view the LASSO as selecting a subset of the regression coefficients for each LASSO parameter. By increasing the LASSO parameter in discrete steps, you obtain a sequence of regression coefficients where the nonzero coefficients at each step correspond to selected parameters.

Early implementations (Tibshirani 1996) of LASSO selection used quadratic programming techniques to solve the constrained least squares problem for each LASSO parameter of interest. Later Osborne, Presnell, and Turlach (2000) developed a "homotopy method" that generates the LASSO solutions for all values of t. Efron et al. (2004) derived a variant of their algorithm for least angle regression that can be used to obtain a sequence of LASSO solutions from which all other LASSO solutions can be obtained by linear interpolation. This algorithm for SELECTION=LASSO is used in PROC GLMSELECT. It can be viewed as a stepwise procedure with a single addition to or deletion from the set of nonzero regression coefficients at any step.

As with the other selection methods supported by PROC GLMSELECT, you can specify a criterion to choose among the models at each step of the LASSO algorithm with the CHOOSE= option. You can also specify a stopping criterion with the STOP= option. See the discussion in the section "Forward Selection (FORWARD)" on page 2717 for additional details. The model degrees of freedom PROC GLMSELECT uses at any step of the LASSO are simply the number of nonzero regression coefficients in the model at that step. Efron et al. (2004) cite empirical evidence for doing this but do not give any mathematical justification for this choice.

A modification of LASSO selection suggested in Efron et al. (2004) uses the LASSO algorithm to select the set of covariates in the model at any step, but uses ordinary least squares regression with just these covariates to obtain the regression coefficients. You can request this hybrid method by specifying the LSCOEFFS suboption of SELECTION=LASSO.

Model Selection Issues

Many authors caution against the use of "automatic variable selection" methods and describe pitfalls that plague many such methods. For example, Harrell (2001) states that "stepwise variable selection has been a very popular technique for many years, but if this procedure had just been proposed as a statistical method, it would most likely be rejected because it violates every principle of statistical estimation and hypothesis testing." He lists and discusses several of these issues and cites a variety of studies that highlight these problems. He also notes that many of these issues are not restricted to stepwise selection, but affect forward selection and backward elimination, as well as methods based on all-subset selection.

In their introductory chapter, Burnham and Anderson (2002) discuss many issues involved in model selection. They also strongly warn against "data dredging," which they describe as "the process of analyzing data with few or no a priori questions, by subjectively and iteratively searching the data for patterns and 'significance'." However, Burnham and Anderson also discuss the desirability of finding parsimonious models. They note that using "full models" that contain many insignificant predictors might avoid some of the inferential problems arising in models with automatically selected variables but will lead to overfitting the particular sample data and produce a model that performs poorly in predicting data not used in training the model.

One problem in the traditional implementations of forward, backward, and stepwise selection methods is that they are based on sequential testing with specified entry (SLE) and stay (SLS) significance levels. However, it is known that the "F-to-enter" and "F-to-delete" statistics do not follow an F distribution (Draper, Guttman, and Kanemasu 1971). Hence the SLE and SLS values cannot reliably be viewed as probabilities. One way to address this difficulty is to replace hypothesis testing as a means of selecting a model with information criteria or out-of-sample prediction criteria. While Harrell (2001) points out that information criteria were developed for comparing only prespecified models, Burnham and Anderson (2002) note that AIC criteria have routinely been used for several decades for performing model selection in time series analysis.

Problems also arise when the selected model is interpreted as if it were prespecified. There is a "selection bias" in the parameter estimates that is discussed in detail in Miller (2002). This bias occurs because a parameter is more likely to be selected if it is above its expected value than if it is below its expected value. Furthermore, because multiple comparisons are made in obtaining the selected model, the p-values obtained for the selected model are not valid. When a single best model is selected, inference is conditional on that model. Model averaging approaches provide a way to make more stable inferences based on a set of models. Methods for doing this are presented in Burnham and Anderson (2002), and Bayesian approaches are discussed in Raftery, Madigan, and Hoeting (1997).

Despite these difficulties, careful and informed use of variable selection methods still has its place in modern data analysis. For example, Foster and Stine (2004) use a modified version of stepwise selection to build a predictive model for bankruptcy from over 67,000 possible predictors and show that this yields a model whose predictions compare favorably with other recently developed data mining tools. In particular, when the goal is prediction rather than estimation or hypothesis testing, variable selection with careful use of validation to limit both under and over fitting is often a useful starting point of model development.

Criteria Used in Model Selection Methods

PROC GLMSELECT supports a variety of fit statistics that you can specify as criteria for the CHOOSE=, SELECT=, and STOP= options in the MODEL statement. The following statistics are available:

ADJRSQ	adjusted R-square statistic (Darlington 1968; Judge et al. 1985)
AIC	Akaike information criterion (Darlington 1968; Judge et al. 1985)
AICC	corrected Akaike information criterion (Hurvich and Tsai 1989)
BIC	Sawa Bayesian information criterion (Sawa 1978; Judge et al. 1985)
CP	Mallows C_p statistic (Mallows 1973; Hocking 1976)
PRESS	predicted residual sum of squares statistic
SBC	Schwarz Bayesian information criterion (Schwarz 1978; Judge et al. 1985)
SL	significance level of the F statistic used to assess an effect's contribution to the fit when it is added to or removed from a model
VALIDATE	average square error over the validation data

Table 42.6 provides formulas and definitions for the fit statistics.

Table 42.6 Formulas and Definitions for Model Fit Summary Statistics

Statistic	Definition or Formula
n	number of observations
p	number of parameters including the intercept
$\hat{\sigma}^2$	estimate of pure error variance from fitting the full model
SST	total sum of squares corrected for the mean for the dependent variable
SSE	error sum of squares
ASE	$\frac{\text{SSE}}{n}$
MSE	$\frac{\text{SSE}}{n-p}$
R^2	$1-\frac{\text{SSE}}{\text{SST}}$
ADJRSQ	$1-\frac{(n-1)(1-R^2)}{n-p}$
AIC	$n\ln\left(\frac{\text{SSE}}{n}\right)+2p$
AICC	$1+\ln\left(\frac{\text{SSE}}{n}\right)+\frac{2(p+1)}{n-p-2}$
BIC	$n\ln\left(\frac{\text{SSE}}{n}\right)+2(p+2)q-2q^2$ where $q=\frac{n\hat{\sigma}^2}{\text{SSE}}$
CP (C_p)	$\frac{\text{SSE}}{\hat{\sigma}^2}+2p-n$
PRESS	$\sum_{i=1}^{n}\frac{r_i^2}{(1-h_i)^2}$ where r_i = residual at observation i and h_i = leverage of observation i = $\mathbf{x}_i(\mathbf{X}'\mathbf{X})^{-}\mathbf{x}_i'$
RMSE	$\sqrt{\text{MSE}}$
SBC	$n\ln\left(\frac{\text{SSE}}{n}\right)+p\ln(n)$

Changes in Formulas for AIC and AICC

The formulas used for the AIC and AICC statistics have been changed in SAS 9.2. However, the models selected at each step of the selection process and the final selected model are unchanged from the experimental download release of PROC GLMSELECT, even in the case where you specify AIC or AICC in the SELECT=, CHOOSE=, and STOP= options in the MODEL statement. The reason for making this change is to make the connection between the AIC statistic and the AICC statistic more transparent.

In the context of linear regression, several different versions of the formulas for AIC and AICC appear in the statistics literature. However, for a fixed number of observations, these different

versions differ by additive and positive multiplicative constants. Because the model selected to yield a minimum of a criterion is not affected if the criterion is changed by additive and positive multiplicative constants, these changes in the formula for AIC and AICC do not affect the selection process.

The following section provides details about these changes. Formulas used in the experimental download release are denoted with a superscript of (d) and n, p and SSE are defined in Table 42.6.

In the experimental download release of PROC GLMSELECT the following formulas are used for AIC (Darlington 1968; Judge et al. 1985) and AICC (Hurvich, Simonoff, and Tsai 1998):

$$\mathrm{AIC}^{(d)} = n\log\left(\frac{\mathrm{SSE}}{n}\right) + 2p$$

and

$$\mathrm{AICC}^{(d)} = \log\left(\frac{\mathrm{SSE}}{n}\right) + 1 + \frac{2(p+1)}{n-p-2}$$

The definitions of AIC and AICC used in this release are found in Hurvich and Tsai (1989). These formulas are

$$\mathrm{AIC} = n\log\left(\frac{\mathrm{SSE}}{n}\right) + 2p + n + 2$$

and

$$\mathrm{AICC} = \mathrm{AIC} + \frac{2(p+1)(p+2)}{n-p-2}$$

Hurvich and Tsai (1989) show that the formula for AICC can also be written as

$$\mathrm{AICC} = n\log\left(\frac{\mathrm{SSE}}{n}\right) + \frac{n(n+p)}{n-p-2}$$

The relationships between the alternative forms of the formulas are

$$\mathrm{AIC} = \mathrm{AIC}^{(d)} + n + 2$$

$$\mathrm{AICC} = n\ \mathrm{AICC}^{(d)}$$

CLASS Variable Parameterization

Consider a model with one classification variable A with four levels, 1, 2, 5, and 7. Details of the possible choices for the PARAM= option follow.

EFFECT — Three columns are created to indicate group membership of the nonreference levels. For the reference level, all three dummy variables have a value of −1. For instance, if the reference level is 7 (REF=7), the design matrix columns for A are as follows.

Effect Coding			
	Design Matrix		
A	A1	A2	A5
1	1	0	0
2	0	1	0
5	0	0	1
7	−1	−1	−1

Parameter estimates of classification main effects that use the effect coding scheme estimate the difference in the effect of each nonreference level compared to the average effect over all four levels.

GLM — As in PROC GLM, four columns are created to indicate group membership. The design matrix columns for A are as follows.

GLM Coding				
	Design Matrix			
A	A1	A2	A5	A7
1	1	0	0	0
2	0	1	0	0
5	0	0	1	0
7	0	0	0	1

Parameter estimates of classification main effects that use the GLM coding scheme estimate the difference in the effects of each level compared to the last level.

ORDINAL

THERMOMETER Three columns are created to indicate group membership of the higher levels of the effect. For the first level of the effect (which for A is 1), all three dummy variables have a value of 0. The design matrix columns for A are as follows.

Ordinal Coding			
	Design Matrix		
A	A2	A5	A7
1	0	0	0
2	1	0	0
5	1	1	0
7	1	1	1

The first level of the effect is a control or baseline level. Parameter estimates of classification main effects that use the ORDINAL coding scheme estimate the effect on the response as the ordinal factor is set to each succeeding level. When the parameters for an ordinal main effect have the same sign, the response effect is monotonic across the levels.

POLYNOMIAL

POLY Three columns are created. The first represents the linear term (x), the second represents the quadratic term (x^2), and the third represents the cubic term (x^3), where x is the level value. If the classification levels are not numeric, they are translated into 1, 2, 3, ... according to their sorting order. The design matrix columns for A are as follows.

Polynomial Coding			
	Design Matrix		
A	APOLY1	APOLY2	APOLY3
1	1	1	1
2	2	4	8
5	5	25	125
7	7	49	343

REFERENCE

REF Three columns are created to indicate group membership of the nonreference levels. For the reference level, all three dummy variables have a value of 0. For instance, if the reference level is 7 (REF=7), the design matrix columns for A are as follows.

Reference Coding			
	Design Matrix		
A	A1	A2	A5
1	1	0	0
2	0	1	0
5	0	0	1
7	0	0	0

Parameter estimates of CLASS main effects that use the reference coding scheme estimate the difference in the effect of each nonreference level compared to the effect of the reference level.

ORTHEFFECT The columns are obtained by applying the Gram-Schmidt orthogonalization to the columns for PARAM=EFFECT. The design matrix columns for A are as follows.

Orthogonal Effect Coding			
	Design Matrix		
A	AOEFF1	AOEFF2	AOEFF3
1	1.41421	−0.81650	−0.57735
2	0.00000	1.63299	−0.57735
5	0.00000	0.00000	1.73205
7	−1.41421	−0.81649	−0.57735

ORTHORDINAL

ORTHOTHERM The columns are obtained by applying the Gram-Schmidt orthogonalization to the columns for PARAM=ORDINAL. The design matrix columns for A are as follows.

Orthogonal Ordinal Coding			
	Design Matrix		
A	AOORD1	AOORD2	AOORD3
1	−1.73205	0.00000	0.00000
2	0.57735	−1.63299	0.00000
5	0.57735	0.81650	−1.41421
7	0.57735	0.81650	1.41421

ORTHPOLY The columns are obtained by applying the Gram-Schmidt orthogonalization to the columns for PARAM=POLY. The design matrix columns for A are as follows.

Orthogonal Polynomial Coding			
	Design Matrix		
A	AOPOLY1	AOPOLY2	AOPOLY5
1	−1.153	0.907	−0.921
2	−0.734	−0.540	1.473
5	0.524	−1.370	−0.921
7	1.363	1.004	0.368

ORTHREF The columns are obtained by applying the Gram-Schmidt orthogonalization to the columns for PARAM=REFERENCE. The design matrix columns for A are as follows.

Orthogonal Reference Coding			
	Design Matrix		
A	AOREF1	AOREF2	AOREF3
1	1.73205	0.00000	0.00000
2	−0.57735	1.63299	0.00000
5	−0.57735	−0.81650	1.41421
7	−0.57735	−0.81650	−1.41421

The following example illustrates several features of the CLASS statement.

```
data codingExample;
   drop i;
   do i=1 to 1000;
     c1 = 1 + mod(i,6);
     if      i < 50  then c2 = 'very low ';
     else if i < 250 then c2 = 'low';
     else if i < 500 then c2 = 'medium';
     else if i < 800 then c2 = 'high';
     else                 c2 = 'very high';
     x1 = ranuni(1);
     x2 = ranuni(1);
     y = x1 + 10*(c1=3) +5*(c1=5) +rannor(1);
     output;
   end;
run;
proc glmselect data=codingExample;
   class c1(param=ref split) c2(param=ordinal order=data) /
          delimiter = ',' showcoding;
   model y = c1 c2 x1 x2/orderselect;
run;
```

Figure 42.11 Class Level Information

```
                          The GLMSELECT Procedure

                          Class Level Information

          Class      Levels      Values

          c1              6 *    1,2,3,4,5,6
          c2              5      very low,low,medium,high,very high

                       * Associated Parameters Split
```

The "Class Level Information" table shown in Figure 42.11 is produced by default whenever you specify a CLASS statement. Note that because the levels of the variable "c2" contain embedded blanks, the DELIMITER="," option has been specified. The SHOWCODING option requests the display of the "Class Level Coding" table shown in Figure 42.12. An ordinal parameterization is used for "c2" because its levels have a natural order. Furthermore, because these levels appear in their natural order in the data, you can preserve this order by specifying the ORDER=*data* option.

Figure 42.12 Class Level Coding

Class Level Coding

c1	Design Variables				
Level	1	2	3	4	5
1	1	0	0	0	0
2	0	1	0	0	0
3	0	0	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1
6	0	0	0	0	0

The SPLIT option has been specified for the classification variable "c1." This permits the parameters associated with the effect "c1" to enter or leave the model individually. The "Parameter Estimates" table in Figure 42.13 shows that for this example the parameters corresponding to only levels 3 and 5 of "c1" are in the selected model. Finally, note that the ORDERSELECT option in the MODEL statement specifies that the parameters are displayed in the order in which they first entered the model.

Figure 42.13 Parameter Estimates

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value
Intercept	1	−0.216680	0.068650	−3.16
c1_3	1	10.160900	0.087898	115.60
c1_5	1	5.018015	0.087885	57.10
x1	1	1.315468	0.109772	11.98

Macro Variables Containing Selected Models

Often you might want to perform post-selection analysis by using other SAS procedures. To facilitate this, PROC GLMSELECT saves the list of selected effects in a macro variable. This list does not explicitly include the intercept so that you can use it in the MODEL statement of other SAS/STAT regression procedures.

The following table describes the macro variables that PROC GLMSELECT creates. Note that when BY processing is used, one macro variable, indexed by the BY group number, is created for each BY group.

Macro Variable	Description
No BY processing	
_GLSIND1	Selected model
BY processing	
_GLSNUMBYS	Number of BY groups
_GLSIND1	Selected model for BY group 1
_GLSIND2	Selected model for BY group 2
...	

You can use the macro variable _GLSIND as a synonym for _GLSIND1. If you do not use BY processing, _GLSNUMBYS is still defined and has the value 1.

To aid in associating indexed macro variables with the appropriate observations when BY processing is used, PROC GLMSELECT creates a variable _BY_ in the output data set specified in an OUTPUT statement (see the section "OUTPUT Statement" on page 2712) that tags observations with an index that matches the index of the appropriate macro variable.

The following statements create a data set with two BY groups and run PROC GLMSELECT to select a model for each BY group.

```
data one(drop=i j);
  array x{5} x1-x5;
  do i=1 to 1000;
    classVar = mod(i,4)+1;
    do j=1 to 5;
      x{j} = ranuni(1);
    end;
    if i<400 then do;
         byVar = 'group 1';
         y     = 3*classVar+7*x2+5*x2*x5+rannor(1);
       end;
    else do;
         byVar = 'group 2';
         y     = 2*classVar+x5+rannor(1);
       end;
    output;
  end;
run;

proc glmselect data=one;
  by     byVar;
  class  classVar;
  model  y = classVar x1|x2|x3|x4|x5 @2 /
                selection=stepwise(stop=aicc);
  output out=glmselectOutput;
run;
```

The preceding PROC GLMSELECT step produces three macro variables:

Macro Variable	Value	Description
_GLSNUMBYS	2	Number of BY groups
_GLSIND1	classVar x2 x2*x5	Selected model for the first BY group
_GLSIND2	classVar x5	Selected model for the second BY group

You can now leverage these macro variables and the output data set created by PROC GLMSELECT to perform post-selection analyses that match the selected models with the appropriate BY-group observations. For example, the following statements create and run a macro that uses PROC GLM to perform LSMeans analyses.

```
%macro LSMeansAnalysis;
   %do i=1 %to &_GLSNUMBYS;
      title1  "Analysis Using the Selected Model for BY group number &i";
      title2 "Selected Effects: &&_GLSIND&i";

      ods select LSMeans;
      proc glm data=glmselectOutput(where = (_BY_ = &i));
         class classVar;
         model y = &&_GLSIND&i;
         lsmeans classVar;
      run;quit;
   %end;
%mend;
%LSMeansAnalysis;
```

The LSMeans analysis output from PROC GLM is shown in Output 42.14.

Figure 42.14 LS-Means Analyses for Selected Models

```
Analysis Using the Selected Model for BY group number 1
Selected Effects: classVar x2 x2*x5

The GLM Procedure
Least Squares Means

class
Var          y LSMEAN

1           7.8832052
2          10.9528618
3          13.9412216
4          16.7929355
```

Figure 42.14 *continued*

```
Analysis Using the Selected Model for BY group number 2
Selected Effects: classVar x5

The GLM Procedure
Least Squares Means

class
Var          y LSMEAN

1          2.46805014
2          4.52102826
3          6.53369479
4          8.49354763
```

Building the SSCP Matrix

Traditional implementations of FORWARD and STEPWISE selection methods start by computing the augmented crossproduct matrix for all the specified effects. This initial crossproduct matrix is updated as effects enter or leave the current model by sweeping the columns corresponding to the parameters of the entering or departing effects. Building the starting crossproduct matrix can be done with a single pass through the data and requires $O(m^2)$ storage and $O(nm^2)$ work, where n is the number of observations and m is the number of parameters. If k selection steps are done, then the total work sweeping effects in and out of the model is $O(km^2)$. When $n >> m$, the work required is dominated by the time spent forming the crossproduct matrix. However, when m is large (tens of thousands), just storing the crossproduct matrix becomes intractable even though the number of selected parameters might be small. Note also that when interactions of classification effects are considered, the number of parameters considered can be large, even though the number of effects considered is much smaller.

When the number of selected parameters is smaller than the total number of parameters, it turns out that many of the crossproducts are not needed in the selection process. Let y denote the dependent variable, and suppose at some step of the selection process that X denotes the $n \times p$ design matrix columns corresponding to the currently selected model. Let $Z = Z_1, Z_2, \ldots, Z_{m-p}$ denote the design matrix columns corresponding to the $m - p$ effects not yet in the model. Then in order to compute the reduction in the residual sum of squares when Z_j is added to the model, the only additional crossproducts needed are $Z_j'y$, $Z_j'X$, and $Z_j'Z_j$. Note that it is not necessary to compute any of $Z_j'Z_i$ with $i \neq j$ and if $p << m$, and this yields a substantial saving in both memory required and computational work. Note, however, that this strategy does require a pass through the data at any step where adding an effect to the model is considered.

PROC GLMSELECT supports both of these strategies for building the crossproduct matrix. You can choose which of these strategies to use by specifying the BUILDSSCP=FULL or BUILDSSCP=INCREMENTAL option in the PERFORMANCE statement. If you request BACKWARD selection, then the full SSCP matrix is required. Similarly, if you request the BIC or CP criterion as the SELECT=, CHOOSE=, or STOP= criterion, or if you request the display of one or both of these criteria with the STATS=BIC, STATS=CP, or STATS=ALL option, then the full

model needs to be computed. If you do not specify the BUILDSSCP= option, then PROC GLMSELECT switches to the incremental strategy if the number of effects is greater than one hundred. This default strategy is designed to give good performance when the number of selected parameters is less than about 20% of the total number of parameters. Hence if you choose options that you know will cause the selected model to contain a significantly higher percentage of the total number of candidate parameters, then you should consider specifying BUILDSSCP=FULL. Conversely, if you specify fewer than 100 effects in the MODEL statement but many of these effects have a large number of associated parameters, then specifying BUILSSCP=INCREMENTAL might result in improved performance.

Parallel BY-Group Computation

The BY-group processing in PROC GLMSELECT is multithreaded, enabling parallel processing of BY groups when more than one processor is available. Before effect selection begins, PROC GLMSELECT preprocesses the entire input data set and writes the preprocessed data to one or more utility files. When you specify BY-group processing and there are multiple CPUs available, one utility file is created for each processor and each BY group is assigned to one of these utility files. Once this preprocessing phase is complete, each utility file is read and all the BY groups it contains are processed in a dedicated thread. Finally, once model selection has been completed for each thread, then the results are displayed sequentially from a single thread.

You can control the use of threading in PROC GLMSELECT and other multithreaded procedures by using the SAS system options THREADS | NOTHREADS and CPUCOUNT=. You can override these SAS system options by specifying THREADS | NOTHREADS and CPUCOUNT= in the PERFORMANCE statement. Note that if NOTHREADS is in effect, or CPUCOUNT=1, or no BY processing is used, then PROC GLMSELECT will use singly threaded code. If BY groups are to be processed in multiple threads, then the memory required is proportional to the number of threads used. PROC GLMSELECT tries to predict how much memory is required for model selection and reduces the number of threads it uses if there is insufficient memory for simultaneously processing more threads. You can find out the number of threads actually used by PROC GLMSELECT in the "Performance Settings" table that you request by using the DETAILS option in the PERFORMANCE statement.

The speedup you will obtain processing BY groups in parallel depends on several factors. First, if the time required to perform model selection on each BY group is small, then the multithreading overhead might negate any gains obtained by parallel processing. Another limiting factor is the I/O speed. When PROC GLMSELECT builds crossproduct matrices incrementally (see the section "Building the SSCP Matrix" on page 2735), data in each BY group are read multiple times. If there is insufficient I/O bandwidth, then the parallel processing can stall as CPUs wait on I/O. Optimal results will be obtained if each of the utility files used by PROC GLMSELECT is assigned to its own I/O controller. The locations of utility files can be controlled by using the SAS system option UTILLOC, which must be specified when SAS is invoked. For information about the UTILLOC option and additional information about parallel processing in SAS, see the chapter "Support for Parallel Processing" in *SAS Language Reference: Concepts.*

Using Validation and Test Data

When you have sufficient data, you can subdivide your data into three parts called the training, validation, and test data. During the selection process, models are fit on the training data, and the prediction error for the models so obtained is found by using the validation data. This prediction error on the validation data can be used to decide when to terminate the selection process or to decide what effects to include as the selection process proceeds. Finally, once a selected model has been obtained, the test set can be used to assess how the selected model generalizes on data that played no role in selecting the model.

In some cases you might want to use only training and test data. For example, you might decide to use an information criterion to decide what effects to include and when to terminate the selection process. In this case no validation data are required, but test data can still be useful in assessing the predictive performance of the selected model. In other cases you might decide to use validation data during the selection process but forgo assessing the selected model on test data. Hastie, Tibshirani, and Friedman (2001) note that it is difficult to give a general rule on how many observations you should assign to each role. They note that a typical split might be 50% for training and 25% each for validation and testing.

PROC GLMSELECT provides several methods for partitioning data into training, validation, and test data. You can provide data for each role in separate data sets that you specify with the DATA=, TESTDATA=, and VALDATA= options in the PROC GLMSELECT procedure. An alternative method is to use a PARTITION statement to logically subdivide the DATA= data set into separate roles. You can name the fractions of the data that you want to reserve as test data and validation data. For example, specifying

```
proc glmselect data=inData;
  partition fraction(test=0.25 validate=0.25);
  ...
run;
```

randomly subdivides the “inData” data set, reserving 50% for training and 25% each for validation and testing.

In some cases you might need to exercise more control over the partitioning of the input data set. You can do this by naming a variable in the input data set as well as a formatted value of that variable that correspond to each role. For example, specifying

```
proc glmselect data=inData;
  partition roleVar=group(test='group 1' train='group 2')
  ...
run;
```

assigns all roles observations in the “inData” data set based on the value of the variable named group in that data set. Observations where the value of group is ’group 1’ are assigned for testing, and those with value ’group 2’ are assigned to training. All other observations are ignored.

You can also combine the use of the PARTITION statement with named data sets for specifying data roles. For example,

```
proc glmselect data=inData testData=inTest;
  partition fraction(validate=0.4);
  ...
run;
```

reserves 40% of the "inData" data set for validation and uses the remaining 60% for training. Data for testing is supplied in the "inTest" data set. Note that in this case, because you have supplied a TESTDATA= data set, you cannot reserve additional observations for testing with the PARTITION statement.

When you use a PARTITION statement, the output data set created with an OUTPUT statement contains a character variable _ROLE_ whose values "TRAIN," "TEST," and "VALIDATE" indicate the role of each observation. _ROLE_ is blank for observations that were not assigned to any of these three roles. When the input data set specified in the DATA= option in the PROC GLMSELECT statement contains an _ROLE_ variable and no PARTITION statement is used, and TESTDATA= and VALDATA= are not specified, then the _ROLE_ variable is used to define the roles of each observation. This is useful when you want to rerun PROC GLMSELECT but use the same data partitioning as in a previous PROC GLMSELECT step. For example, the following statements use the same data for testing and training in both PROC GLMSELECT steps:

```
proc glmselect data=inData;
  partition fraction(test=0.5);
  model y=x1-x10/selection=forward;
  output out=outDataForward;
run;

proc glmselect data=outDataForward;
  model y=x1-x10/selection=backward;
run;
```

When you have reserved observations for training, validation, and testing, a model fit on the training data is scored on the validation and test data, and the average squared error, denoted by ASE, is computed separately for each of these subsets. The ASE for each data role is the error sum of squares for observations in that role divided by the number of observations in that role.

Using the Validation ASE as the STOP= Criterion

If you have provided observations for validation, then you can specify STOP=VALIDATE as a suboption of the SELECTION= option in the MODEL statement. At step k of the selection process, the best candidate effect to enter or leave the current model is determined. Note that here "best candidate" means the effect that gives the best value of the SELECT= criterion that need not be based on the validation data. The validation ASE for the model with this candidate effect added is computed. If this validation ASE is greater than the validation ASE for the model at step k, then the selection process terminates at step k.

Using the Validation ASE as the CHOOSE= Criterion

When you specify the CHOOSE=VALIDATE suboption of the SELECTION= option in the MODEL statement, the validation ASE is computed for the models at each step of the selection process. The model at the first step yielding the smallest validation ASE is selected.

Using the Validation ASE as the SELECT= Criterion

You request the validation ASE as the selection criterion by specifying the SELECT=VALIDATE suboption of the SELECTION= option in the MODEL statement. At step k of the selection process, the validation ASE is computed for each model where a candidate for entry is added or candidate for removal is dropped. The selected candidate for entry or removal is the one that yields a model with the minimal validation ASE.

Cross Validation

Deciding when to stop a selection method is a crucial issue in performing effect selection. Predictive performance of candidate models on data not used in fitting the model is one approach supported by PROC GLMSELECT for addressing this problem (see the section "Using Validation and Test Data" on page 2737). However, in some cases, you might not have sufficient data to create a sizable training set and a validation set that represent the predictive population well. In these cases, cross validation is an attractive alternative for estimating prediction error.

In k-fold cross validation, the data are split into k roughly equal-sized parts. One of these parts is held out for validation, and the model is fit on the remaining $k-1$ parts. This fitted model is used to compute the predicted residual sum of squares on the omitted part, and this process is repeated for each of k parts. The sum of the k predicted residual sum of squares so obtained is the estimate of the prediction error that is denoted by CVPRESS. Note that computing the CVPRESS statistic for k-fold cross validation requires fitting k different models, and so the work and memory requirements increase linearly with the number of cross validation folds.

You can use the CVMETHOD= option in the MODEL statement to specify the method for splitting the data into k parts. CVMETHOD=BLOCK(k) requests that the k parts be made of blocks of floor(n/k) or floor(n/k) + 1 successive observations, where n is the number of observations. CVMETHOD=SPLIT(k) requests that parts consist of observations $\{1, k+1, 2k+1, 3k+1, \ldots\}$, $\{2, k+2, 2k+2, 3k+2, \ldots\}$, $\ldots$, $\{k, 2k, 3k, \ldots\}$. CVMETHOD=RANDOM(k) partitions the data into random subsets each with roughly floor(n/k) observations. Finally, you can use the formatted value of an input data set variable to define the parts by specifying CVMETHOD=*variable*. This last partitioning method is useful in cases where you need to exercise extra control over how the data are partitioned by taking into account factors such as important but rare observations that you want to "spread out" across the various parts.

You can request details of the CVPRESS computations by specifying the CVDETAILS= option in the MODEL statement. When you use cross validation, the output data set created with an OUTPUT statement contains an integer-valued variable, _CVINDEX_, whose values indicate the subset to which an observation is assigned.

The widely used special case of n-fold cross validation when you have n observations is known as *leave-one-out* cross validation. In this case, each omitted part consists of one observation, and CVPRESS statistic can be efficiently obtained without refitting the model n times. In this case, the CVPRESS statistic is denoted simply by PRESS and is given by

$$\text{PRESS} = \sum_{i=1}^{n} \left(\frac{r_i}{1 - h_i} \right)^2$$

where r_i is the residual and h_i is the leverage of the ith observation. You can request *leave-one-out* cross validation by specifying PRESS instead of CV with the options SELECT=, CHOOSE=, and STOP= in the MODEL statement. For example, if the number of observations in the data set is 100, then the following two PROC GLMSELECT steps are mathematically equivalent, but the second step is computed much more efficiently:

```
proc glmselect;
    model y=x1-x10/selection=forward(stop=CV) cvMethod=split(100);
run;

proc glmselect;
    model y=x1-x10/selection=forward(stop=PRESS);
run;
```

Hastie, Tibshirani, and Friedman (2001) include a discussion about choosing the cross validation fold. They note that as an estimator of true prediction error, cross validation tends to have decreasing bias but increasing variance as the number of folds increases. They recommend five- or tenfold cross validation as a good compromise. By default, PROC GLMSELECT uses CVMETHOD=RANDOM(5) for cross validation.

Using Cross Validation as the STOP= Criterion

You request cross validation as the stopping criterion by specifying the STOP=CV suboption of the SELECTION= option in the MODEL statement. At step k of the selection process, the best candidate effect to enter or leave the current model is determined. Note that here "best candidate" means the effect that gives the best value of the SELECT= criterion that need not be the CV criterion. The CVPRESS score for the model with this candidate effect added or removed is determined. If this CVPRESS score is greater than the CVPRESS score for the model at step k, then the selection process terminates at step k.

Using Cross Validation as the CHOOSE= Criterion

When you specify the CHOOSE=CV suboption of the SELECTION= option in the MODEL statement, the CVPRESS score is computed for the models at each step of the selection process. The model at the first step yielding the smallest CVPRESS score is selected.

Using Cross Validation as the SELECT= Criterion

You request cross validation as the selection criterion by specifying the SELECT=CV suboption of the SELECTION= option in the MODEL statement. At step k of the selection process, the CVPRESS score is computed for each model where a candidate for entry is added or a candidate for removal is dropped. The selected candidate for entry or removal is the one that yields a model with the minimal CVPRESS score. Note that at each step of the selection process, this requires forming the CVPRESS statistic for all possible candidate models at the next step. Since forming the CVPRESS statistic for k-fold requires fitting k models, using cross validation as the selection criterion is computationally very demanding compared to using other selection criteria.

Displayed Output

The following sections describe the displayed output produced by PROC GLMSELECT. The output is organized into various tables, which are discussed in the order of appearance. Note that the contents of a table might change depending on the options you specify.

Model Information

The "Model Information" table displays basic information about the data sets and the settings used to control effect selection. These settings include the following:

- the selection method
- the criteria used to select effects, stop the selection, and choose the selected model
- the effect hierarchy enforced

For ODS purposes, the name of the "Model Information" table is "ModelInfo."

Performance Settings

The "Performance Settings" table displays settings that affect performance. These settings include whether threading is enabled and the number of CPUs available as well as the method used to build the crossproduct matrices. This table is displayed only if you specify the DETAILS option in the PERFORMANCE statement. For ODS purposes, the name of the "Performance Settings" table is "PerfSettings."

Number of Observations

The "Number of Observations" table displays the number of observations read from the input data set and the number of observations used in the analysis. If you specify a FREQ statement, the table

also displays the sum of frequencies read and used. If you use a PARTITION statement, the table also displays the number of observations used for each data role. If you specify TESTDATA= or VALDATA= data sets in the PROC GLMSELECT statement, then "Number of Observations" tables are also produced for these data sets. For ODS purposes, the name of the "Number of Observations" table is "NObs."

Class Level Information

The "Class Level Information" table lists the levels of every variable specified in the CLASS statement. For ODS purposes, the name of the "Class Level Information" table is "ClassLevelInfo."

Class Level Coding

The "Class Level Coding" table shows the coding used for variables specified in the CLASS statement. For ODS purposes, the name of the "Class Level Coding" table is "ClassLevelCoding."

Dimensions

The "Dimensions" table displays information about the number of effects and the number of parameters from which the selected model is chosen. If you use split classification variables, then this table also includes the number of effects after splitting is taken into account. For ODS purposes, the name of the "Dimensions" table is "Dimensions."

Candidates

The "Candidates" table displays the effect names and values of the criterion used to select entering or departing effects at each step of the selection process. The effects are displayed in sorted order from best to worst of the selection criterion. You request this table with the DETAILS= option in the MODEL statement. For ODS purposes, the name of the "Candidates" table is "Candidates."

Selection Summary

The "Selection Summary" table displays details about the sequence of steps of the selection process. For each step, the effect that was entered or dropped is displayed along with the statistics used to select the effect, stop the selection, and choose the selected model. You can request that additional statistics be displayed with the STATS= option in the MODEL statement. For all criteria that you can use for model selection, the steps at which the optimal values of these criteria occur are also indicated. For ODS purposes, the name of the "Selection Summary" table is "SelectionSummary."

Stop Reason

The "Stop Reason" table displays the reason why the selection stopped. To facilitate programmatic use of this table, an integer code is assigned to each reason and is included if you output this table by using an ODS OUTPUT statement. The reasons and their associated codes follow:

Code	Stop Reason
1	maximum number of steps done
2	specified number of steps done
3	specified number of effects in model
4	stopping criterion at local optimum
5	model is an exact fit
6	all entering effects are linearly dependent on those in the model
7	all effects are in the model
8	all effects have been dropped
9	requested full least squares fit completed
10	stepwise selection is cycling
11	dropping any effect does not improve the selection criterion
12	no effects are significant at the specified SLE or SLS levels
13	adding or dropping any effect does not improve the selection criterion
14	all remaining effects are required

For ODS purposes, the name of the "Stop Reason" table is "StopReason."

Stop Details

The "Stop Details" table compares the optimal value of the stopping criterion at the final model with how it would change if the best candidate effect were to enter or leave the model. For ODS purposes, the name of the "Stop Details" table is "StopDetails."

Selected Effects

The "Selected Effects" table displays a string containing the list of effects in the selected model. For ODS purposes, the name of the "Selected Effects" table is "SelectedEffects."

ANOVA

The "ANOVA" table displays an analysis of variance for the selected model. This table includes the following:

- the Source of the variation, Model for the fitted regression, Error for the residual error, and C Total for the total variation after correcting for the mean. The Uncorrected Total Variation is produced when the NOINT option is used.

- the degrees of freedom (DF) associated with the source
- the Sum of Squares for the term
- the Mean Square, the sum of squares divided by the degrees of freedom
- the F Value for testing the hypothesis that all parameters are zero except for the intercept. This is formed by dividing the mean square for Model by the mean square for Error.
- the Prob>F, the probability of getting a greater F statistic than that observed if the hypothesis is true. Note that these p-values are displayed only if you specify the "SHOWPVALUES" option in the MODEL statement. These p-values are generally liberal because they are not adjusted for the fact that the terms in the model have been selected.

You can request "ANOVA" tables for the models at each step of the selection process with the DETAILS= option in the MODEL statement. For ODS purposes, the name of the "ANOVA" table is "ANOVA."

Fit Statistics

The "Fit Statistics" table displays fit statistics for the selected model. The statistics displayed include the following:

- Root MSE, an estimate of the standard deviation of the error term. It is calculated as the square root of the mean square error.
- Dep Mean, the sample mean of the dependent variable
- R-square, a measure between 0 and 1 that indicates the portion of the (corrected) total variation attributed to the fit rather than left to residual error. It is calculated as SS(Model) divided by SS(Total). It is also called the *coefficient of determination*. It is the square of the multiple correlation—in other words, the square of the correlation between the dependent variable and the predicted values.
- Adj R-Sq, the adjusted R^2, a version of R^2 that has been adjusted for degrees of freedom. It is calculated as

$$\bar{R}^2 = 1 - \frac{(n-i)(1-R^2)}{n-p}$$

  where i is equal to 1 if there is an intercept and 0 otherwise, n is the number of observations used to fit the model, and p is the number of parameters in the model.
- fit criteria AIC, AICC, BIC, CP, and PRESS if they are used in the selection process or are requested with the STATS= option. See the section "Criteria Used in Model Selection Methods" on page 2725 for details and Table 42.6 for the formulas for evaluating these criteria.
- the CVPRESS statistic when cross validation is used in the selection process. See the section "Cross Validation" on page 2739 for details.
- the average square errors (ASE) on the training, validation, and test data. See the section "Using Validation and Test Data" on page 2737 for details.

You can request "Fit Statistics" tables for the models at each step of the selection process with the DETAILS= option in the MODEL statement. For ODS purposes, the name of the "Fit Statistics" table is "FitStatistics."

Cross Validation Details

The "Cross Validation Details" table displays the following:

- the fold number
- the number of observations used for fitting
- the number of observations omitted
- the predicted residual sum of squares on the omitted observations

You can request this table with the CVDETAILS= option in the MODEL statement whenever cross validation is used in the selection process. This table is displayed for the selected model, but you can request this table at each step of the selection process by using the DETAILS= option in the MODEL statement. For ODS purposes, the name of the "Cross Validation Details" table is "CVDetails."

Parameter Estimates

The "Parameter Estimates" table displays the parameters in the selected model and their estimates. The information displayed for each parameter in the selected model includes the following:

- the parameter label that includes the effect name and level information for effects containing classification variables
- the degrees of freedom (DF) for the parameter. There is one degree of freedom unless the model is not full rank.
- the parameter estimate
- the standard error, which is the estimate of the standard deviation of the parameter estimate
- T for H0: Parameter=0, the t test that the parameter is zero. This is computed as the parameter estimate divided by the standard error.
- the Prob > |T|, the probability that a t statistic would obtain a greater absolute value than that observed given that the true parameter is zero. This is the two-tailed significance probability. Note that these p-values are displayed only if you specify the "SHOWPVALUES" option in the MODEL statement. These p-values are generally liberal because they are not adjusted for the fact that the terms in the model have been selected.

If cross validation is used in the selection process, then you can request that estimates of the parameters for each cross validation fold be included in the "Parameter Estimates" table by using the CVDETAILS= option in the MODEL statement. You can request "Parameter Estimates" tables for the models at each step of the selection process with the DETAILS= option in the MODEL statement. For ODS purposes, the name of the "Parameter Estimates" table is "ParameterEstimates."

Score Information

For each SCORE statement, the "Score Information" table displays the names of the score input and output data sets, and the number of observations that were read and successfully scored. For ODS purposes, the name of the "Score Information" table is "ScoreInfo."

Timing Breakdown

The "Timing Breakdown" table displays a broad breakdown of where time was spent in the PROC GLMSELECT step. This table is displayed only if you specify the DETAILS option in the PERFORMANCE statement. If multithreaded BY-group processing is employed, then the number of threads used for the various phases of the computation is displayed. For ODS purposes, the name of the "Timing Breakdown" table is "Timing."

ODS Table Names

PROC GLMSELECT assigns a name to each table it creates. You can use these names to reference the table when you use the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in Table 42.7.

For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 42.7 ODS Tables Produced by PROC GLMSELECT

ODS Table Name	Description	Statement	Option
ANOVA	selected model ANOVA table	MODEL	default
BSplineDetails	B-spline basis details	EFFECT	DETAILS
Candidates	entry/removal effect ranking	MODEL	DETAILS=
ClassLevelCoding	classification variable coding	CLASS	SHOWCODING
ClassLevelInfo	classification variable levels	CLASS	default
CollectionLevelInfo	levels of collection effects	EFFECT	DETAILS
CVDetails	cross validation PRESS by fold	MODEL	CVDETAILS=
Dimensions	number of effects and parameters	MODEL	default
FitStatistics	selected model fit statistics	MODEL	default
MMLevelInfo	levels of MM effects	EFFECT	DETAILS
ModelInfo	model information	MODEL	default
NObs	number of observations	MODEL	default
ParameterNames	labels for column names in the design matrix	PROC	OUTDESIGN(names)
ParameterEstimates	selected model parameter estimates	MODEL	default
PerfSettings	performance settings	PERFORMANCE	DETAILS
PolynomialDetails	polynomial details	EFFECT	DETAILS
PolynomialScaling	polynomial scaling	EFFECT	DETAILS

Table 42.7 *continued*

ODS Table Name	Description	Statement	Option
ScoreInfo	score request information	SCORE	default
SelectedEffects	list of selected effects	MODEL	default
SelectionSummary	selection summary	MODEL	default
StopDetails	stopping criterion details	MODEL	default
StopReason	reason why selection stopped	MODEL	default
Timing	timing details	PERFORMANCE	DETAILS
TPFSplineDeatils	TPF spline basis details	EFFECT	DETAILS

ODS Graphics

This section describes the use of ODS for creating statistical graphs with the GLMSELECT procedure. To request these graphs you must specify the ODS GRAPHICS statement and request plots with the PLOTS= option in the PROC GLMSELECT statement. For more information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS." The following sections describe the ODS graphical displays produced by PROC GLMSELECT. The examples use the Baseball data set that is described in the section "Getting Started: GLMSELECT Procedure" on page 2684.

Candidates Plot

You request the "Candidates Plot" by specifying the PLOTS=CANDIDATES option in the PROC GLMSELECT statement and the DETAILS=STEPS option in the MODEL statement. This plot shows the values of selection criterion for the candidate effects for entry or removal, sorted from best to worst from left to right across the plot. The leftmost candidate displayed is the effect selected for entry or removal at that step. You can use this plot to see at what steps the decision about which effect to add or drop is clear-cut. See Figure 42.5 for an example.

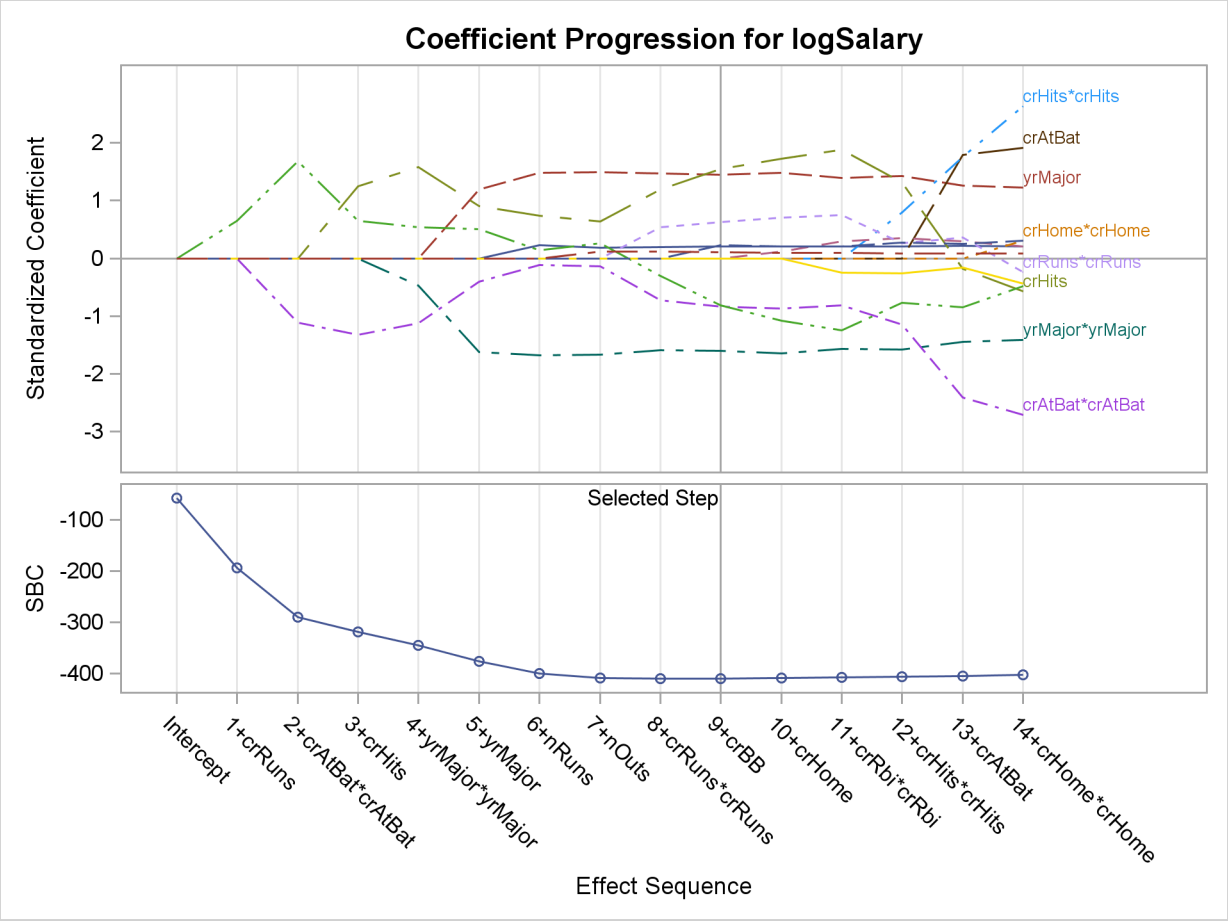
Coefficient Panel

When you specify the PLOTS=COEFFICIENTS option in the PROC GLMSELECT statement, PROC GLMSELECT produces a panel of two plots showing how the standardized coefficients and the criterion used to choose the final model evolve as the selection progresses. The following statements provide an example:

```
ods graphics on;

proc glmselect data=baseball plots=coefficients;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor|yrMajor crAtBat|crAtBat crHits|crHits
                    crHome|crHome crRuns|crRuns crRbi|crRbi
                    crBB|crBB league division nOuts nAssts nError /
                    selection=forward(stop=AICC CHOOSE=SBC);
run;
```

Figure 42.15 shows the requested graphic. The upper plot in the panel displays the standardized coefficients as a function of the step number. You can request standardized coefficients in the parameter estimates tables by specifying the STB option in the MODEL statement, but this option is not required to produce this plot. To help in tracing the changes in a parameter, the standardized coefficients for each parameter are connected by lines. Coefficients corresponding to effects that are not in the selected model at a step are zero and hence not observable. For example, consider the parameter crAtBat*crAtBat in Output 42.15. Because crAtBat*crAtBat enters the model at step 2, the line that represents this parameter starts rising from zero at step 1 when crRuns enters the model. Parameters that are nonzero at the final step of the selection are labeled if their magnitudes are greater than 1% of the range of the magnitudes of all the nonzero parameters at this step. To avoid collision, labels corresponding to parameters with similar values at the final step might get suppressed. You can control when this label collision avoidance occurs by using the LABELGAP= suboption of the PLOTS=COEFFICIENTS option. Planned enhancements to the automatic label collision avoidance algorithm will obviate the need for this option in future releases of the GLMSELECT procedure.

Figure 42.15 Coefficient Panel



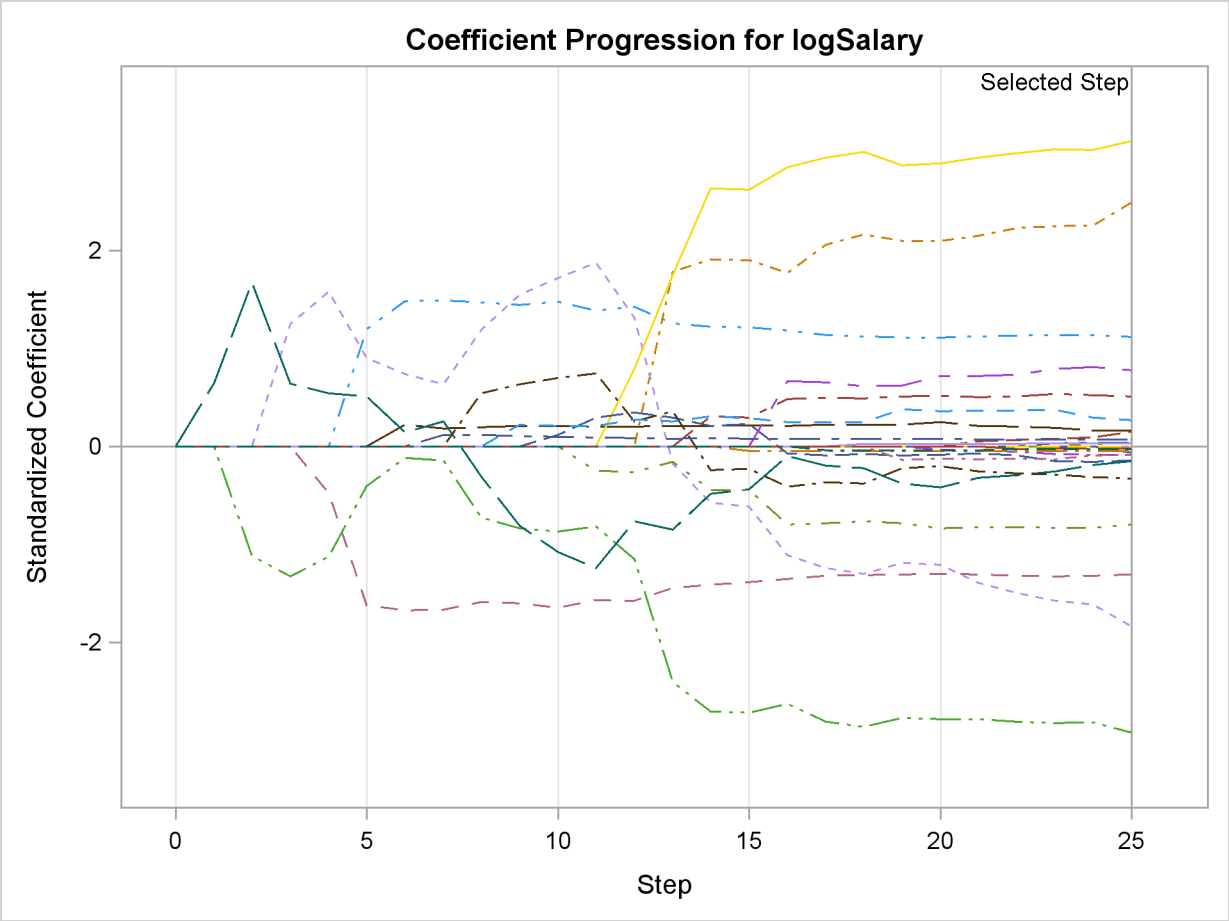
The lower plot in the panel shows how the criterion used to choose among the examined models progresses. The selected step occurs at the optimal value of this criterion. In this example, this criterion is the SBC criterion and it achieves its minimal value at step 9 of the forward selection.

In some cases, particularly when the final step contains a large number of parameters, you might be interested in using this plot only to discern if and when the parameters in the model are essential unchanged beyond a certain step. In such cases, you might want to suppress the labeling of the parameters and use a numeric axis on the horizontal axis of the plot. You can do this using the STEPAXIS= and MAXPARMLABEL= suboptions of the PLOTS=CRITERIA option. The following statements provide an example:

```
proc glmselect data=baseball
     plots(unpack maxparmlabel=0 stepaxis=number)=coefficients;

  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor|yrMajor crAtBat|crAtBat crHits|crHits
                    crHome|crHome crRuns|crRuns crRbi|crRbi
                    crBB|crBB league division nOuts nAssts nError /
                    selection=forward(stop=none);
run;
```

The UNPACK = option requests that the plots of the coefficients and CHOOSE= criterion be shown in separate plots. The STEPAXIS=NUMBER option requests a numeric horizontal axis showing step number, and the MAXPAMLABEL=0 option suppresses the labels for the parameters. The "Coefficient Plot" is shown in Figure 42.16. You can see that the standardized coefficients do not vary greatly after step 16.

Figure 42.16 Coefficient Plot



Criterion Panel

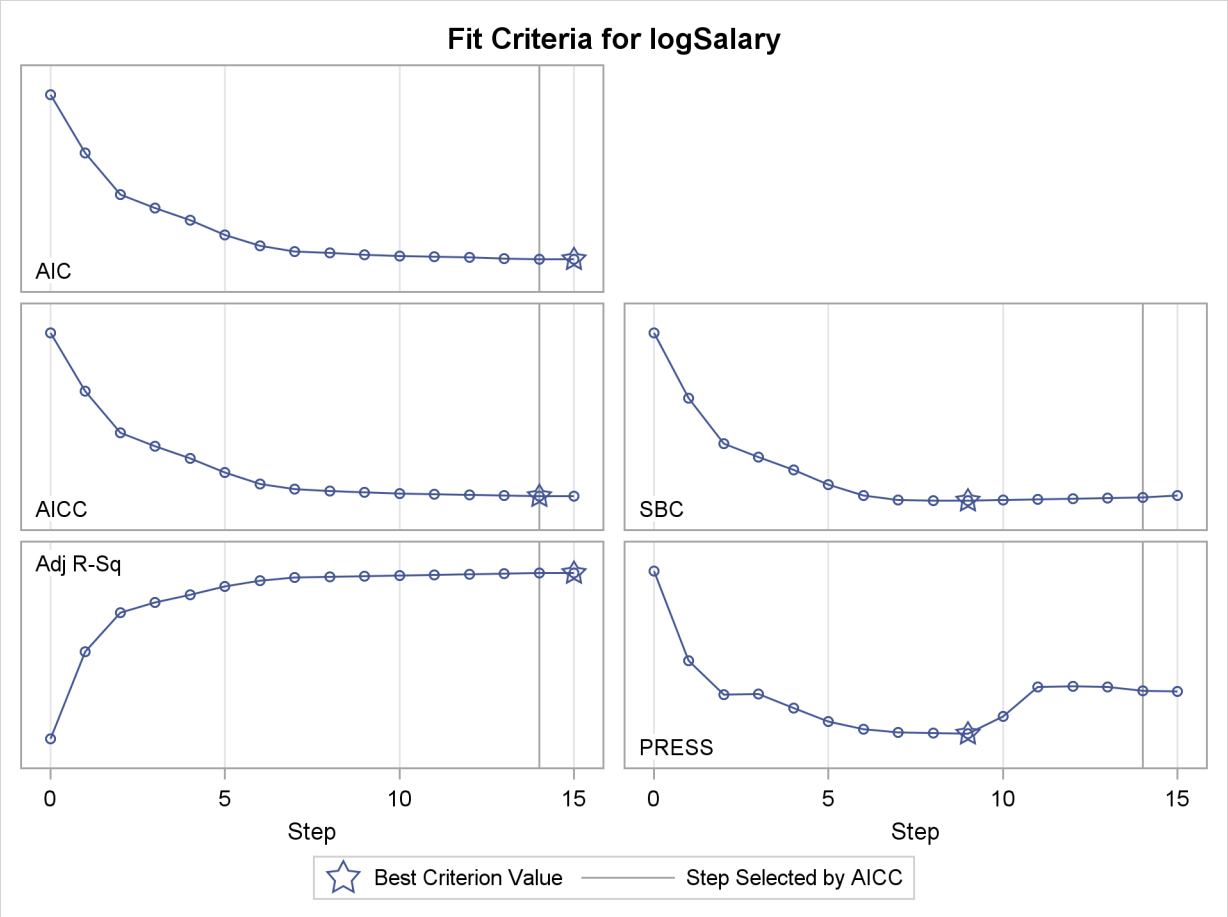
You request the criterion panel by specifying the PLOTS=CRITERIA option in the PROC GLMSELECT statement. This panel displays the progression of the ADJRSQ, AIC, AICC, and SBC criteria, as well as any other criteria that are named in the CHOOSE=, SELECT=, STOP=, or STATS= option in the MODEL statement.

The following statements provide an example:

```
proc glmselect data=baseball plots=criteria;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor|yrMajor crAtBat|crAtBat crHits|crHits
                    crHome|crHome crRuns|crRuns crRbi|crRbi
                    crBB|crBB league division nOuts nAssts nError /
                    selection=forward(steps=15 choose=AICC)
                    stats=PRESS;
run;
```

Figure 42.17 shows the requested criterion panel. Note that the PRESS criterion is included in the panel because it is named in the STATS= option in the MODEL statement. The selected step is displayed as a vertical reference line on the plot of each criterion, and the legend indicates which of these criteria is used to make the selection. If the selection terminates for a reason other than optimizing a criterion displayed on this plot, then the legend will not report a reason for the selected step. The optimal value of each criterion is indicated with the "Star" marker. Note that it is possible that a better value of a criterion might have been reached had more steps of the selection process been done.

Figure 42.17 Criterion Panel



Average Square Error Plot

You request the average square error plot by specifying the PLOTS=ASE option in the PROC GLMSELECT statement. This plot shows the progression of the average square error (ASE) evaluated separately on the training data, and the test and validation data whenever these data are provided with the TESTDATA= and VALDATA= options or are produced by using a PARTITION statement. You use the plot to detect when overfitting the training data occurs. The ASE decreases monotonically on the training data as parameters are added to a model. However, the average square error on test and validation data typically starts increasing when overfitting occurs. See Output 42.1.9 and Output 42.2.6 for examples.

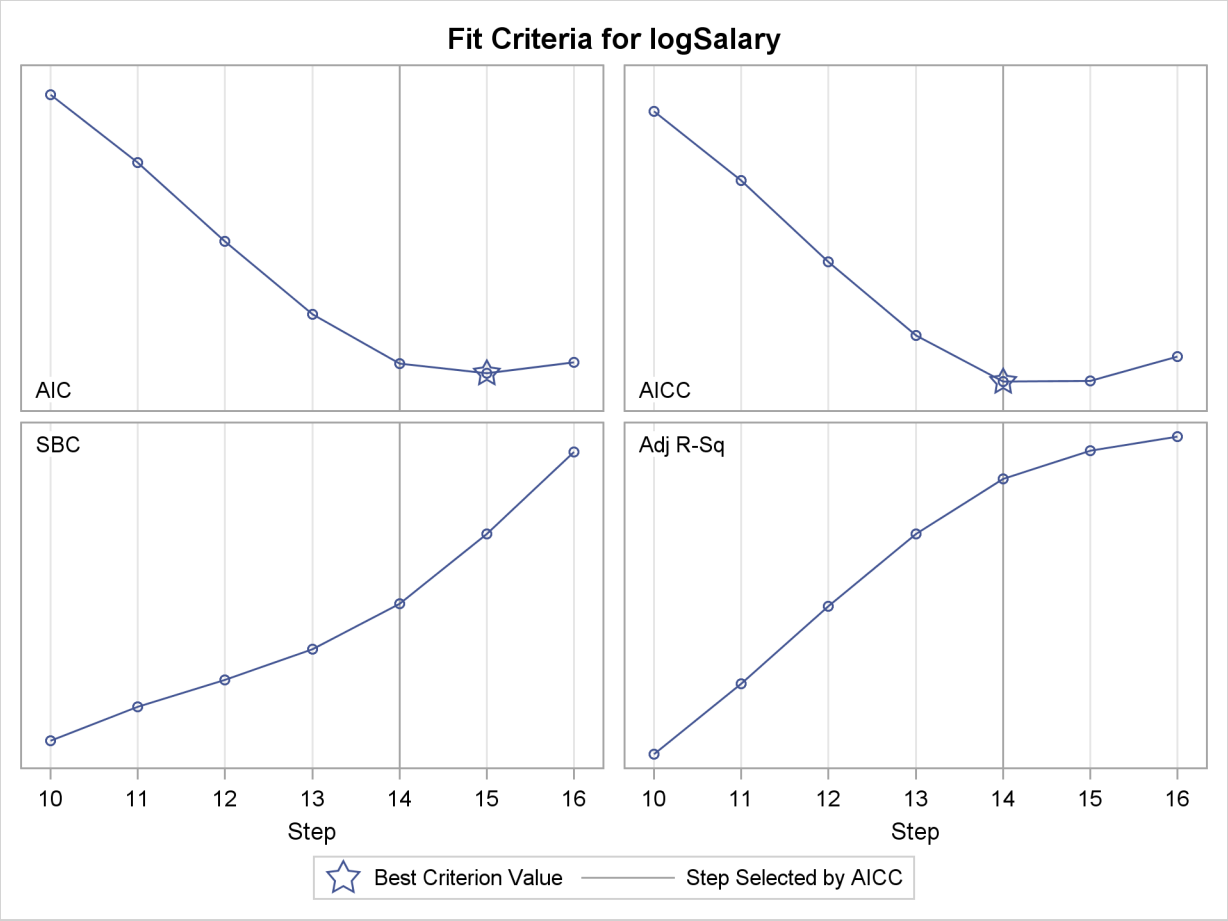
Examining Specific Step Ranges

The coefficient panel, criterion panel, and average square error plot display information for all the steps examined in the selection process. In some cases, you might want to focus attention on just a particular step range. For example, it is hard to discern the variation in the criteria displayed in Figure 42.17 near the selected step because the variation in these criteria in the steps close to the selected step is small relative to the variation across all steps. You can request a range of steps to display using the STARTSTEP= and ENDSTEP= suboptions of the PLOTS= option. You can specify these options as both global and specific plot options, with the specific options taking precedence if both are specified. The following statements provide an example:

```
proc glmselect data=baseball plots=criteria(startstep=10 endstep=16);
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor|yrMajor crAtBat|crAtBat crHits|crHits
                    crHome|crHome crRuns|crRuns crRbi|crRbi
                    crBB|crBB league division nOuts nAssts nError /
                    selection=forward(stop=none choose=AICC);
run;

ods graphics off;
```

Figure 42.18 shows the progression of the fit criteria between steps 10 and 16. Note that if the optimal value of a criterion does not occur in this specified step range, then no optimal marker appears for that criterion. The plot of the SBC criterion in Figure 42.18 is one such case.

Figure 42.18 Criterion Panel for Specified Step Range



ODS Graph Names

PROC GLMSELECT assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when using ODS. The names are listed in Table 42.8.

To request these graphs you must specify the ODS GRAPHICS statement. For more information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS."

Table 42.8 ODS Graphics Produced by PROC GLMSELECT

ODS Graph Name	Plot Description	PLOTS Option
AdjRSqPlot	Adjusted R-square by step	CRITERIA(UNPACK)
AICCPlot	Corrected Akaike information criterion by step	CRITERIA(UNPACK)
AICPlot	Akaike information criterion by step	CRITERIA(UNPACK)
ASEPlot	Average square errors by step	ASE
BICPlot	Sawa's Bayesian information criterion by step	CRITERIA(UNPACK)
CandidatesPlot	SELECT criterion by effect	CANDIDATES
ChooseCriterionPlot	CHOOSE criterion by step	COEFFICIENTS(UNPACK)

Table 42.8 *continued*

ODS Graph Name	Plot Description	PLOTS= Option
CoefficientPanel	Coefficients and CHOOSE criterion by step	COEFFICIENTS
CoefficientPlot	Coefficients by step	COEFFICIENTS(UNPACK)
CPPlot	Mallows C_p by step	CRITERIA(UNPACK)
CriterionPanel	Fit criteria by step	CRITERIA
CVPRESSPlot	Cross validation predicted RSS by step	CRITERIA(UNPACK)
PRESSPlot	Predicted RSS by step	CRITERIA(UNPACK)
SBCPlot	Schwarz Bayesian information criterion by step	CRITERIA(UNPACK)
ValidateASEPlot	Average square error on validation data by step	CRITERIA(UNPACK)

Examples: GLMSELECT Procedure

Example 42.1: Modeling Baseball Salaries Using Performance Statistics

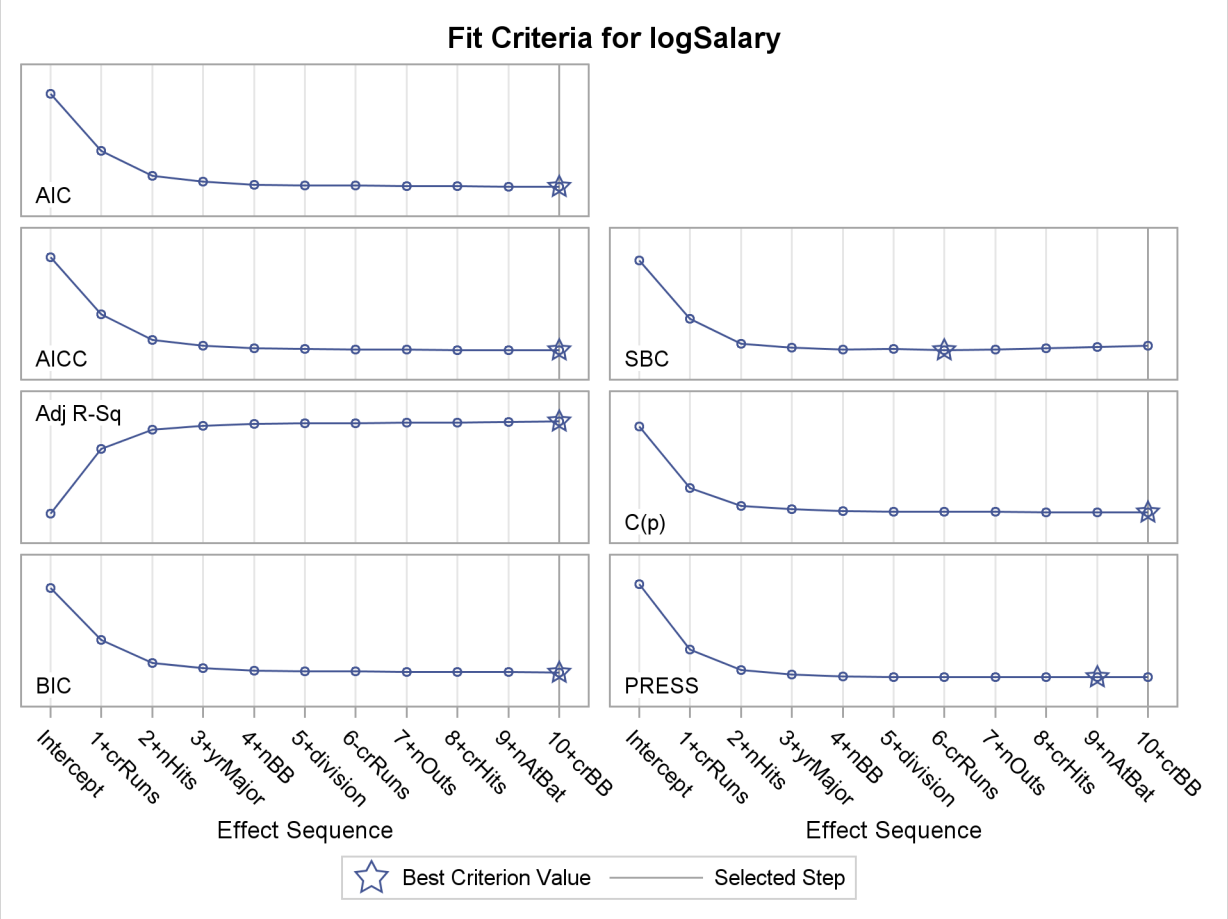
This example continues the investigation of the baseball data set introduced in the section "Getting Started: GLMSELECT Procedure" on page 2684. In that example, the default stepwise selection method based on the SBC criterion was used to select a model. In this example, model selection that uses other information criteria and out-of-sample prediction criteria is explored.

PROC GLMSELECT provides several selection algorithms that you can customize by specifying criteria for selecting effects, stopping the selection process, and choosing a model from the sequence of models at each step. For more details on the criteria available, see the section "Criteria Used in Model Selection Methods" on page 2725. The SELECT=SL suboption of the SELECTION= option in the MODEL statement in the following code requests the traditional hypothesis test-based stepwise selection approach, where effects in the model that are not significant at the stay significance level (SLS) are candidates for removal and effects not yet in the model whose addition is significant at the entry significance level (SLE) are candidates for addition to the model.

```
ods graphics on;

proc glmselect data=baseball plot=CriterionPanel;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor crAtBat crHits crHome crRuns crRbi
                    crBB league division nOuts nAssts nError
                  / selection=stepwise(select=SL) stats=all;
run;
```

The default SLE and SLS values of 0.15 might not be appropriate for these data. One way to investigate alternative ways to stop the selection process is to assess the sequence of models in terms of model fit statistics. The STATS=ALL option in the MODEL statement requests that all model fit statistics for assessing the sequence of models of the selection process be displayed. To help in the interpretation of the selection process, you can use graphics supported by PROC GLMSELECT. You enable these graphical displays by specifying the ODS GRAPHICS statement. For general information about ODS graphics, see Chapter 21, "Statistical Graphics Using ODS." With ODS graphics enabled, the PLOTS=CRITRERIONPANEL option in the PROC GLMSELECT statement produces the criterion panel shown in Output 42.1.1.

Output 42.1.1 Criterion Panel



You can see in Output 42.1.1 that this stepwise selection process would stop at an earlier step if you use the Schwarz Bayesian information criterion (SBC) or predicted residual sum of squares (PRESS) to assess the selected models as stepwise selection progresses. You can use the CHOOSE= suboption of the SELECTION= option in the MODEL statement to specify the criterion you want to use to select among the evaluated models. The following statements use the PRESS statistic to choose among the models evaluated during the stepwise selection.

```
proc glmselect data=baseball;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor crAtBat crHits crHome crRuns crRbi
                    crBB league division nOuts nAssts nError
                  / selection=stepwise(select=SL choose=PRESS);
run;
```

Note that the selected model is the model at step 9. By default, PROC GLMSELECT displays the selected model, ANOVA and fit statistics, and parameter estimates for the selected model. These are shown in Output 42.1.2.

Output 42.1.2 Details of Selected Model

The GLMSELECT Procedure
Selected Model

The selected model, based on PRESS, is the model at Step 9.

Effects: Intercept nAtBat nHits nBB yrMajor crHits division nOuts

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value
Model	7	124.67715	17.81102	55.07
Error	255	82.47658	0.32344	
Corrected Total	262	207.15373		

Root MSE	0.56872
Dependent Mean	5.92722
R-Square	0.6019
Adj R-Sq	0.5909
AIC	-23.98522
AICC	-23.27376
PRESS	88.55275
SBC	-260.40799

Parameter Estimates

Parameter		DF	Estimate	Standard Error	t Value
Intercept		1	4.176133	0.150539	27.74
nAtBat		1	-0.001468	0.000946	-1.55
nHits		1	0.011078	0.002983	3.71
nBB		1	0.007226	0.002115	3.42
yrMajor		1	0.070056	0.018911	3.70
crHits		1	0.000247	0.000143	1.72
division	East	1	0.143082	0.070972	2.02
division	West	0	0	.	.
nOuts		1	0.000241	0.000134	1.81

Even though the model that is chosen to give the smallest value of the PRESS statistic is the model at step 9, the stepwise selection process continues to the step where the stopping condition based on entry and stay significance levels is met. If you use the PRESS statistic as the stopping criterion, the stepwise selection process stops at step 9. This ability to stop at the first extremum of the criterion you specify can significantly reduce the amount of computation done, especially in the cases where you are selecting from a large number of effects. The following statements request stopping based on the PRESS statistic. The stop reason and stop details tables are shown in Output 42.1.3.

```
proc glmselect data=baseball plot=Coefficients;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                yrMajor crAtBat crHits crHome crRuns crRbi
                crBB league division nOuts nAssts nError
              / selection=stepwise(select=SL stop=PRESS);
run;
```

Output 42.1.3 Stopping Based on PRESS Statistic

The GLMSELECT Procedure

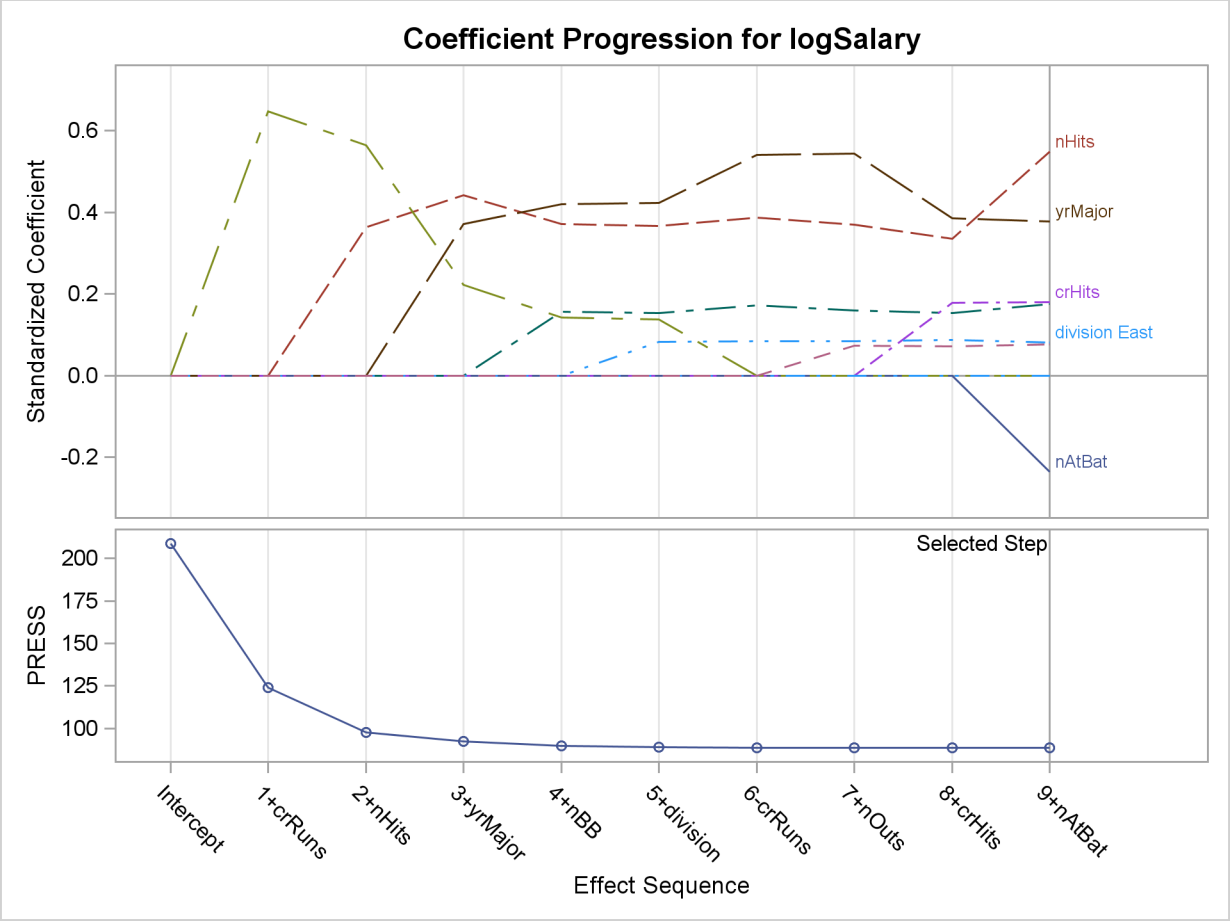
Selection stopped at a local minimum of the PRESS criterion.

Stop Details

Candidate For	Effect	Candidate PRESS		Compare PRESS
Entry	crBB	88.6321	>	88.5528
Removal	nAtBat	88.6866	>	88.5528

The PLOTS=COEFFICIENTS specification in the PROC GLMSELECT statement requests a plot that enables you to visualize the selection process.

Output 42.1.4 Coefficient Progression Plot



Output 42.1.4 shows the standardized coefficients of all the effects selected at some step of the stepwise method plotted as a function of the step number. This enables you to assess the relative importance of the effects selected at any step of the selection process as well as providing information as to when effects entered the model. The lower plot in the panel shows how the criterion used to choose the selected model changes as effects enter or leave the model.

Model selection is often done in order to obtain a parsimonious model that can be used for prediction on new data. An ever-present danger is that of selecting a model that overfits the "training" data used in the fitting process, yielding a model with poor predictive performance. Using cross validation is one way to assess the predictive performance of the model. Using k-fold cross validation, the training data are subdivided into k parts, and at each step of the selection process, models are obtained on each of the k subsets of the data obtained by omitting one of these parts. The cross validation predicted residual sum of squares, denoted CV PRESS, is obtained by summing the squares of the residuals when each of these submodels is scored on the data omitted in fitting the submodel. Note that the PRESS statistic corresponds to the special case of "leave-one-out" cross validation.

In the preceding example, the PRESS statistic was used to choose among models that were chosen based on entry and stay significance levels. In the following statements, the SELECT=CVPRESS suboption of the SELECTION= option in the MODEL statement requests that the CV PRESS statis-

tic itself be used as the selection criterion. The DROP=COMPETITIVE suboption requests that additions and deletions be considered simultaneously when deciding whether to add or remove an effect. At any step, the CV PRESS statistic for all models obtained by deleting one effect from the model or adding one effect to the model is computed. Among these models, the one yielding the smallest value of the CV PRESS statistic is selected and the process is repeated from this model. The stepwise selection terminates if all additions or deletions increase the CV PRESS statistic. The CVMETHOD=SPLIT(5) option requests five-fold cross validation with the five subsets consisting of observations $\{1, 6, 11, \ldots\}$, $\{2, 7, 12, \ldots\}$, and so on.

```
proc glmselect data=baseball plot=Candidates;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor crAtBat crHits crHome crRuns crRbi
                    crBB league division nOuts nAssts nError
                  / selection=stepwise(select=CV drop=competitive)
                    cvMethod=split(5);
run;
```

The selection summary table is shown in Output 42.1.5. By comparing Output 42.1.5 and Output 42.6 you can see that the sequence of models produced is different from the sequence when the stepwise selection is based on the SBC statistic.

Output 42.1.5 Stepwise Selection Based on Cross Validation

The GLMSELECT Procedure

Stepwise Selection Summary

Step	Effect Entered	Effect Removed	Number Effects In	Number Parms In	CV PRESS
0	Intercept		1	1	208.9638
1	crRuns		2	2	122.5755
2	nHits		3	3	96.3949
3	yrMajor		4	4	92.2117
4	nBB		5	5	89.5242
5		crRuns	4	4	88.6917
6	league		5	5	88.0417
7	nError		6	6	87.3170
8	division		7	7	87.2147
9	nHome		8	8	87.0960*

* Optimal Value Of Criterion

If you have sufficient data, another way you can assess the predictive performance of your model is to reserve part of your data for testing your model. You score the model obtained using the training data on the test data and assess the predictive performance on these data that had no role in the selection process. You can also reserve part of your data to validate the model you obtain in the training process. Note that the validation data are not used in obtaining the coefficients of the model, but they are used to decide when to stop the selection process to limit overfitting.

PROC GLMSELECT enables you to partition your data into disjoint subsets for training validation and testing roles. This partitioning can be done by using random proportions of the data, or you can designate a variable in your data set that defines which observations to use for each role. See the section "PARTITION Statement" on page 2713 for more details.

The following statements randomly partition the baseball data set, using 50% for training, 30% for validation, and 20% for testing. The model selected at each step is scored on the validation data, and the average residual sums of squares (ASE) is evaluated. The model yielding the lowest ASE on the validation data is selected. The ASE on the test data is also evaluated, but these data play no role in the selection process. Note that a seed for the pseudo-random number generator is specified in the PROC GLMSELECT statement.

```
proc glmselect data=baseball plots=(CriterionPanel ASE) seed=1;
  partition fraction(validate=0.3 test=0.2);
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                yrMajor crAtBat crHits crHome crRuns crRbi
                crBB league division nOuts nAssts nError
               / selection=forward(choose=validate stop=10);
run;
```

Output 42.1.6 Number of Observations Table

The GLMSELECT Procedure

Number of Observations Read	322
Number of Observations Used	263
Number of Observations Used for Training	132
Number of Observations Used for Validation	80
Number of Observations Used for Testing	51

Output 42.1.6 shows the number of observation table. You can see that of the 263 observations that were used in the analysis, 132 (50.2%) observations were used for model training, 80 (30.4%) for model validation, and 51 (19.4%) for model testing.

Output 42.1.7 Selection Summary and Stop Reason

The GLMSELECT Procedure

Forward Selection Summary

Step	Effect Entered	Number Effects In	Number Parms In	SBC	ASE
0	Intercept	1	1	-30.8531	0.7628
1	crRuns	2	2	-93.9367	0.4558
2	nHits	3	3	-126.2647	0.3439
3	yrMajor	4	4	-128.7570	0.3252
4	nBB	5	5	-132.2409*	0.3052
5	division	6	6	-130.7794	0.2974
6	nOuts	7	7	-128.5897	0.2914
7	nRBI	8	8	-125.7825	0.2868
8	nHome	9	9	-124.7709	0.2786
9	nAtBat	10	10	-121.3767	0.2754

* Optimal Value Of Criterion

Forward Selection Summary

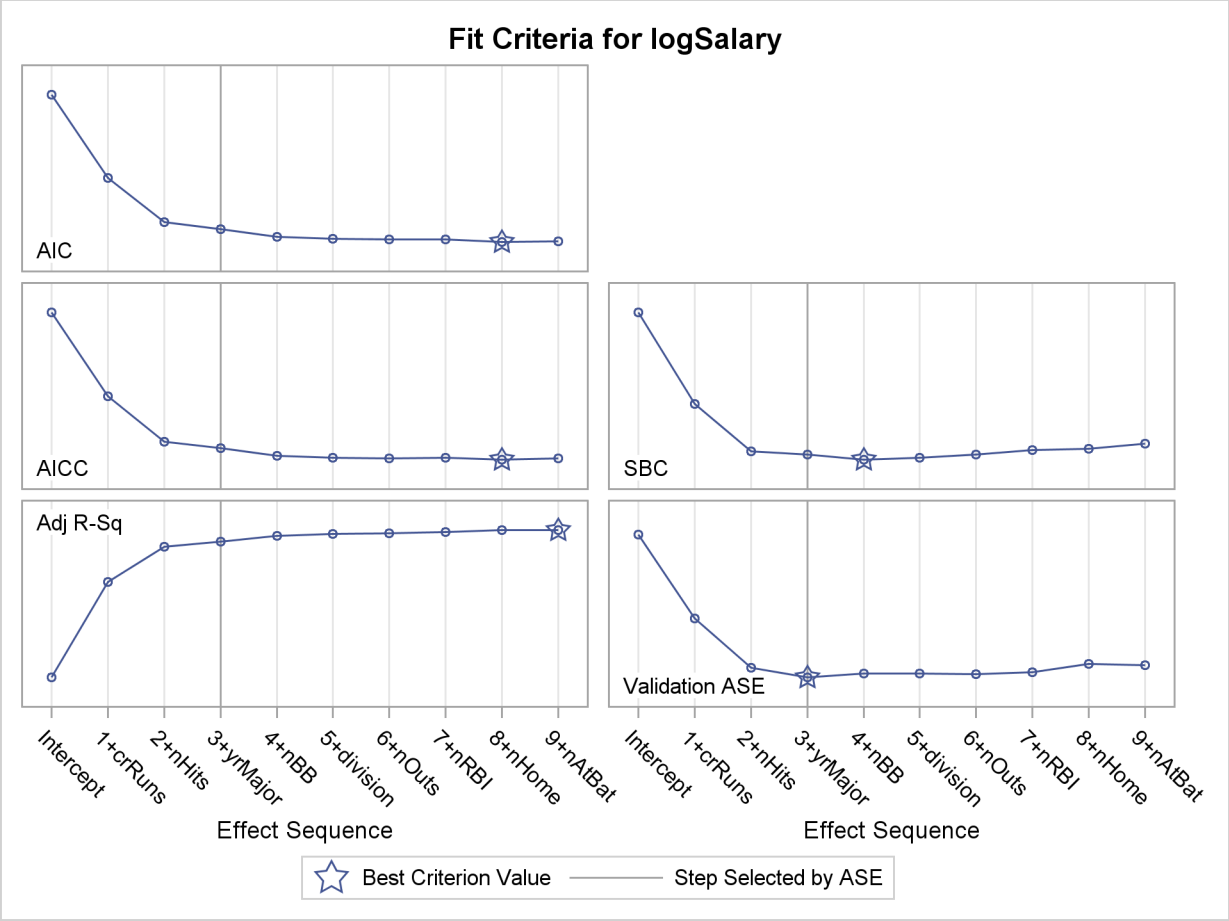
Step	Effect Entered	Validation ASE	Test ASE
0	Intercept	0.7843	0.8818
1	crRuns	0.4947	0.4210
2	nHits	0.3248	0.4697
3	yrMajor	0.2920*	0.4614
4	nBB	0.3065	0.4297
5	division	0.3050	0.4218
6	nOuts	0.3028	0.4186
7	nRBI	0.3097	0.4489
8	nHome	0.3383	0.4533
9	nAtBat	0.3337	0.4580

* Optimal Value Of Criterion

Selection stopped at the first model containing the specified number of effects (10).

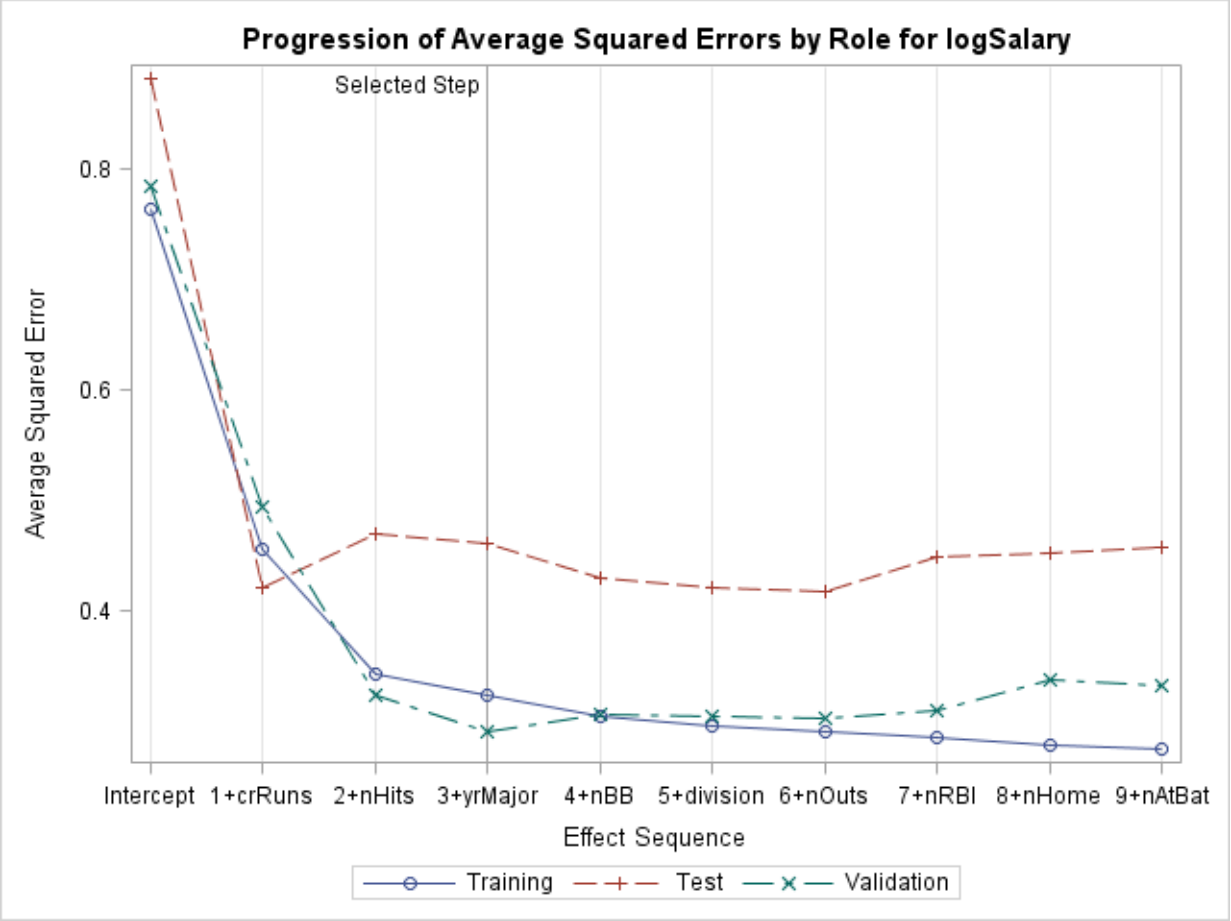
Output 42.1.7 shows the selection summary table and the stop reason. The forward selection stops at step 9 since the model at this step contains 10 effects, and so it satisfies the stopping criterion requested with the "STOP=10" suboption. However, the selected model is the model at step 3, where the validation ASE, the CHOOSE= criterion, achieves its minimum.

Output 42.1.8 Criterion Panel



The criterion panel in Output 42.1.8 shows how the various criteria evolved as the stepwise selection method proceeded. Note that other than the ASE evaluated on the validation data, these criteria are evaluated on the training data.

Output 42.1.9 Average Square Errors by Role



Finally, the ASE plot in Output 42.1.9 shows how the average square error evolves on the training, validation, and test data. Note that while the ASE on the training data continued decreasing as the selection steps proceeded, the ASE on the test and validation data behave more erratically.

LASSO selection, pioneered by Tibshirani (1996), is a constrained least squares method that can be viewed as a stepwise-like method where effects enter and leave the model sequentially. You can find additional details about the LASSO method in the section "Lasso Selection (LASSO)" on page 2723. Note that when classification effects are used with LASSO, the design matrix columns for all effects containing classification variables can enter or leave the model individually. The following statements perform LASSO selection for the baseball data. The LASSO selection summary table is shown in Output 42.1.10.

```
 proc glmselect data=baseball plot=CriterionPanel ;
  class league division;
  model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                    yrMajor crAtBat crHits crHome crRuns crRbi
                    crBB league division nOuts nAssts nError
                  / selection=lasso(choose=CP steps=20);
run;

ods graphics off;
```

Output 42.1.10 Selection Summary for LASSO Selection

The GLMSELECT Procedure

LASSO Selection Summary

Step	Effect Entered	Effect Removed	Number Effects In	CP
0	Intercept		1	375.9275
1	crRuns		2	328.6492
2	crHits		3	239.5392
3	nHits		4	134.0374
4	nBB		5	111.6638
5	crRbi		6	81.7296
6	yrMajor		7	75.0428
7	nRBI		8	30.4494
8	division_East		9	29.9913
9	nOuts		10	25.1656
10		crRuns	9	18.7295
11		crRbi	8	15.1683
12	nError		9	16.6233
13	nHome		10	16.3741
14	league_American		11	14.8794
15		nRBI	10	8.8477*
16	crBB		11	9.2242
17	crRuns		12	10.7608
18	nAtBat		13	11.6266
19	nAssts		14	11.8572
20	crAtBat		15	13.4020

* Optimal Value Of Criterion

LASSO Selection Summary

Step	Effect Entered	Effect Removed	SBC
0	Intercept		-57.2041
1	crRuns		-72.8102
2	crHits		-111.5450
3	nHits		-170.1414
4	nBB		-181.5853
5	crRbi		-200.1160
6	yrMajor		-201.5014
7	nRBI		-236.7565
8	division_East		-233.5248
9	nOuts		-234.5631

* Optimal Value Of Criterion

Output 42.1.10 *continued*

The GLMSELECT Procedure

LASSO Selection Summary

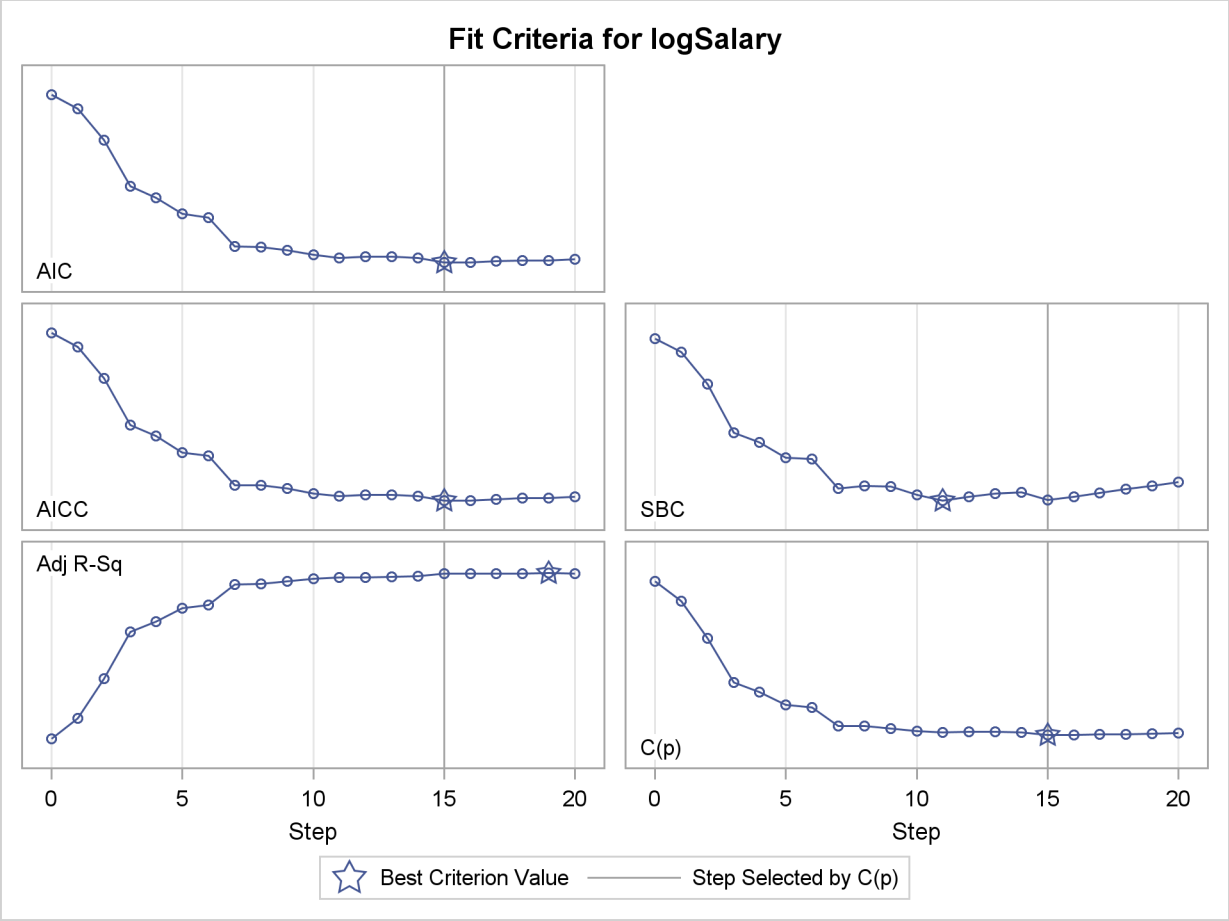
Step	Effect Entered	Effect Removed	SBC
10		crRuns	−244.5223
11		crRbi	−251.6560*
12	nError		−246.6311
13	nHome		−243.3298
14	league_American		−241.3253
15		nRBI	−251.0743
16	crBB		−247.2030
17	crRuns		−242.1184
18	nAtBat		−237.7433
19	nAssts		−234.0495
20	crAtBat		−228.9628

* Optimal Value Of Criterion

Selection stopped at the specified number of steps (20).

Note that effects enter and leave sequentially. In this example, the STEPS= suboption of the SELECTION= option specifies that 20 steps of LASSO selection be done. You can see how the various model fit statistics evolved in Output 42.1.11.

Output 42.1.11 Criterion Panel



The CHOOSE=CP suboption specifies that the selected model be the model at step 15 that yields the optimal value of Mallows' C(p) statistic. Details of this selected model are shown in Output 42.1.12.

Output 42.1.12 Selected Model

```
                    The GLMSELECT Procedure
                         Selected Model

        The selected model, based on C(p), is the model at Step 15.


Effects: Intercept nHits nHome nBB yrMajor crHits league_American division_East
         nOuts nError

                        Analysis of Variance

                                    Sum of           Mean
Source                  DF        Squares         Square    F Value

Model                    9      125.24302       13.91589      42.98
Error                  253       81.91071        0.32376
Corrected Total        262      207.15373
```

Output 42.1.12 *continued*

Root MSE	0.56900
Dependent Mean	5.92722
R-Square	0.6046
Adj R-Sq	0.5905
AIC	-21.79589
AICC	-20.74409
BIC	-283.91417
C(p)	8.84767
SBC	-251.07435

Parameter Estimates

Parameter	DF	Estimate
Intercept	1	4.204236
nHits	1	0.006942
nHome	1	0.002785
nBB	1	0.005727
yrMajor	1	0.067054
crHits	1	0.000249
league_American	1	-0.079607
division_East	1	0.134723
nOuts	1	0.000183
nError	1	-0.007213

Example 42.2: Using Validation and Cross Validation

This example shows how you can use both test set and cross validation to monitor and control variable selection. It also demonstrates the use of split classification variables.

The following statements produce analysis and test data sets. Note that the same statements are used to generate the observations that are randomly assigned for analysis and test roles in the ratio of approximately two to one.

```
data analysisData testData;
   drop i j c3Num;
   length c3$ 7;

   array x{20} x1-x20;

   do i=1 to 1500;
      do j=1 to 20;
         x{j} = ranuni(1);
      end;

      c1 = 1 + mod(i,8);
      c2 = ranbin(1,3,.6);

      if      i < 50   then do; c3 = 'tiny';     c3Num=1;end;
      else if i < 250  then do; c3 = 'small';    c3Num=1;end;
      else if i < 600  then do; c3 = 'average';  c3Num=2;end;
      else if i < 1200 then do; c3 = 'big';      c3Num=3;end;
      else                  do; c3 = 'huge';     c3Num=5;end;

      y = 10 + x1 + 2*x5 + 3*x10 + 4*x20  + 3*x1*x7 + 8*x6*x7
             + 5*(c1=3)*c3Num + 8*(c1=7)  + 5*rannor(1);

      if ranuni(1) < 2/3 then output analysisData;
                         else output testData;
  end;
run;
```

Suppose you suspect that the dependent variable depends on both main effects and two-way interactions. You can use the following statements to select a model:

```
ods graphics on;

proc glmselect data=analysisData testdata=testData
               seed=1 plots(stepAxis=number)=(criterionPanel ASEPlot);
   partition fraction(validate=0.5);
   class c1 c2 c3(order=data);
   model y =  c1|c2|c3|x1|x2|x3|x4|x5|x5|x6|x7|x8|x9|x10
             |x11|x12|x13|x14|x15|x16|x17|x18|x19|x20 @2
           / selection=stepwise(choose = validate
                                select = sl)
             hierarchy=single stb;
run;
```

Note that a TESTDATA= data set is named in the PROC GLMSELECT statement and that a PARTITION statement is used to randomly assign half the observations in the analysis data set for model validation and the rest for model training. You find details about the number of observations used for each role in the number of observations tables shown in Output 42.2.1.

Output 42.2.1 Number of Observations Tables

```
                    The GLMSELECT Procedure

             Observation Profile for Analysis Data

Number of Observations Read                          1010
Number of Observations Used                          1010
Number of Observations Used for Training              510
Number of Observations Used for Validation            500
```

The "Class Level Information" and "Dimensions" tables are shown in Output 42.2.2. The "Dimensions" table shows that at each step of the selection process, 278 effects are considered as candidates for entry or removal. Since several of these effects have multilevel classification variables as members, there are 661 parameters.

Output 42.2.2 Class Level Information and Problem Dimensions

```
                  Class Level Information

Class      Levels    Values

c1              8    1 2 3 4 5 6 7 8
c2              4    0 1 2 3
c3              5    tiny small average big huge


                     Dimensions

                Number of Effects       278
                Number of Parameters    661
```

The model statement options request stepwise selection with the default entry and stay significance levels used for both selecting entering and departing effects and stopping the selection method. The CHOOSE=VALIDATE suboption specifies that the selected model is chosen to minimize the predicted residual sum of squares when the models at each step are scored on the observations reserved for validation. The HIERARCHY=SINGLE option specifies that interactions can enter the model only if the corresponding main effects are already in the model, and that main effects cannot be dropped from the model if an interaction with such an effect is in the model. These settings are listed in the model information table shown in Output 42.2.3.

Output 42.2.3 Model Information

The GLMSELECT Procedure

Data Set	WORK.ANALYSISDATA
Test Data Set	WORK.TESTDATA
Dependent Variable	y
Selection Method	Stepwise
Select Criterion	Significance Level
Stop Criterion	Significance Level
Choose Criterion	Validation ASE
Entry Significance Level (SLE)	0.15
Stay Significance Level (SLS)	0.15
Effect Hierarchy Enforced	Single
Random Number Seed	1

The stop reason and stop details tables are shown in Output 42.2.4. Note that because the STOP= suboption of the SELECTION= option was not explicitly specified, the stopping criterion used is the selection criterion, namely significance level.

Output 42.2.4 Stop Details

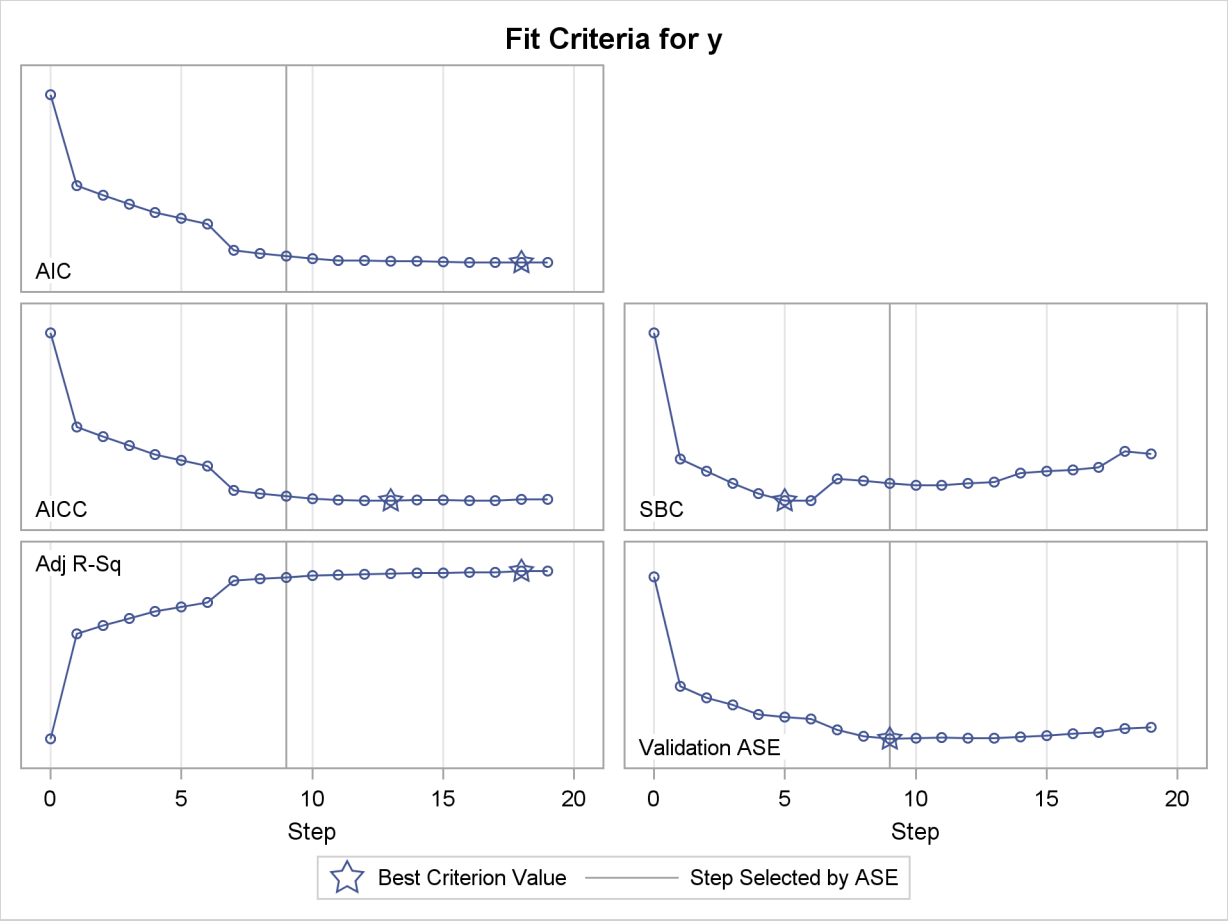
Selection stopped because the candidate for entry has SLE > 0.15 and the candidate for removal has SLS < 0.15.

Stop Details

Candidate For	Effect	Candidate Significance		Compare Significance	
Entry	x2*x5	0.1742	>	0.1500	(SLE)
Removal	x5*x10	0.0534	<	0.1500	(SLS)

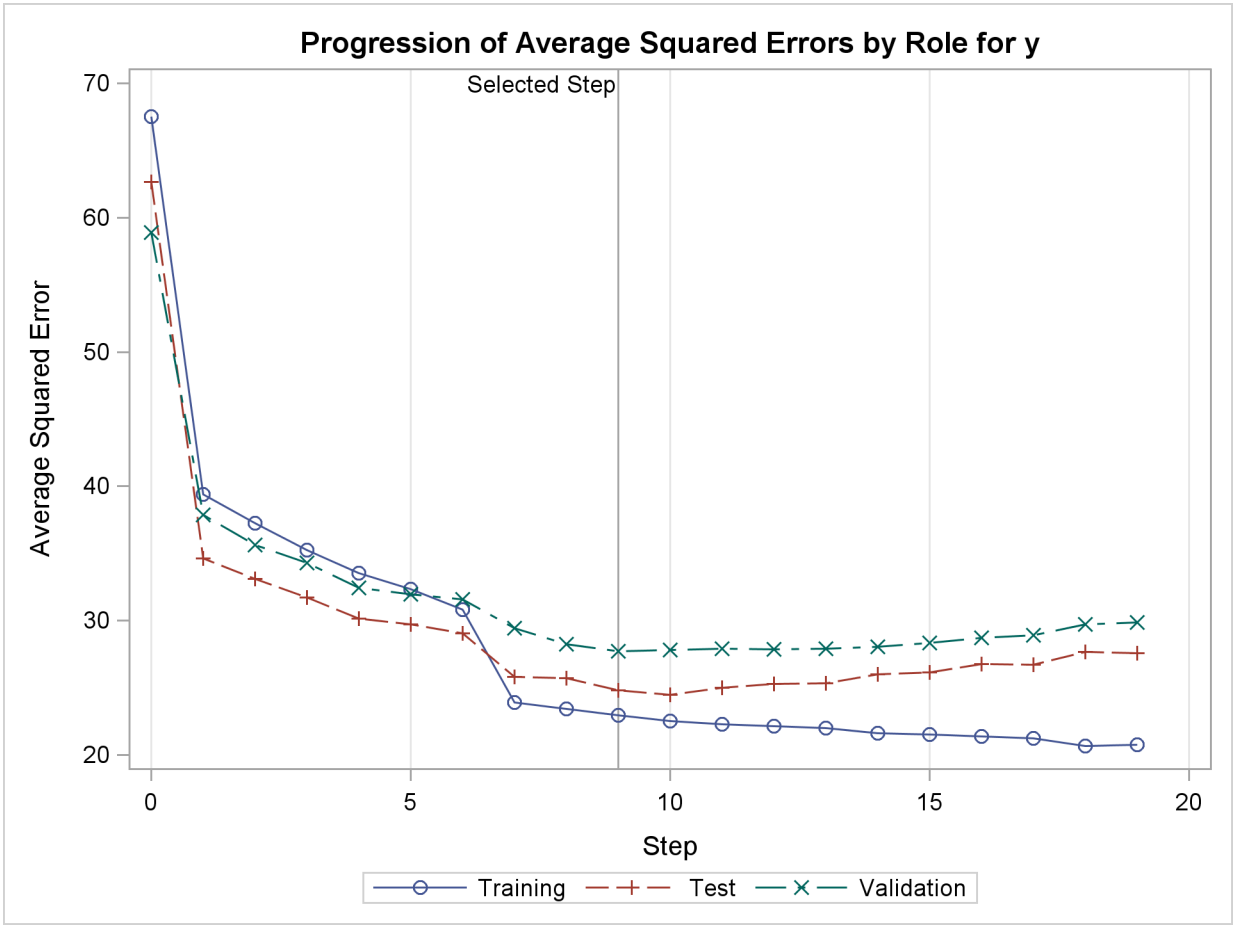
The criterion panel in Output 42.2.5 shows how the various fit criteria evolved as the stepwise selection method proceeded. Note that other than the ASE evaluated on the validation data, these criteria are evaluated on the training data. You see that the minimum of the validation ASE occurs at step 9, and hence the model at this step is selected.

Output 42.2.5 Criterion Panel



Output 42.2.6 shows how the average squared error (ASE) evolved on the training, validation, and test data. Note that while the ASE on the training data decreases monotonically, the errors on both the validation and test data start increasing beyond step 9. This indicates that models after step 9 are beginning to overfit the training data.

Output 42.2.6 Average Squared Errors



Output 42.2.7 shows the selected effects, analysis of variance, and fit statistics tables for the selected model. Output 42.2.8 shows the parameter estimates table.

Output 42.2.7 Selected Model Details

The GLMSELECT Procedure
Selected Model

The selected model, based on Validation ASE, is the model at Step 9.

Effects: Intercept c1 c3 c1*c3 x1 x5 x6 x7 x10 x20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value
Model	44	22723	516.43621	20.49
Error	465	11722	25.20856	
Corrected Total	509	34445		

Root MSE	5.02081
Dependent Mean	21.09705
R-Square	0.6597
Adj R-Sq	0.6275
AIC	2200.75319
AICC	2210.09228
SBC	1879.30167
ASE (Train)	22.98427
ASE (Validate)	27.71105
ASE (Test)	24.82947

Output 42.2.8 Parameter Estimates

Parameter Estimates

Parameter		DF	Estimate	Standardized Estimate	Standard Error	t Value
Intercept		1	6.867831	0	1.524446	4.51
c1	1	1	0.226602	0.008272	2.022069	0.11
c1	2	1	-1.189623	-0.048587	1.687644	-0.70
c1	3	1	25.968930	1.080808	1.693593	15.33
c1	4	1	1.431767	0.054892	1.903011	0.75
c1	5	1	1.972622	0.073854	1.664189	1.19
c1	6	1	-0.094796	-0.004063	1.898700	-0.05
c1	7	1	5.971432	0.250037	1.846102	3.23
c1	8	0	0	0	.	.
c3	tiny	1	-2.919282	-0.072169	2.756295	-1.06
c3	small	1	-4.635843	-0.184338	2.218541	-2.09
c3	average	1	0.736805	0.038247	1.793059	0.41
c3	big	1	-1.078463	-0.063580	1.518927	-0.71
c3	huge	0	0	0	.	.
c1*c3	1 tiny	1	-2.449964	-0.018632	4.829146	-0.51
c1*c3	1 small	1	5.265031	0.069078	3.470382	1.52
c1*c3	1 average	1	-3.489735	-0.064365	2.850381	-1.22
c1*c3	1 big	1	0.725263	0.017929	2.516502	0.29
c1*c3	1 huge	0	0	0	.	.
c1*c3	2 tiny	1	5.455122	0.050760	4.209507	1.30
c1*c3	2 small	1	7.439196	0.131499	2.982411	2.49
c1*c3	2 average	1	-0.739606	-0.014705	2.568876	-0.29
c1*c3	2 big	1	3.179351	0.078598	2.247611	1.41
c1*c3	2 huge	0	0	0	.	.
c1*c3	3 tiny	1	-19.266847	-0.230989	3.784029	-5.09
c1*c3	3 small	1	-15.578909	-0.204399	3.266216	-4.77
c1*c3	3 average	1	-18.119398	-0.395770	2.529578	-7.16
c1*c3	3 big	1	-10.650012	-0.279796	2.205331	-4.83
c1*c3	3 huge	0	0	0	.	.
c1*c3	4 tiny	0	0	0	.	.
c1*c3	4 small	1	4.432753	0.047581	3.677008	1.21
c1*c3	4 average	1	-3.976295	-0.091632	2.625564	-1.51
c1*c3	4 big	1	-1.306998	-0.033003	2.401064	-0.54
c1*c3	4 huge	0	0	0	.	.
c1*c3	5 tiny	1	6.714186	0.062475	4.199457	1.60
c1*c3	5 small	1	1.565637	0.022165	3.182856	0.49
c1*c3	5 average	1	-4.286085	-0.068668	2.749142	-1.56
c1*c3	5 big	1	-2.046468	-0.045949	2.282735	-0.90
c1*c3	5 huge	0	0	0	.	.
c1*c3	6 tiny	1	5.135111	0.039052	4.754845	1.08
c1*c3	6 small	1	4.442898	0.081945	3.079524	1.44
c1*c3	6 average	1	-2.287870	-0.056559	2.601384	-0.88
c1*c3	6 big	1	1.598086	0.043542	2.354326	0.68
c1*c3	6 huge	0	0	0	.	.
c1*c3	7 tiny	1	1.108451	0.010314	4.267509	0.26

Output 42.2.8 *continued*

Parameter Estimates

Parameter		DF	Estimate	Standardized Estimate	Standard Error	t Value
c1*c3	7 small	1	7.441059	0.119214	3.135404	2.37
c1*c3	7 average	1	1.796483	0.038106	2.630570	0.68
c1*c3	7 big	1	3.324160	0.095173	2.303369	1.44
c1*c3	7 huge	0	0	0	.	.
c1*c3	8 tiny	0	0	0	.	.
c1*c3	8 small	0	0	0	.	.
c1*c3	8 average	0	0	0	.	.
c1*c3	8 big	0	0	0	.	.
c1*c3	8 huge	0	0	0	.	.
x1		1	2.713527	0.091530	0.836942	3.24
x5		1	2.810341	0.098303	0.816290	3.44
x6		1	4.837022	0.167394	0.810402	5.97
x7		1	5.844394	0.207035	0.793775	7.36
x10		1	2.463916	0.087712	0.794599	3.10
x20		1	4.385924	0.156155	0.787766	5.57

The magnitudes of the standardized estimates and the t statistics of the parameters of the effect "c1" reveal that only levels "3" and "7" of this effect contribute appreciably to the model. This suggests that a more parsimonious model with similar or better predictive power might be obtained if parameters corresponding to the levels of "c1" are allowed to enter or leave the model independently. You request this with the SPLIT option in the CLASS statement as shown in the following statements:

```
proc glmselect data=analysisData testdata=testData
               seed=1 plots(stepAxis=number)=all;
   partition fraction(validate=0.5);
   class c1(split) c2 c3(order=data);
   model y =  c1|c2|c3|x1|x2|x3|x4|x5|x5|x6|x7|x8|x9|x10
             |x11|x12|x13|x14|x15|x16|x17|x18|x19|x20 @2
           / selection=stepwise(stop   = validate
                                select = sl)
             hierarchy=single;
   output out=outData;
run;
```

The "Class Level Information" and "Dimensions" tables are shown in Output 42.2.9. The "Dimensions" table shows that while the model statement specifies 278 effects, after splitting the parameters corresponding to the levels of "c1," there are 439 split effects that are considered for entry or removal at each step of the selection process. Note that the total number of parameters considered is not affected by the split option.

Output 42.2.9 Class Level Information and Problem Dimensions

```
                        The GLMSELECT Procedure

                        Class Level Information

Class       Levels      Values

c1               8 *    1 2 3 4 5 6 7 8
c2               4      0 1 2 3
c3               5      tiny small average big huge

                * Associated Parameters Split

                          Dimensions

                Number of Effects                  278
                Number of Effects after Splits     439
                Number of Parameters               661
```

The stop reason and stop details tables are shown in Output 42.2.10. Since the validation ASE is specified as the stopping criterion, the selection stops at step 11, where the validation ASE achieves a local minimum and the model at this step is the selected model.

Output 42.2.10 Stop Details

```
Selection stopped at a local minimum of the residual sum of squares of the
validation data.

                                Stop Details

          Candidate                    Candidate          Compare
          For          Effect    Validation ASE          Validation ASE

          Entry        x18              25.9851    >     25.7462
          Removal      x6*x7            25.7611    >     25.7462
```

You find details of the selected model in Output 42.2.11. The list of selected effects confirms that parameters corresponding to levels "3" and "7" only of "c1" are in the selected model. Notice that the selected model with classification variable "c1" split contains 18 parameters, whereas the selected model without splitting "c1" has 45 parameters. Furthermore, by comparing the fit statistics in Output 42.2.7 and Output 42.2.11, you see that this more parsimonious model has smaller prediction errors on both the validation and test data.

Output 42.2.11 Details of the Selected Model

The GLMSELECT Procedure
Selected Model

The selected model is the model at the last step (Step 11).

Effects: Intercept c1_3 c1_7 c3 c1_3*c3 x1 x5 x6 x7 x6*x7 x10 x20

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value
Model	17	22111	1300.63200	51.88
Error	492	12334	25.06998	
Corrected Total	509	34445		

Root MSE	5.00699
Dependent Mean	21.09705
R-Square	0.6419
Adj R-Sq	0.6295
AIC	2172.72685
AICC	2174.27787
SBC	1736.94624
ASE (Train)	24.18515
ASE (Validate)	25.74617
ASE (Test)	22.57297

When you use a PARTITION statement to subdivide the analysis data set, an output data set created with the OUTPUT statement contains a variable named "_ROLE_" that shows the role each observation was assigned to. See the section "OUTPUT Statement" on page 2712 and the section "Using Validation and Test Data" on page 2737 for additional details.

The following statements use PROC PRINT to produce Output 42.2.12, which shows the first five observations of the outData data set.

```
proc print data=outData(obs=5);
run;
```

Output 42.2.12 Output Data Set with _ROLE_ Variable

Obs	c3	x1	x2	x3	x4	x5	x6	x7	x8
1	tiny	0.18496	0.97009	0.39982	0.25940	0.92160	0.96928	0.54298	0.53169
2	tiny	0.47579	0.84499	0.63452	0.59036	0.58258	0.37701	0.72836	0.50660
3	tiny	0.51132	0.43320	0.17611	0.66504	0.40482	0.12455	0.45349	0.19955
4	tiny	0.42071	0.07174	0.35849	0.71143	0.18985	0.14797	0.56184	0.27011
5	tiny	0.42137	0.03798	0.27081	0.42773	0.82010	0.84345	0.87691	0.26722

Obs	x9	x10	x11	x12	x13	x14	x15	x16	x17
1	0.04979	0.06657	0.81932	0.52387	0.85339	0.06718	0.95702	0.29719	0.27261
2	0.93121	0.92912	0.58966	0.29722	0.39104	0.47243	0.67953	0.16809	0.16653
3	0.57484	0.73847	0.43981	0.04937	0.52238	0.34337	0.02271	0.71289	0.93706
4	0.32520	0.56918	0.04259	0.43921	0.91744	0.52584	0.73182	0.90522	0.57600
5	0.30602	0.39705	0.34905	0.76593	0.54340	0.61257	0.55291	0.73591	0.37186

Obs	x18	x19	x20	c1	c2	y	_ROLE_	p_y
1	0.68993	0.97676	0.22651	2	1	11.4391	VALIDATE	18.5069
2	0.87110	0.29879	0.93464	3	1	31.4596	TRAIN	26.2188
3	0.44599	0.94694	0.71290	4	3	16.4294	VALIDATE	17.0979
4	0.18794	0.33133	0.69887	5	3	15.4815	VALIDATE	16.1567
5	0.64565	0.55718	0.87504	6	2	26.0023	TRAIN	24.6358

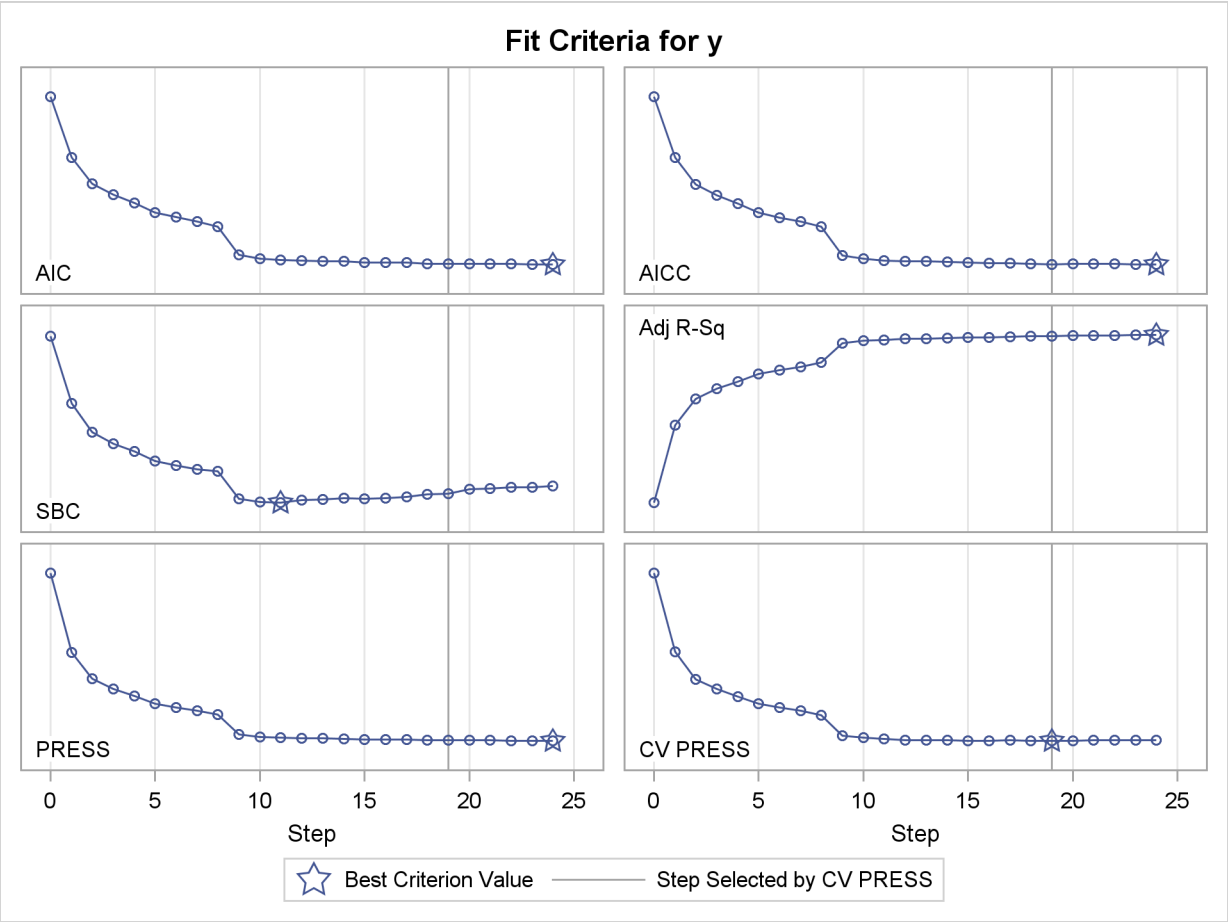
Cross validation is often used to assess the predictive performance of a model, especially for when you do not have enough observations for test set validation. See the section "Cross Validation" on page 2739 for further details. The following statements provide an example where cross validation is used as the CHOOSE= criterion.

```
proc glmselect data=analysisData testdata=testData
               plots(stepAxis=number)=(criterionPanel ASEPlot);
   class c1(split) c2 c3(order=data);
   model y =  c1|c2|c3|x1|x2|x3|x4|x5|x5|x6|x7|x8|x9|x10
             |x11|x12|x13|x14|x15|x16|x17|x18|x19|x20 @2
           / selection = stepwise(choose = cv
                                  select = sl)
             stats     = press
             cvMethod  = split(5)
             cvDetails = all
             hierarchy = single;
    output out=outData;
run;
```

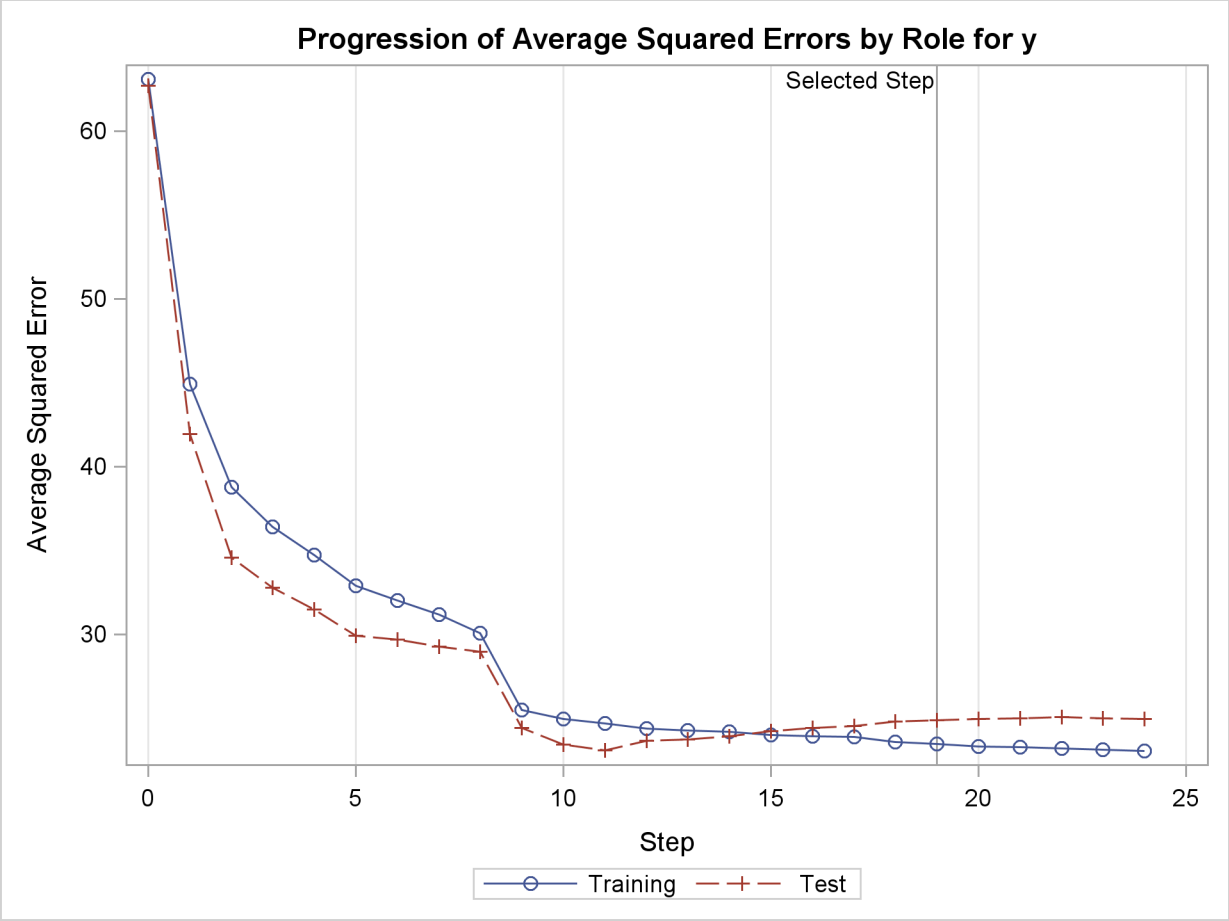
The CVMETHOD=SPLIT(5) option in the MODEL statement requests five-fold cross validation with the five subsets consisting of observations $\{1, 6, 11, \ldots\}$, $\{2, 7, 12, \ldots\}$, and so on. The STATS=PRESS option requests that the leave-one-out cross validation predicted residual sum of squares (PRESS) also be computed and displayed at each step, even though this statistic is not used in the selection process.

Output 42.2.13 shows how several fit statistics evolved as the selection process progressed. The five-fold CV PRESS statistic achieves its minimum at step 19. Note that this gives a larger model than was selected when the stopping criterion was determined using validation data. Furthermore, you see that the PRESS statistic has not achieved its minimum within 25 steps, so an even larger model would have been selected based on leave-one-out cross validation.

Output 42.2.13 Criterion Panel



Output 42.2.14 Average Squared Error Plot



The CVDETAILS=ALL option in the MODEL statement requests the "Cross Validation Details" table in Output 42.2.15 and the cross validation parameter estimates that are included in the "Parameter Estimates" table in Output 42.2.16. For each cross validation index, the predicted residual sum of squares on the observations omitted is shown in the "Cross Validation Details" table and the parameter estimates of the corresponding model are included in the "Parameter Estimates" table. By default, these details are shown for the selected model, but you can request this information at every step with the DETAILS= option in the MODEL statement. You use the "_CVINDEX_" variable in the output data set shown in Output 42.2.17 to find out which observations in the analysis data are omitted for each cross validation fold.

Output 42.2.15 Breakdown of CV Press Statistic by Fold

Cross Validation Details

Index	---Observations--- Fitted	Left Out	CV PRESS
1	808	202	5059.7375
2	808	202	4278.9115
3	808	202	5598.0354
4	808	202	4950.1750
5	808	202	5528.1846
Total			25293.5024

Output 42.2.16 Cross Validation Parameter Estimates

Parameter Estimates

Parameter		--------Cross Validation Estimates-------- 1	2	3	4	5
Intercept		10.7617	10.1200	9.0254	13.4164	12.3352
c1_3		28.2715	27.2977	27.0696	28.6835	27.8070
c1_7		7.6530	7.6445	7.9257	7.4217	7.6862
c3	tiny	-3.1103	-4.4041	-5.1793	-8.4131	-7.2096
c3	small	2.2039	1.5447	1.0121	-0.3998	1.4927
c3	average	0.3021	-1.3939	-1.2201	-3.3407	-2.1467
c3	big	-0.9621	-1.2439	-1.6092	-3.7666	-3.4389
c3	huge	0	0	0	0	0
c1_3*c3	tiny	-21.9104	-21.7840	-22.0173	-22.6066	-21.9791
c1_3*c3	small	-20.8196	-20.2725	-19.5850	-20.4515	-20.7586
c1_3*c3	average	-16.8500	-15.1509	-15.0134	-15.3851	-13.4339
c1_3*c3	big	-12.7212	-12.1554	-12.0354	-12.3282	-13.0174
c1_3*c3	huge	0	0	0	0	0
x1		0.9238	1.7286	2.5976	-0.2488	1.2093
x1*c3	tiny	-1.5819	-1.1748	-3.2523	-1.7016	-2.7624
x1*c3	small	-3.7669	-3.2984	-2.9755	-1.8738	-4.0167
x1*c3	average	2.2253	2.4489	1.5675	4.0948	2.0159
x1*c3	big	0.9222	0.5330	0.7960	2.6061	1.2694
x1*c3	huge	0	0	0	0	0
x5		-1.3562	0.5639	0.3022	-0.4700	-2.5063
x6		-0.9165	-3.2944	-1.2163	-2.2063	-0.5696
x7		5.2295	5.3015	6.2526	4.1770	5.8364
x6*x7		6.4211	7.5644	6.1182	7.0020	5.8730
x10		1.9591	1.4932	0.7196	0.6504	-0.3989
x5*x10		3.6058	1.7274	4.3447	2.4388	3.8967
x15		-0.0079	0.6896	1.6811	0.0136	0.1799
x15*c1_3		-3.5022	-2.7963	-2.6003	-4.2355	-4.7546
x7*x15		-5.1438	-5.8878	-5.9465	-3.6155	-5.3337
x18		-2.1347	-1.5656	-2.4226	-4.0592	-1.4985
x18*c3	tiny	2.2988	1.1931	2.6491	6.1615	5.6204
x18*c3	small	4.6033	3.2359	4.4183	5.5923	1.7270
x18*c3	average	-2.3712	-2.5392	-0.6361	-1.1729	-1.6481
x18*c3	big	2.3160	1.4654	2.7683	3.0487	2.5768
x18*c3	huge	0	0	0	0	0
x6*x18		3.0716	4.2036	4.1354	4.9196	2.7165
x20		4.1229	4.5773	4.5774	4.6555	4.2655

The following statements display the first eight observations in the outData data set.

```
proc print data=outData(obs=8);
run;
```

Output 42.2.17 First Eight Observations in the Output Data Set

Obs	c3	x1	x2	x3	x4	x5	x6	x7	x8
1	tiny	0.18496	0.97009	0.39982	0.25940	0.92160	0.96928	0.54298	0.53169
2	tiny	0.47579	0.84499	0.63452	0.59036	0.58258	0.37701	0.72836	0.50660
3	tiny	0.51132	0.43320	0.17611	0.66504	0.40482	0.12455	0.45349	0.19955
4	tiny	0.42071	0.07174	0.35849	0.71143	0.18985	0.14797	0.56184	0.27011
5	tiny	0.42137	0.03798	0.27081	0.42773	0.82010	0.84345	0.87691	0.26722
6	tiny	0.81722	0.65822	0.02947	0.85339	0.36285	0.37732	0.51054	0.71194
7	tiny	0.19480	0.81673	0.08548	0.18376	0.33264	0.70558	0.92761	0.29642
8	tiny	0.04403	0.51697	0.68884	0.45333	0.83565	0.29745	0.40325	0.95684

Obs	x9	x10	x11	x12	x13	x14	x15	x16	x17
1	0.04979	0.06657	0.81932	0.52387	0.85339	0.06718	0.95702	0.29719	0.27261
2	0.93121	0.92912	0.58966	0.29722	0.39104	0.47243	0.67953	0.16809	0.16653
3	0.57484	0.73847	0.43981	0.04937	0.52238	0.34337	0.02271	0.71289	0.93706
4	0.32520	0.56918	0.04259	0.43921	0.91744	0.52584	0.73182	0.90522	0.57600
5	0.30602	0.39705	0.34905	0.76593	0.54340	0.61257	0.55291	0.73591	0.37186
6	0.37533	0.22954	0.68621	0.55243	0.58182	0.17472	0.04610	0.64380	0.64545
7	0.22404	0.14719	0.59064	0.46326	0.41860	0.25631	0.23045	0.08034	0.43559
8	0.42194	0.78079	0.33106	0.17210	0.91056	0.26897	0.95602	0.13720	0.27190

Obs	x18	x19	x20	c1	c2	y	_CVINDEX_	p_y
1	0.68993	0.97676	0.22651	2	1	11.4391	1	18.1474
2	0.87110	0.29879	0.93464	3	1	31.4596	2	24.7930
3	0.44599	0.94694	0.71290	4	3	16.4294	3	16.5752
4	0.18794	0.33133	0.69887	5	3	15.4815	4	14.7605
5	0.64565	0.55718	0.87504	6	2	26.0023	5	24.7479
6	0.09317	0.62008	0.07845	7	1	16.6503	1	21.4444
7	0.67020	0.42272	0.49827	1	1	14.0342	2	20.9661
8	0.55692	0.65825	0.68465	2	3	14.9830	3	17.5644

This example demonstrates the usefulness of effect selection when you suspect that interactions of effects are needed to explain the variation in your dependent variable. Ideally, a priori knowledge should be used to decide what interactions to allow, but in some cases this information might not be available. Simply fitting a least squares model allowing all interactions produces a model that overfits your data and generalizes very poorly.

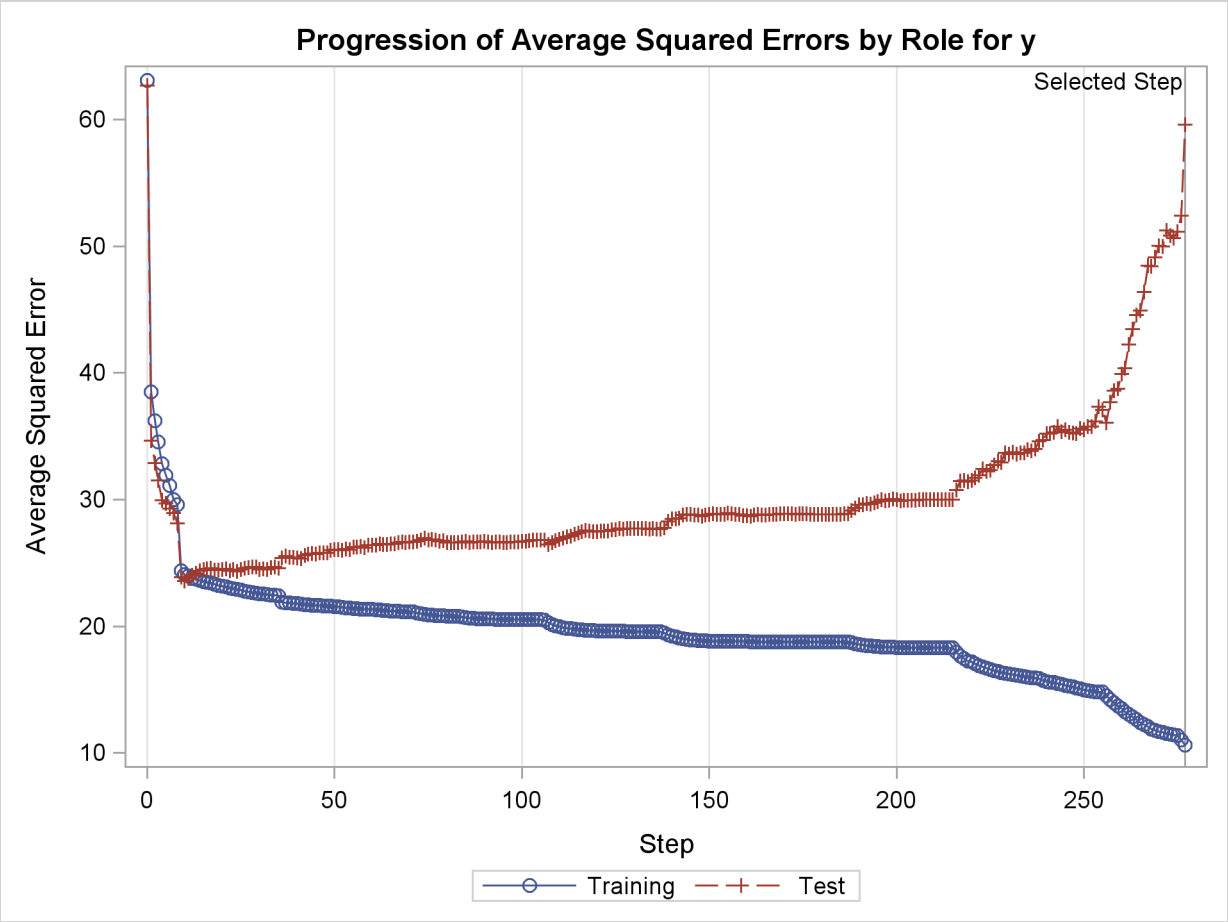
The following statements use forward selection with selection based on the SBC criterion, which is the default selection criterion. At each step, the effect whose addition to the model yields the smallest SBC value is added. The STOP=NONE suboption specifies that this process continue even when the SBC statistic grows whenever an effect is added, and so it terminates at a full least squares model. The BUILDSSCP=FULL option is specified in a PERFORMANCE statement, since building the SSCP matrix incrementally is counterproductive in this case. See the section "BUILDSSCP=FULL" on page 2715 for details. Note that if all you are interested in is a full least squares model, then it is much more efficient to simply specify SELECTION=NONE in the

MODEL statement. However, in this example the aim is to add effects in roughly increasing order of explanatory power.

```
proc glmselect data=analysisData testdata=testData plots=ASEPlot;
   class c1 c2 c3(order=data);
   model y =  c1|c2|c3|x1|x2|x3|x4|x5|x5|x6|x7|x8|x9|x10
             |x11|x12|x13|x14|x15|x16|x17|x18|x19|x20 @2
           / selection=forward(stop=none)
             hierarchy=single;
   performance buildSSCP = full;
run;

ods graphics off;
```

The ASE plot shown in Output 42.2.18 clearly demonstrates the danger in overfitting the training data. As more insignificant effects are added to the model, the growth in test set ASE shows how the predictions produced by the resulting models worsen. This decline is particularly rapid in the latter stages of the forward selection, because the use of the SBC criterion results in insignificant effects with lots of parameters being added after insignificant effects with fewer parameters.

Output 42.2.18 Average Squared Error Plot



Example 42.3: Scatter Plot Smoothing by Selecting Spline Functions

This example shows how you can use model selection to perform scatter plot smoothing. It illustrates how you can use the experimental EFFECT statement to generate a large collection of B-spline basis functions from which a subset is selected to fit scatter plot data.

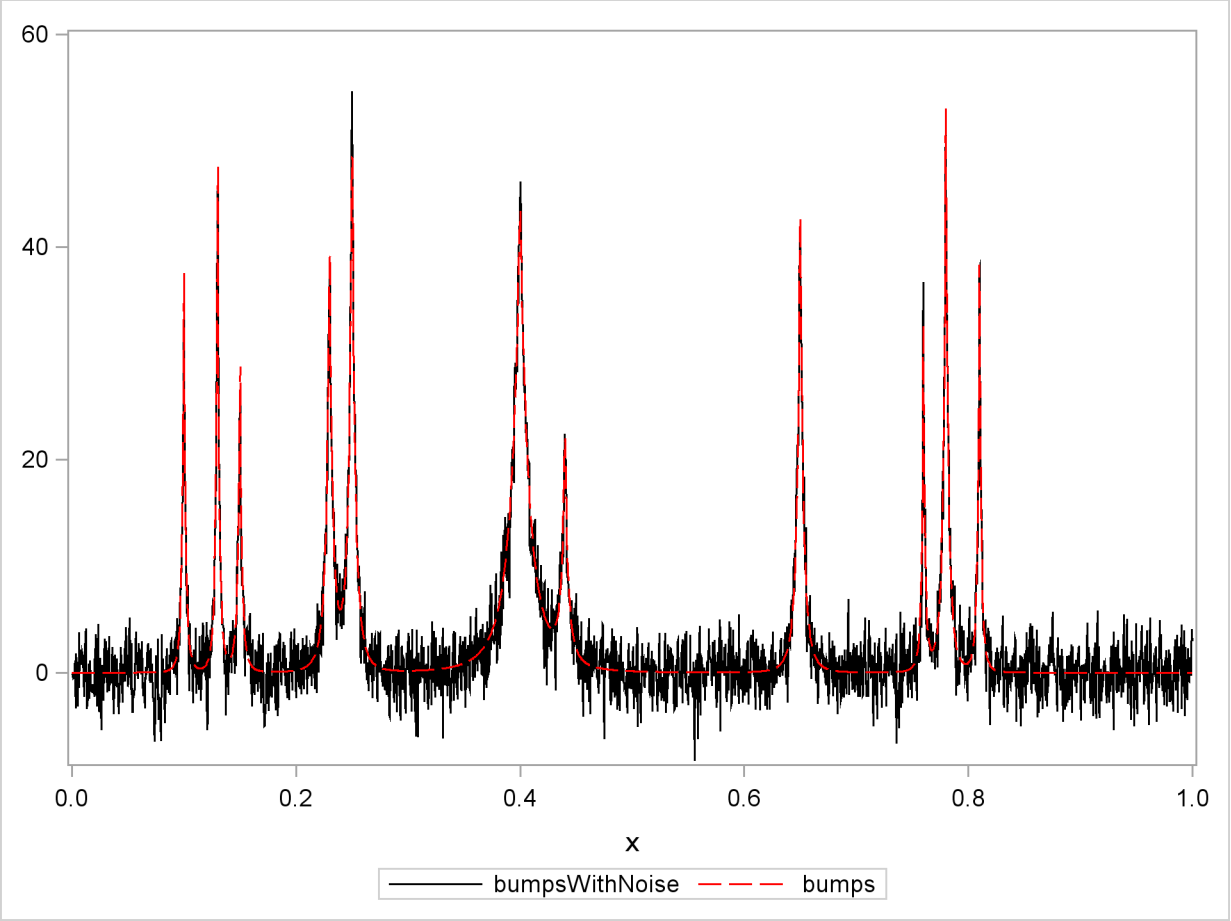
The data for this example come from a set of benchmarks developed by Donoho and Johnstone (1994) that have become popular in the statistics literature. The particular benchmark used is the "Bumps" functions to which random noise has been added to create the test data. The following DATA step, extracted from Sarle (2001), creates the data. The constants are chosen so that the noise-free data have a standard deviation of 7. The standard deviation of the noise is $\sqrt{5}$, yielding bumpsNoise with a signal-to-noise ratio of 3.13 ($7/\sqrt{5}$).

```
%let random=12345;

data DoJoBumps;
   keep x bumps bumpsWithNoise;

   pi = arcos(-1);

   do n=1 to 2048;
      x=(2*n-1)/4096;
      link compute;
      bumpsWithNoise=bumps+rannor(&random)*sqrt(5);
      output;
   end;
stop;

compute:
   array t(11) _temporary_ (.1 .13 .15 .23 .25 .4 .44 .65 .76 .78 .81);
   array b(11) _temporary_ (   4    5    3   4   5 4.2 2.1 4.3  3.1  5.1  4.2);
   array w(11) _temporary_ (.005 .005 .006 .01 .01 .03 .01 .01 .005 .008 .005);

   bumps=0;
   do i=1 to 11;
      bumps=bumps+b[i]*(1+abs((x-t[i])/w[i]))**-4;
   end;
   bumps=bumps*10.528514619;
return;
run;
```

The following statements use the SGPLOT procedure to produce the plot in Output 42.3.1. The plot shows the bumps function superimposed on the function with added noise.

```
proc sgplot data=DoJoBumps;
  yaxis display=(nolabel);
  series  x=x y=bumpsWithNoise/lineattrs=(color=black);
  series  x=x y=bumps/lineattrs=(color=red);
run;
```

Output 42.3.1 Donoho-Johnstone Bumps Function

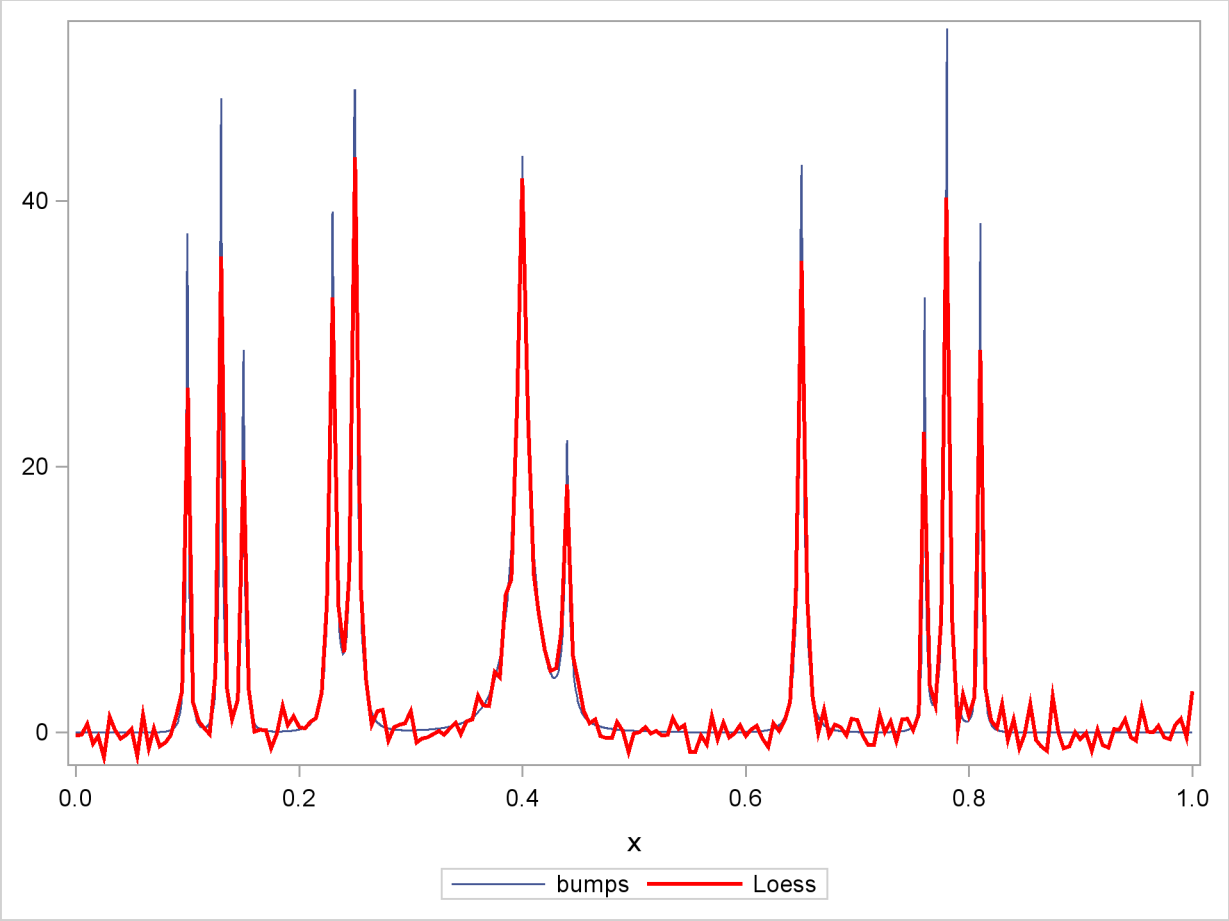


Suppose you want to smooth the noisy data to recover the underlying function. This problem is studied by Sarle (2001), who shows how neural nets can be used to perform the smoothing. The following statements use the LOESS statement in the SGPLOT procedure to show a loess fit superimposed on the noisy data (Output 42.3.2). (See Chapter 50, "The LOESS Procedure," for information about the loess method.)

```
proc sgplot data=DoJoBumps;
   yaxis display=(nolabel);
   series x=x y=bumps;
   loess  x=x y=bumpsWithNoise / lineattrs=(color=red) nomarkers;
run;
```

The algorithm selects a smoothing parameter that is small enough to enable bumps to be resolved. Because there is a single smoothing parameter that controls the number of points for all local fits, the loess method undersmooths the function in the intervals between the bumps.

Output 42.3.2 Loess Fit



Another approach to doing nonparametric fitting is to approximate the unknown underlying function as a linear combination of a set of basis functions. Once you specify the basis functions, then you can use least squares regression to obtain the coefficients of the linear combination. A problem with this approach is that for most data, you do not know a priori what set of basis functions to use. You need to supply a sufficiently rich set to enable the features in the data to be approximated. However, if you use too rich a set of functions, then this approach yields a fit that undersmooths the data and captures spurious features in the noise.

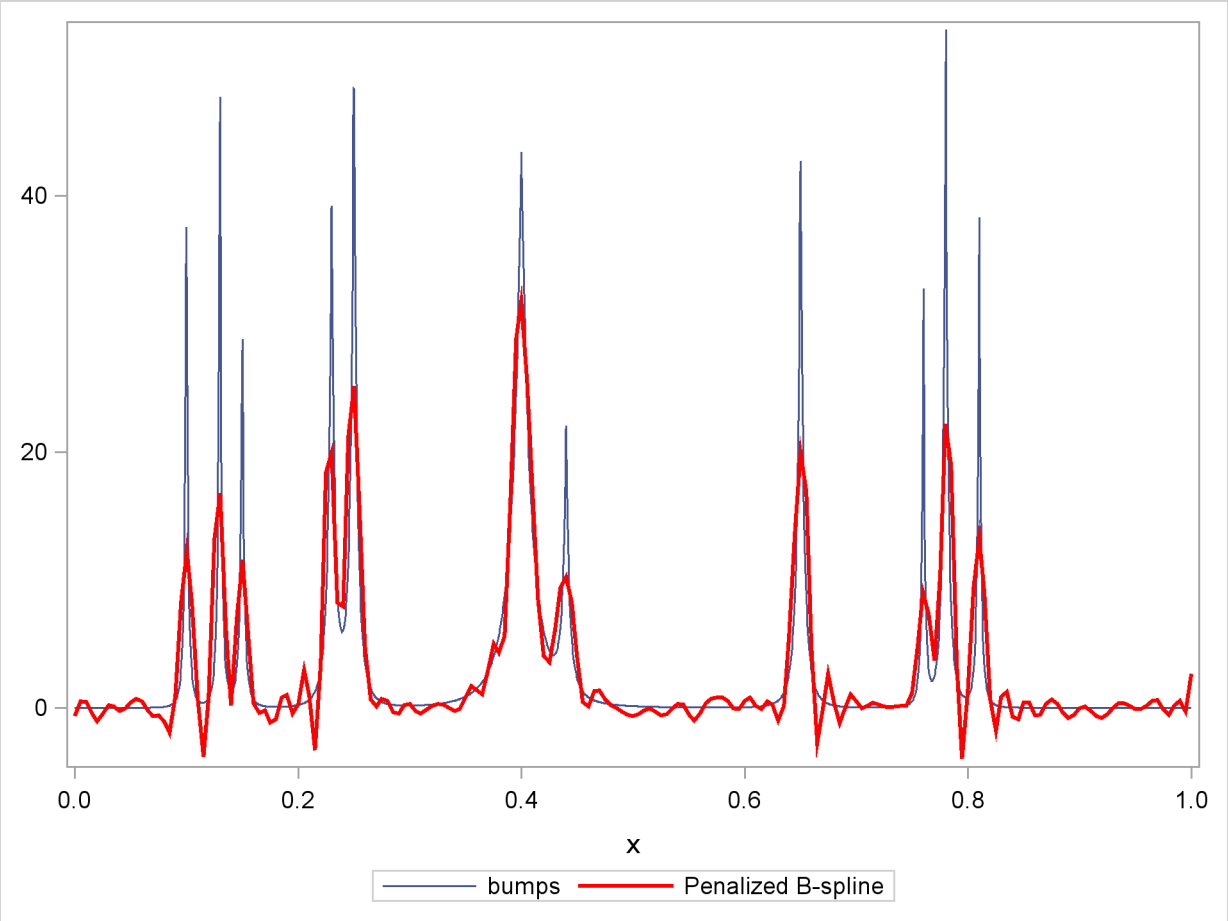
The penalized B-spline method (Eilers and Marx 1996) uses a basis of B-splines (see the section "Constructed Effects and the EFFECT Statement (Experimental)" on page 377 of Chapter 18, "Shared Concepts and Topics") corresponding to a large number of equally spaced knots as the set of approximating functions. To control the potential overfitting, their algorithm modifies the least squares objective function to include a penalty term that grows with the complexity of the fit.

The following statements use the PBSPLINE statement in the SGPLOT procedure to show a penalized B-spline fit superimposed on the noisy data (Output 42.3.3). See Chapter 90, "The TRANSREG Procedure," for details about the implementation of the penalized B-spline method.

```
proc sgplot data=DoJoBumps;
   yaxis display=(nolabel);
   series    x=x y=bumps;
   pbspline  x=x y=bumpsWithNoise /
               lineattrs=(color=red) nomarkers;
run;
```

As in the case of loess fitting, you see undersmoothing in the intervals between the bumps because there is only a single smoothing parameter that controls the overall smoothness of the fit.

Output 42.3.3 Penalized B-spline Fit



An alternative to using a smoothness penalty to control the overfitting is to use variable selection to obtain an appropriate subset of the basis functions. In order to be able to represent features in the data that occur at multiple scales, it is useful to select from B-spline functions defined on just a few knots to capture large scale features of the data as well as B-spline functions defined on many knots to capture fine details of the data. The following statements show how you can use PROC GLMSELECT to implement this strategy:

```
proc glmselect data=dojoBumps;
   effect spl = spline(x / knotmethod=multiscale(endscale=8)
                           split details);
   model bumpsWithNoise=spl;
   output out=out1 p=pBumps;
run;

proc sgplot data=out1;
   yaxis display=(nolabel);
   series    x=x y=bumps;
   series    x=x y=pBumps / lineattrs=(color=red);
run;
```

The KNOTMETHOD=MULTISCALE suboption of the **EFFECT spl = SPLINE** statement provides a convenient way to generate B-spline basis functions at multiple scales. The ENDSCALE=8 option requests that the finest scale use B-splines defined on 2^8 equally spaced knots in the interval $[0, 1]$. Because the cubic B-splines are nonzero over five adjacent knots, at the finest scale, the support of each B-spline basis function is an interval of length about 0.02 (5/256), enabling the bumps in the underlying data to be resolved. The default value is ENDSCALE=7. At this scale you will still be able to capture the bumps, but with less sharp resolution. For these data, using a value of ENDSCALE= greater than eight provides unneeded resolution, making it more likely that basis functions that fit spurious features in the noise are selected.

Output 42.3.4 shows the model information table. Since no options are specified in the MODEL statement, PROC GLMSELECT uses the stepwise method with selection and stopping based on the SBC criterion.

Output 42.3.4 Model Settings

The GLMSELECT Procedure

Data Set	WORK.DOJOBUMPS
Dependent Variable	bumpsWithNoise
Selection Method	Stepwise
Select Criterion	SBC
Stop Criterion	SBC
Effect Hierarchy Enforced	None

The DETAILS suboption in the EFFECT statement requests the display of spline knots and spline basis tables. These tables contain information about knots and basis functions at all scales. The results for scale four are shown in Output 42.3.5 and Output 42.3.6.

Output 42.3.5 Spline Knots

Knots for Spline Effect spl

Knot Number	Scale	Scale Knot Number	Boundary	x
40	4	1	*	-0.11735
41	4	2	*	-0.05855
42	4	3	*	0.00024414
43	4	4		0.05904
44	4	5		0.11783
45	4	6		0.17663
46	4	7		0.23542
47	4	8		0.29422
48	4	9		0.35301
49	4	10		0.41181
50	4	11		0.47060
51	4	12		0.52940
52	4	13		0.58819
53	4	14		0.64699
54	4	15		0.70578
55	4	16		0.76458
56	4	17		0.82337
57	4	18		0.88217
58	4	19		0.94096
59	4	20	*	0.99976
60	4	21	*	1.05855
61	4	22	*	1.11735

Output 42.3.6 Spline Details

Basis Details for Spline Effect spl

Column	Scale	Scale Column	Support		Support Knots
32	4	1	-0.11735	0.05904	1-4
33	4	2	-0.11735	0.11783	1-5
34	4	3	-0.05855	0.17663	2-6
35	4	4	0.00024414	0.23542	3-7
36	4	5	0.05904	0.29422	4-8
37	4	6	0.11783	0.35301	5-9
38	4	7	0.17663	0.41181	6-10
39	4	8	0.23542	0.47060	7-11
40	4	9	0.29422	0.52940	8-12
41	4	10	0.35301	0.58819	9-13
42	4	11	0.41181	0.64699	10-14
43	4	12	0.47060	0.70578	11-15
44	4	13	0.52940	0.76458	12-16
45	4	14	0.58819	0.82337	13-17
46	4	15	0.64699	0.88217	14-18
47	4	16	0.70578	0.94096	15-19
48	4	17	0.76458	0.99976	16-20
49	4	18	0.82337	1.05855	17-21
50	4	19	0.88217	1.11735	18-22
51	4	20	0.94096	1.11735	19-22

The "Dimensions" table in Output 42.3.7 shows that at each step of the selection process, 548 effects are considered as candidates for entry or removal. Note that although the MODEL statement specifies a single constructed effect spl, the SPLIT suboption causes each of the parameters in this constructed effect to be treated as an individual effect.

Output 42.3.7 Dimensions

Dimensions

Number of Effects	548
Number of Parameters	548

Output 42.3.8 shows the parameter estimates for the selected model. You can see that the selected model contains 31 B-spline basis functions and that all the selected B-spline basis functions are from scales four though eight. For example, the first basis function listed in the parameter estimates table is spl_S4:9—the 9th B-spline function at scale 4. You see from Output 42.3.6 that this function is nonzero on the interval $(0.29, 0.52)$.

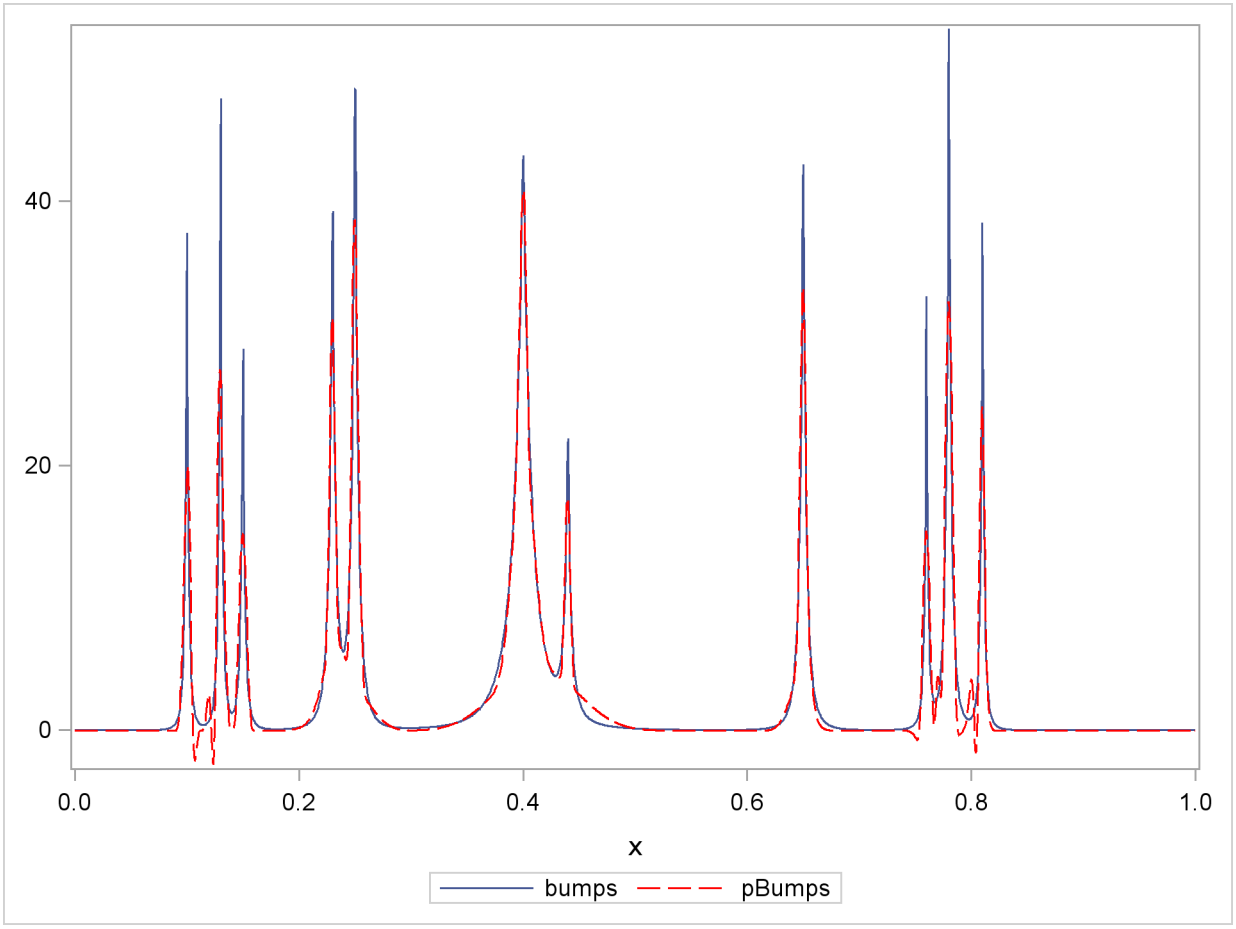
Output 42.3.8 Parameter Estimates

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value
Intercept	1	-0.009039	0.077412	-0.12
spl_S4:9	1	7.070207	0.586990	12.04
spl_S5:10	1	5.323121	1.199824	4.44
spl_S6:17	1	5.222808	1.728910	3.02
spl_S6:28	1	24.562103	1.490639	16.48
spl_S6:44	1	4.930829	1.243552	3.97
spl_S6:52	1	-7.046308	2.487700	-2.83
spl_S7:86	1	9.592742	2.626471	3.65
spl_S7:106	1	16.268550	3.334015	4.88
spl_S8:27	1	10.626586	1.752152	6.06
spl_S8:28	1	27.882444	2.004520	13.91
spl_S8:29	1	-6.129939	1.752151	-3.50
spl_S8:33	1	5.855648	1.766912	3.31
spl_S8:34	1	-11.782303	2.092484	-5.63
spl_S8:35	1	38.705178	2.092486	18.50
spl_S8:36	1	13.823256	1.766916	7.82
spl_S8:40	1	15.975124	1.691679	9.44
spl_S8:41	1	14.898716	1.691679	8.81
spl_S8:61	1	37.441965	2.084375	17.96
spl_S8:66	1	47.484506	1.883409	25.21
spl_S8:67	1	16.811502	1.910358	8.80
spl_S8:104	1	11.098484	1.958676	5.67
spl_S8:105	1	26.704556	2.042735	13.07
spl_S8:115	1	21.102920	1.576185	13.39
spl_S8:169	1	36.572294	2.914521	12.55
spl_S8:197	1	20.869716	1.882529	11.09
spl_S8:198	1	16.210987	2.693183	6.02
spl_S8:200	1	13.113942	3.458187	3.79
spl_S8:202	1	38.463549	2.462314	15.62
spl_S8:203	1	34.164644	1.757908	19.43
spl_S8:209	1	-22.645471	3.598587	-6.29
spl_S8:210	1	29.024741	2.557567	11.35

The OUTPUT statement captures the predicted values in a data set named out1, and Output 42.3.9 shows a fit plot produced by PROC SGPLOT.

Output 42.3.9 Fit by Selecting B-splines



Example 42.4: Multimember Effects and the Design Matrix

This example shows how you can use multimember effects to build predictive models. It also demonstrates several features of the OUTDESIGN= option in the PROC GLMSELECT statement.

The simulated data for this example describe a two-week summer tennis camp. The tennis ability of each camper was assessed and ratings were assigned at the beginning and end of the camp. The camp consisted of supervised group instruction in the mornings with a number of different options in the afternoons. Campers could elect to participate in unsupervised practice and play. Some campers paid for one or more individual lessons from 30 to 90 minutes in length, focusing on forehand and backhand strokes and volleying. The aim of this example is to build a predictive model for the rating improvement of each camper based on the times the camper spent doing each activity and several other variables, including the age, gender, and initial rating of the camper.

The following statements produce the TennisCamp data set:

```
data TennisCamp;
   length forehandCoach $6 backhandCoach $6 volleyCoach $6 gender $1;
   input forehandCoach backhandCoach volleyCoach tLessons tPractice tPlay
         gender inRating nPastCamps age tForehand tBackhand tVolley
         improvement;
   label forehandCoach = "Forehand lesson coach"
         backhandCoach = "Backhand lesson coach"
         volleyCoach   = "Volley   lesson coach"
         tForehand     = "time (1/2 hours) of forehand lesson"
         tBackhand     = "time (1/2 hours) of backhand lesson"
         tVolley       = "time (1/2 hours) of volley lesson"
         tLessons      = "time (1/2 hours) of all lessons"
         tPractice     = "total practice time (hours)"
         tPlay         = "total play time (hours)"
         nPastCamps    = "Number of previous camps attended"
         age           = "age (years)"
         inRating      = "Rating at camp start"
         improvement   = "Rating improvement at end of camp";
datalines;
.        .        Tom      1   30   19   f   44   0   13   0   0   1    6
Greg     .        .        2   12   33   f   48   2   15   2   0   0   14
.        .        Mike     2   12   24   m   53   0   15   0   0   2   13
.        Mike     .        1   12   28   f   48   0   13   0   1   0   11

... more lines ...

.        .        .        0   12   38   m   47   1   15   0   0   0    8
Greg     Tom      Tom      6    3   41   m   48   2   15   2   1   3   19
.        Greg     Mike     5   30   16   m   52   0   13   0   2   3   18
;
```

A multimember effect (see the section "Constructed Effects and the EFFECT Statement (Experimental)" on page 377 of Chapter 18, "Shared Concepts and Topics") is appropriate for modeling the effect of coaches on the campers' improvement, because campers might have worked with multiple coaches. Furthermore, since the time a coach spent with each camper varies, it is

appropriate to use these times to weight each coach's contribution in the multimember effect. It is also important not to exclude campers from the analysis if they did not receive any individual instruction. You can accomplish all these goals by using a multimember effect defined as follows:

```
class forehandCoach backhandCoach volleyCoach;
effect coach = MM(forehandCoach backhandCoach volleyCoach/ noeffect
                  weight=(tForehand tBackhand tVolley));
```

Based on similar previous studies, it is known that the time spent practicing should not be included linearly, because there are diminishing returns and perhaps even counterproductive effects beyond about 25 hours. A spline effect with a single knot at 25 provides flexibility in modeling effect of practice time.

The following statements use PROC GLMSELECT to select effects for the model.

```
proc glmselect data=TennisCamp outdesign=designCamp;
   class forehandCoach backhandCoach volleyCoach gender;

   effect coach    = mm(forehandCoach backhandCoach volleyCoach / noeffect details
                        weight=(tForehand tBackhand tVolley));
   effect practice = spline(tPractice/knotmethod=list(25) details);

   model improvement = coach practice tLessons tPlay age gender
                       inRating nPastCamps;
run;
```

Output 42.4.1 shows the class level and MM level information. The levels of the constructed MM effect are the union of the levels of its constituent classification variables. The MM level information is not displayed by default—you request this table by specifying the DETAILS suboption in the relevant EFFECT statement.

Output 42.4.1 Levels of MM EFFECT Coach

```
                        The GLMSELECT Procedure

                        Class Level Information

          Class               Levels    Values

          forehandCoach            5    Bruna Elaine Greg Mike Tom
          backhandCoach            5    Bruna Elaine Greg Mike Tom
          volleyCoach              5    Andy Bruna Greg Mike Tom
          gender                   2    f m


                        The GLMSELECT Procedure

                   Level Details for MM Effect coach

                 Levels    Values

                      6    Andy Bruna Elaine Greg Mike Tom
```

Output 42.4.2 shows the parameter estimates for the selected model. You can see that the constructed multimember effect coach and the spline effect practice are both included in the selected model. All coaches provided benefit (all the parameters of the multimember effect coach are positive), with Greg and Mike being the most effective.

Output 42.4.2 Parameter Estimates

Parameter Estimates

Parameter		DF	Estimate	Standard Error	t Value
Intercept		1	0.379873	0.513431	0.74
coach	Andy	1	1.444370	0.318078	4.54
coach	Bruna	1	1.446063	0.110179	13.12
coach	Elaine	1	1.312290	0.281877	4.66
coach	Greg	1	3.042828	0.112256	27.11
coach	Mike	1	2.840728	0.121166	23.45
coach	Tom	1	1.248946	0.115266	10.84
practice	1	1	2.538938	1.015772	2.50
practice	2	1	3.837684	1.104557	3.47
practice	3	1	2.574775	0.930816	2.77
practice	4	1	−0.034747	0.717967	−0.05
practice	5	0	0	.	.
tPlay		1	0.139409	0.023043	6.05

Suppose you want to examine regression diagnostics for the selected model. PROC GLMSELECT does not support such diagnostics, so you might want to use the REG procedure to produce these diagnostics. You can overcome the difficulty that PROC REG does not support CLASS and EFFECT statements by using the OUTDESIGN= option in the PROC GLMSELECT statement to obtain the design matrix that you can use as an input data set for further analysis with other SAS procedures.

The following statements use PROC PRINT to produce Output 42.4.3, which shows the first five observations of the design matrix designCamp.

```
proc print data=designCamp(obs=5);
run;
```

Output 42.4.3 First Five Observations of the designCamp Data Set

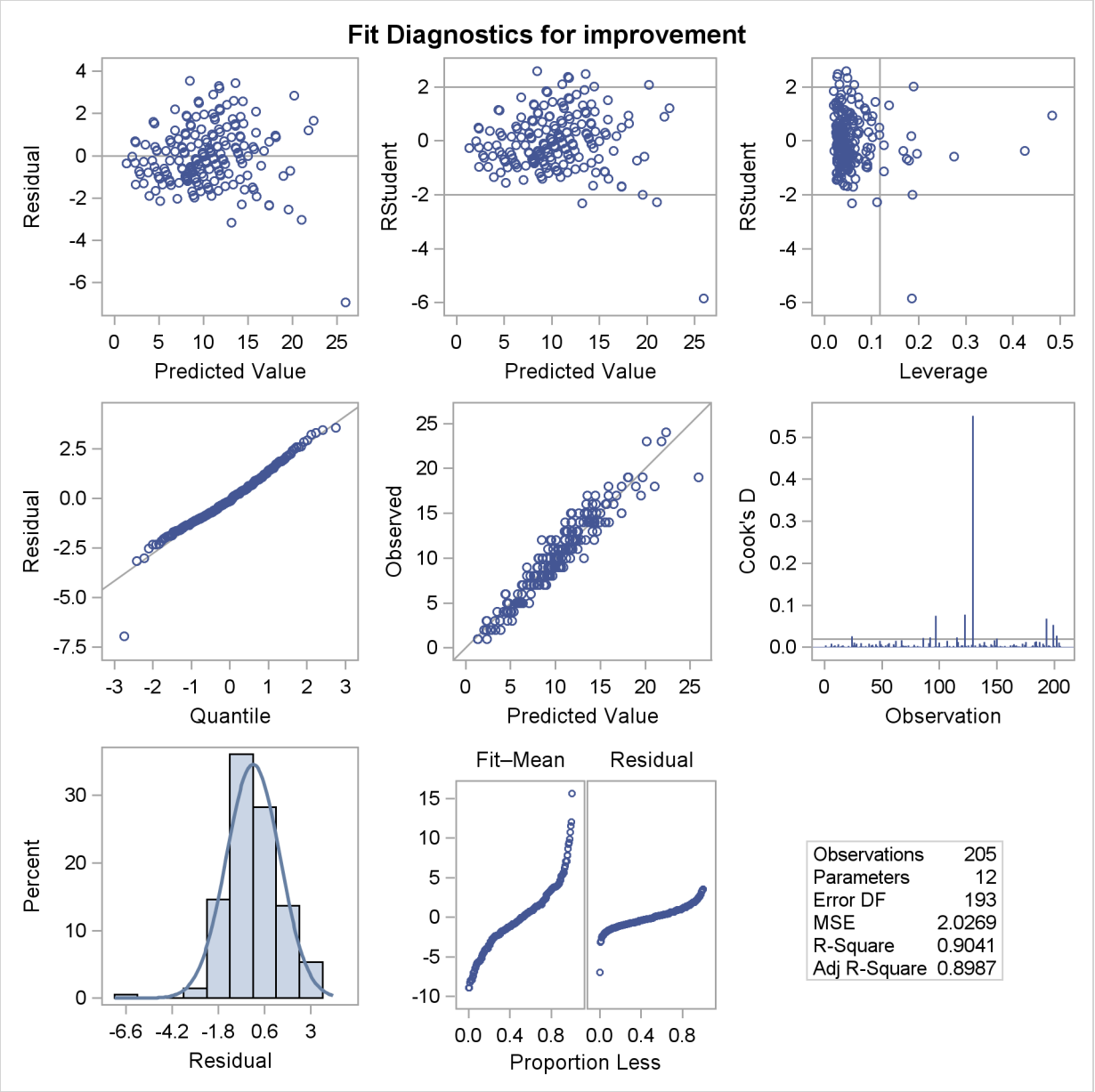
Obs	Intercept	coach_Andy	coach_Bruna	coach_Elaine	coach_Greg	coach_Mike	coach_Tom	tPlay	practice_1	practice_2	practice_3	practice_4	practice_5	improvement
1	1	0	0	0	0	0	1	19	0.00000	0.00136	0.05077	0.58344	0.36443	6
2	1	0	0	0	2	0	0	33	0.20633	0.50413	0.25014	0.03940	0.00000	14
3	1	0	0	0	0	2	0	24	0.20633	0.50413	0.25014	0.03940	0.00000	13
4	1	0	0	0	0	1	0	28	0.20633	0.50413	0.25014	0.03940	0.00000	11
5	1	0	2	0	0	0	0	34	0.16228	0.49279	0.29088	0.05405	0.00000	12

To facilitate specifying the columns of the design matrix corresponding to the selected model, you can use the macro variable named _GLSMOD that PROC GLMSELECT creates whenever you specify the OUTDESIGN= option. The following statements use PROC REG to produce a panel of regression diagnostics corresponding to the model selected by PROC GLMSELECT.

```
ods graphics on;

proc reg data=designCamp;
  model improvement = &_GLSMOD;
quit;

ods graphics off;
```

The regression diagnostics shown in Output 42.4.4 indicate a reasonable model. However, they also reveal the presence of one large outlier and several influential observations that you might want to investigate.

Output 42.4.4 Fit Diagnostics



Sometimes you might want to use subsets of the columns of the design matrix. In such cases, it might be convenient to produce a design matrix with generic names for the columns. You might also want a design matrix containing the columns corresponding to the full model that you specify in the MODEL statement. By default, the design matrix includes only the columns that correspond to effects in the selected model. The following statements show how to do this.

```
   proc glmselect data=TennisCamp
        outdesign(fullmodel prefix=parm names)=designCampGeneric;
   class forehandCoach backhandCoach volleyCoach gender;

   effect coach    = mm(forehandCoach backhandCoach volleyCoach / noeffect details
                        weight=(tForehand tBackhand tVolley));
   effect practice = spline(tPractice/knotmethod=list(25) details);

   model improvement = coach practice tLessons tPlay age gender
                       inRating nPastCamps;
run;
```

The PREFIX=parm suboption of the OUTDESIGN= option specifies that columns in the design matrix be given the prefix parm with a trailing index. The NAMES suboption requests the table in Output 42.4.5 that associates descriptive labels with the names of columns in the design matrix. Finally, the FULLMODEL suboption specifies that the design matrix include columns corresponding to all effects specified in the MODEL statement.

Output 42.4.5 Descriptive Names of Design Matrix Columns

The GLMSELECT Procedure
Selected Model

Parameter Names

Name	Parameter
parm1	Intercept
parm2	coach Andy
parm3	coach Bruna
parm4	coach Elaine
parm5	coach Greg
parm6	coach Mike
parm7	coach Tom
parm8	practice 1
parm9	practice 2
parm10	practice 3
parm11	practice 4
parm12	practice 5
parm13	tLessons
parm14	tPlay
parm15	age
parm16	gender f
parm17	gender m
parm18	inRating
parm19	nPastCamps

The following statements produce Output 42.4.6, displaying the first five observations of the designCampGeneric data set:

```
proc print data=designCampGeneric(obs=5);
run;
```

Output 42.4.6 First Five Observations of designCampGeneric Data Set

Obs	parm1	parm2	parm3	parm4	parm5	parm6	parm7	parm8	parm9	parm10	parm11	parm12	parm13	parm14	parm15	parm16	parm17	parm18	parm19	improve
1	1	0	0	0	0	0	1	0.00000	0.00136	0.05077	0.58344	0.36443	1	19	13	1	0	44	0	6
2	1	0	0	0	2	0	0	0.20633	0.50413	0.25014	0.03940	0.00000	2	33	15	1	0	48	2	14
3	1	0	0	0	0	2	0	0.20633	0.50413	0.25014	0.03940	0.00000	2	24	15	0	1	53	0	13
4	1	0	0	0	0	1	0	0.20633	0.50413	0.25014	0.03940	0.00000	1	28	13	1	0	48	0	11
5	1	0	2	0	0	0	0	0.16228	0.49279	0.29088	0.05405	0.00000	2	34	16	1	0	57	0	12

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Syntax Index

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