SAS/OR® 13.1 User’s Guide
Network Optimization Algorithms
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Credits

Documentation

Writing Matthew Galati
Editing Anne Baxter
Documentation Support Tim Arnold, Melanie Gratton, Daniel Underwood
Technical Review Manoj Chari, Charles B. Kelly, Leo Lopes, Michelle Opp, Bengt Pederson, Rob Pratt

Software

PROC OPTNET Matthew Galati

Support Groups

Software Testing Charles B. Kelly, Yu-Min Lin
Technical Support Tonya Chapman
Chapter 1
What’s New in SAS/OR 13.1

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Overview

SAS/OR 13.1 includes new features and enhancements to current features in optimization, discrete-event simulation, and constraint programming. In addition to ongoing improvements in the performance of the linear, mixed integer, quadratic, and general nonlinear optimization solver algorithms, these changes expand the range of problems you can address, make it easier to use the SAS/OR modeling and solution methods, deepen integration with other SAS analytic capabilities, and more fully utilize your available computational resources.

Highlights of Enhancements in SAS/OR 13.1

Highlights of the SAS/OR enhancements include the following:

• PROC OPTMODEL adds:
  – direct access to network optimization and analysis algorithms (Experimental)
  – parallel execution of solver invocations in a COFOR loop
  – support for function definition via PROC FCMP in Base SAS software

• PROC OPTLSO adds:
  – multiobjective optimization
  – support for the use of array-structured data in function definition (with PROC FCMP)
Chapter 1: What's New in SAS/OR 13.1

- The mixed integer linear programming (MILP) solver adds the option to execute in parallel on multiple computational cores (Experimental)

- SAS Simulation Studio adds:
  - support for custom block icons
  - improvements to the simulation clock display
  - enhancements to the Submodel block interface
  - other interface improvements

Procedure Enhancements

The CLP Procedure

The CLP procedure uses constraint logic programming methods to solve general and scheduling-oriented constraint satisfaction problems; it can also solve optimization problems. In SAS/OR 13.1, the OBJ statement, which specifies an objective function to be added to a constraint satisfaction problem, attains production status. This statement enables PROC CLP to solve optimization problems that include intricate logical constraints.

The OPTLSO Procedure

The OPTLSO procedure uses local and global search methods to solve optimization problems without making any simplifying assumptions about the nature or the behavior of the objective or constraint functions. In SAS/OR 13.1, PROC OPTLSO adds the ability to specify more than one objective by using the OBJECTIVE= option in the PROC OPTLSO statement. For multiple objectives, PROC OPTLSO returns a set of Pareto-optimal points, which constitute an efficient frontier. This feature expands the range of problems that can be addressed with PROC OPTLSO and enables you to explore the trade-offs that can exist between your identified objective functions. The definition of the Pareto-optimal set is always based on the values of the objective functions; it also includes feasibility considerations when constraints are present. You can limit the size of this set by specifying a value for the PARETOMAX= option in the PROC OPTLSO statement.

The OPTMODEL Procedure

The OPTMODEL procedure provides an interactive environment in which you can build and solve a wide range of optimization models. In SAS/OR 13.1, PROC OPTMODEL adds the (experimental) network solver, providing direct access to the set of 11 network analysis and optimization algorithms that are also accessible via PROC OPTNET. This addition makes it easier to solve network-oriented problems with PROC OPTMODEL, especially as a component of a larger solution or other analytical process. The SOLVE WITH NETWORK statement invokes the network solver. Unlike other solvers that PROC OPTMODEL uses, the network solver operates directly on arrays and sets. You do not need to explicitly define variables,
constraints, and objectives to use the network solver because PROC OPTMODEL declares the appropriate objects internally as needed. You specify the names of arrays and sets that define your network-structured inputs and outputs as options in the SOLVE WITH NETWORK statement. In addition to input and output, options in the SOLVE WITH NETWORK statement define processing and diagnostic controls and specify which algorithm to execute.

PROC OPTMODEL also now enables you to run optimization solver invocations in parallel, in iterations of a concurrent FOR loop that is specified using the COFOR statement. The COFOR statement operates in the same manner as the FOR statement, except that with a COFOR statement PROC OPTMODEL can execute the SOLVE statement concurrently with other statements. The execution of the COFOR sub-statement is interleaved between loop iterations so that other iterations can be processed while an iteration waits for a SOLVE statement to complete. Multiple solvers can run concurrently. This interleaving is managed so that in many cases a FOR loop can be replaced by a COFOR loop to achieve concurrency with minimal or no other changes to the code. For problems in which the SOLVE statement execution accounts for the majority of the time needed to execute a loop iteration, use of the COFOR statement can produce significant reductions in overall time needed.

PROC OPTMODEL can now also call functions and subroutines that have been defined and compiled using the FCMP procedure in Base SAS software. This capability enables reuse of previously defined functions and subroutines, deepening integration with other SAS analytic procedures from which they can also be called. You can use an FCMP function anywhere a function is otherwise permitted in PROC OPTMODEL. You can use the CALL statement in PROC OPTMODEL to call FCMP subroutines. An FCMP subroutine can return data by updating PROC OPTMODEL parameters (numeric and string) that are passed as arguments in the corresponding CALL statement and declared using the OUTARGS statement in the PROC FCMP subroutine definition. In addition to numeric and string parameters, you can pass PROC OPTMODEL arrays to PROC FCMP functions and subroutines that accept matrix arguments.

---

### Linear and Nonlinear Optimization with PROC OPTLP and PROC OPTMODEL

In SAS/OR 13.1, the concurrent solve capability for linear and nonlinear optimization attains production status. You can specify this capability in the ALGORITHM=CONCURRENT option in the PROC OPTLP statement or in the SOLVE statement in PROC OPTMODEL. This feature executes all available solution algorithms in parallel, as permitted by the number of computational cores available. The first algorithm to finish returns its solution.

---

### Mixed Integer Linear Optimization with PROC OPTMILP and PROC OPTMODEL

In SAS/OR 13.1, the mixed integer linear programming (MILP) solver adds the (experimental) ability to execute the branch-and-cut solution algorithm in parallel on multiple computational cores in single-machine mode. To enable parallel processing of the branch-and-cut algorithm, you need to specify the PARALLEL=1 option in the PROC OPTMILP statement or in the SOLVE statement in PROC OPTMODEL.
SAS Simulation Studio 13.1

SAS Simulation Studio 13.1, a component of SAS/OR 13.1 for Windows environments, makes several enhancements to its graphical discrete-event simulation modeling and analysis interface. You can now specify a custom image to replace the default icon for any block instance; the new **Visual** tab on each block’s Block Properties dialog box enables you to specify a custom image and customize the block label. The Simulation Clock display has been augmented to graphically indicate the status of a model (running, paused, or stopped/completed).

The Definition view for a Submodel window adds its own block template, which appears when you place the cursor over a small image of the template near the top of the Submodel window. This feature makes it easier for you to add blocks to the definition of a submodel. In SAS Simulation Studio 13.1 it is also easier for you to edit expressions by using a dedicated Edit Expressions window. This window opens from many Block Properties dialog boxes for blocks in which you can specify expressions such as attribute rules. In addition, the calculation of statistics in the Resource Stats Collector block has been improved and expanded. Lastly, SAS Simulation Studio 13.1 streamlines the method for specifying a DataStreamDescription factor or an InStreamPolicy port value for a Numeric Source block in order to control the probability distribution that is sampled in the block.
Chapter 2
The OPTNET Procedure

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Overview: OPTNET Procedure

The OPTNET procedure includes a number of graph theory, combinatorial optimization, and network analysis algorithms. The algorithm classes are listed in Table 2.1.

<table>
<thead>
<tr>
<th>Algorithm Class</th>
<th>PROC OPTNET Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biconnected components</td>
<td>BICONCOMP</td>
</tr>
<tr>
<td>Maximal cliques</td>
<td>CLIQUE</td>
</tr>
<tr>
<td>Connected components</td>
<td>CONCOMP</td>
</tr>
<tr>
<td>Cycle detection</td>
<td>CYCLE</td>
</tr>
<tr>
<td>Weighted matching</td>
<td>LINEAR_ASSIGNMENT</td>
</tr>
<tr>
<td>Minimum-cost network flow</td>
<td>MINCOSTFLOW</td>
</tr>
<tr>
<td>Minimum cut</td>
<td>MINCUT</td>
</tr>
<tr>
<td>Minimum spanning tree</td>
<td>MINSPANTREE</td>
</tr>
<tr>
<td>Shortest path</td>
<td>SHORTPATH</td>
</tr>
<tr>
<td>Transitive closure</td>
<td>TRANSITIVE_CLOSURE</td>
</tr>
<tr>
<td>Traveling salesman</td>
<td>TSP</td>
</tr>
</tbody>
</table>

You can use the OPTNET procedure to analyze relationships between entities. These relationships are typically defined by using a graph. A graph \( G = (N, A) \) is defined over a set \( N \) of nodes and a set \( A \) of arcs. A node is an abstract representation of some entity (or object), and an arc defines some relationship (or connection) between two nodes. The terms node and vertex are often interchanged in describing an entity. The term arc is often interchanged with the term edge or link when describing a connection.

Getting Started: OPTNET Procedure

Since graphs are abstract objects, their analyses have applications in many different fields of study, including social sciences, linguistics, biology, transportation, marketing, and so on. This document shows a few potential applications through simple examples.
This section shows an introductory example for getting started with the OPTNET procedure. For more detail about the input formats expected and the various algorithms available, see the sections “Details: OPTNET Procedure” on page 35 and “Examples: OPTNET Procedure” on page 101.

Road Network Shortest Path

Consider the following road network between a SAS employee’s home in Raleigh, NC, and the SAS headquarters in Cary, NC.

In this road network (graph), the links are the roads and the nodes are intersections between roads. For each road, you assign a link attribute in the variable time_to_travel to describe the number of minutes that it takes to drive from one node to another. The following data were collected using Google Maps (Google 2011), which gives an approximate number of minutes to traverse between two points, based on the length of the road and the typical speed during normal traffic patterns:

```plaintext
data LinkSetInRoadNC10am;
  input start_inter $1-20 end_inter $20-40 miles miles_per_hour;
datalines;
  614CapitalBlvd Capital/WadeAve 0.6 25
  614CapitalBlvd Capital/US70W 0.6 25
  614CapitalBlvd Capital/US440W 3.0 45
  Capital/WadeAve WadeAve/RaleighExpy 3.0 40
  Capital/US70W US70W/US440W 3.2 60
  US70W/US440W US440W/RaleighExpy 2.7 60
  Capital/US440W US440W/RaleighExpy 6.7 60
  US440W/RaleighExpy RaleighExpy/US40W 3.0 60
  WadeAve/RaleighExpy RaleighExpy/US40W 3.0 60
  RaleighExpy/US40W US40W/HarrisonAve 1.3 55
  US40W/HarrisonAve SASCampusDrive 0.5 25;

data LinkSetInRoadNC10am;
  set LinkSetInRoadNC10am;
  time_to_travel = miles * 1/miles_per_hour * 60;
run;
```

Using PROC OPTNET, you want to find the route that yields the shortest path between home (614 Capital Blvd) and the SAS headquarters (SAS Campus Drive). This can be done with the SHORTPATH statement as follows:

```plaintext
proc optnet
  data_links = LinkSetInRoadNC10am;
data_links_var
    from = start_inter
to = end_inter
weight = time_to_travel;
shortpath
  out_paths = ShortPath
source = "614CapitalBlvd"
sink = "SASCampusDrive";
run;
```
For more information about shortest path algorithms in PROC OPTNET, see the section “Shortest Path” on page 72. Figure 2.1 displays the output data set ShortPath, which shows the best route to take to minimize travel time at 10:00 a.m. This route is also shown in Google Maps in Figure 2.2.

**Figure 2.1** Shortest Path for Road Network at 10:00 A.M.

<table>
<thead>
<tr>
<th>order</th>
<th>start_inter</th>
<th>end_inter</th>
<th>time_to_ travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>614CapitalBlvd</td>
<td>Capital/WadeAve</td>
<td>1.4400</td>
</tr>
<tr>
<td>2</td>
<td>Capital/WadeAve</td>
<td>WadeAve/RaleighExpy</td>
<td>4.5000</td>
</tr>
<tr>
<td>3</td>
<td>WadeAve/RaleighExpy</td>
<td>RaleighExpy/US40W</td>
<td>3.0000</td>
</tr>
<tr>
<td>4</td>
<td>RaleighExpy/US40W</td>
<td>US40W/HarrisonAve</td>
<td>1.4182</td>
</tr>
<tr>
<td>5</td>
<td>US40W/HarrisonAve</td>
<td>SASCampusDrive</td>
<td>1.2000</td>
</tr>
</tbody>
</table>

**Figure 2.2** Shortest Path for Road Network at 10:00 A.M. in Google Maps

Now suppose that it is rush hour (5:00 p.m.) and the time to traverse the roads has changed because of traffic patterns. You want to find the route that is the shortest path for going home from SAS headquarters under different speed assumptions due to traffic. The following data set lists approximate travel times and speeds for driving in the opposite direction:

```plaintext
data LinkSetInRoadNC5pm;
  input start_inter $1-20 end_inter $20-40 miles miles_per_hour;
datalines;
614CapitalBlvd Capital/WadeAve 0.6 25
614CapitalBlvd Capital/US70W 0.6 25
614CapitalBlvd Capital/US440W 3.0 45
Capital/WadeAve WadeAve/RaleighExpy 3.0 25 /*high traffic*/
Capital/US70W US70W/US440W 3.2 60
US70W/US440W US440W/RaleighExpy 2.7 60
```
The following statements are similar to the first PROC OPTNET run, except that they use the data set LinkSetInRoadNC5pm and the SOURCE and SINK option values are reversed:

```plaintext
proc optnet
  data_links  = LinkSetInRoadNC5pm;
  data_links_var
    from  = start_inter
    to    = end_inter
    weight  = time_to_travel;
  shortpath
    out_paths  = ShortPath
    source  = "SASCampusDrive"
    sink    = "614CapitalBlvd";
run;
```

Now, the output data set ShortPath, shown in Figure 2.3, shows the best route for going home. Because the traffic on Wade Avenue is usually heavy at this time of day, the route home is different from the route to work.

**Figure 2.3** Shortest Path for Road Network at 5:00 P.M.

<table>
<thead>
<tr>
<th>order</th>
<th>start_inter</th>
<th>end_inter</th>
<th>time_to_travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SASCampusDrive</td>
<td>US40W/HarrisonAve</td>
<td>1.2000</td>
</tr>
<tr>
<td>2</td>
<td>US40W/HarrisonAve</td>
<td>RaleighExpy/US40W</td>
<td>1.4182</td>
</tr>
<tr>
<td>3</td>
<td>RaleighExpy/US40W</td>
<td>US440W/RaleighExpy</td>
<td>3.0000</td>
</tr>
<tr>
<td>4</td>
<td>US440W/RaleighExpy</td>
<td>US70W/US440W</td>
<td>2.7000</td>
</tr>
<tr>
<td>6</td>
<td>Capital/US70W</td>
<td>614CapitalBlvd</td>
<td>1.4400</td>
</tr>
</tbody>
</table>

*********

12.9582
This new route is shown in Google Maps in Figure 2.4.

![Figure 2.4 Shortest Path for Road Network at 5:00 P.M. in Google Maps](image)

Syntax: OPTNET Procedure

```sas
PROC OPTNET options ;

Data Input Statements:
DATA_LINKS_VAR < options > ;
DATA_MATRIX_VAR column1 <,column2,...> ;
DATA_NODES_VAR < options > ;

Algorithm Statements:
BICONCOMP < option > ;
CLIQUE < options > ;
CONCOMP < option > ;
CYCLE < options > ;
LINEAR_ASSIGNMENT < options > ;
MINCOSTFLOW < option > ;
MINCUT < options > ;
MINSPANTREE < options > ;
SHORTPATH < options > ;
TRANSITIVE_CLOSURE < option > ;
TSP < option > ;
```

PROC OPTNET statements are divided into three main categories: the PROC statement, the data input statements, and the algorithm statements. The PROC statement invokes the procedure and sets option values that are used across multiple algorithms. The data input statements control the names of the variables that
PROC OPTNET expects in the data input. The algorithm statements determine which algorithms are run and set options for each individual algorithm.

The section “Functional Summary” on page 11 provides a quick reference for each of the options for each statement. Each statement is then described in more detail in its own section; the PROC OPTNET statement is described first, and sections that describe all other statements are presented in alphabetical order.

### Functional Summary

Table 2.2 summarizes the statements and options available with PROC OPTNET.

<table>
<thead>
<tr>
<th>Description</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PROC OPTNET Options</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Input</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the link data set</td>
<td>DATA_LINKS=</td>
</tr>
<tr>
<td>Specifies the matrix data set</td>
<td>DATA_MATRIX=</td>
</tr>
<tr>
<td>Specifies the node data set</td>
<td>DATA_NODES=</td>
</tr>
<tr>
<td>Specifies the node subset data set</td>
<td>DATA_NODES_SUB=</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the link output data set</td>
<td>OUT_LINKS=</td>
</tr>
<tr>
<td>Specifies the node output data set</td>
<td>OUT_NODES=</td>
</tr>
<tr>
<td><strong>Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the graph direction</td>
<td>GRAPH_DIRECTION=</td>
</tr>
<tr>
<td>Specifies the internal graph format</td>
<td>GRAPH_INTERNAL_FORMAT=</td>
</tr>
<tr>
<td>Includes self links</td>
<td>INCLUDE_SELFLINK</td>
</tr>
<tr>
<td>Specifies the overall log level</td>
<td>LOGLEVEL=</td>
</tr>
<tr>
<td>Specifies whether time units are in CPU time or real time</td>
<td>TIMETYPE=</td>
</tr>
</tbody>
</table>

**Data Input Statements**

**DATA_LINKS_VAR Options**

| Specifies the data set variable name for the from nodes | FROM= |
| Specifies the data set variable name for the link flow lower bounds | LOWER= |
| Specifies the data set variable name for the to nodes | TO= |
| Specifies the data set variable name for the link flow upper bounds | UPPER= |
| Specifies the data set variable name for the link weights | WEIGHT= |

**DATA_MATRIX_VAR**

| Specifies the data set variable names for the matrix |                  |

**DATA_NODES_VAR Options**

| Specifies the data set variable name for the nodes | NODE= |
| Specifies the data set variable name for node weights | WEIGHT= |
| Specifies the data set variable name for auxiliary node weights | WEIGHT2= |

**Algorithm Statements**

**BICONCOMP Option**

| Specifies the log level for biconnected components | LOGLEVEL= |
### Table 2.2 (continued)

<table>
<thead>
<tr>
<th>Description</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CLIQUE Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the log level for clique calculations</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum number of cliques to return during clique calculations</td>
<td><strong>MAXCLIQUES</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum amount of time to spend calculating cliques</td>
<td><strong>MAXTIME</strong>=</td>
</tr>
<tr>
<td>Specifies the output data set for cliques</td>
<td><strong>OUT</strong>=</td>
</tr>
<tr>
<td><strong>CONCOMP Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the algorithm to use for connected components</td>
<td><strong>ALGORITHM</strong>=</td>
</tr>
<tr>
<td>Specifies the log level for connected components</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td><strong>CYCLE Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the log level for the cycle algorithm</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum number of cycles to return during cycle calculations</td>
<td><strong>MAXCYCLES</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum length for the cycles found</td>
<td><strong>MAXLENGTH</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum link weight for the cycles found</td>
<td><strong>MAXLINKWEIGHT</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum node weight for the cycles found</td>
<td><strong>MAXNODEWEIGHT</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum amount of time to spend calculating cycles</td>
<td><strong>MAXTIME</strong>=</td>
</tr>
<tr>
<td>Specifies the minimum length for the cycles found</td>
<td><strong>MINLENGTH</strong>=</td>
</tr>
<tr>
<td>Specifies the minimum link weight for the cycles found</td>
<td><strong>MINLINKWEIGHT</strong>=</td>
</tr>
<tr>
<td>Specifies the minimum node weight for the cycles found</td>
<td><strong>MINNODEWEIGHT</strong>=</td>
</tr>
<tr>
<td>Specifies the mode for the cycle calculations</td>
<td><strong>MODE</strong>=</td>
</tr>
<tr>
<td>Specifies the output data set for cycles</td>
<td><strong>OUT</strong>=</td>
</tr>
<tr>
<td><strong>LINEAR_ASSIGNMENT Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the data set variable names for the linear assignment identifiers</td>
<td><strong>ID</strong>=( )</td>
</tr>
<tr>
<td>Specifies the log level for the linear assignment algorithm</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td>Specifies the output data set for linear assignment</td>
<td><strong>OUT</strong>=</td>
</tr>
<tr>
<td>Specifies the data set variable names for costs (or weights)</td>
<td><strong>WEIGHT</strong>=( )</td>
</tr>
<tr>
<td><strong>MINCOSTFLOW Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the iteration log frequency</td>
<td><strong>LOGFREQ</strong>=</td>
</tr>
<tr>
<td>Specifies the log level for the minimum-cost network flow algorithm</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum amount of time to spend calculating the optimal flow</td>
<td><strong>MAXTIME</strong>=</td>
</tr>
<tr>
<td><strong>MINCUT Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the log level for the minimum cut algorithm</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum number of cuts to return from the algorithm</td>
<td><strong>MAXNUMCUTS</strong>=</td>
</tr>
<tr>
<td>Specifies the maximum weight of the cuts to return from the algorithm</td>
<td><strong>MAXWEIGHT</strong>=</td>
</tr>
<tr>
<td>Specifies the output data set for minimum cut</td>
<td><strong>OUT</strong>=</td>
</tr>
<tr>
<td><strong>MINSPANTREE Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the log level for the minimum spanning tree algorithm</td>
<td><strong>LOGLEVEL</strong>=</td>
</tr>
<tr>
<td>Specifies the output data set for minimum spanning tree</td>
<td><strong>OUT</strong>=</td>
</tr>
<tr>
<td><strong>SHORTPATH Options</strong></td>
<td></td>
</tr>
<tr>
<td>Specifies the iteration log frequency (nodes)</td>
<td><strong>LOGFREQ</strong>=</td>
</tr>
</tbody>
</table>
Table 2.2  (continued)

<table>
<thead>
<tr>
<th>Description</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specifies the log level for shortest paths</td>
<td>LOGLEVEL=</td>
</tr>
<tr>
<td>Specifies the output data set for shortest paths</td>
<td>OUT_PATHS=</td>
</tr>
<tr>
<td>Specifies the output data set for shortest path summaries</td>
<td>OUT_WEIGHTS=</td>
</tr>
<tr>
<td>Specifies the type of output for shortest paths results</td>
<td>PATHS=</td>
</tr>
<tr>
<td>Specifies the sink node for shortest paths calculations</td>
<td>SINK=</td>
</tr>
<tr>
<td>Specifies the source node for shortest paths calculations</td>
<td>SOURCE=</td>
</tr>
<tr>
<td>Specifies whether to use weights in calculating shortest paths</td>
<td>USEWEIGHT=</td>
</tr>
<tr>
<td>Specifies the data set variable name for the auxiliary link weights</td>
<td>WEIGHT2=</td>
</tr>
</tbody>
</table>

**TRANSITIVE_CLOSURE Options**

- Specifies the log level for transitive closure                           | LOGLEVEL=                     |
- Specifies the output data set for transitive closure results             | OUT=                          |

**TSP Options**

- Specifies the stopping criterion based on the absolute objective gap     | ABSOBJGAP=                    |
- Specifies the level of conflict search                                   | CONFLICTSEARCH=               |
- Specifies the cutoff value for branch-and-bound node removal             | CUTOFF=                       |
- Specifies the overall cut strategy level                                  | CUTSTRATEGY=                  |
- Emphasizes feasibility or optimality                                     | EMPHASIS=                     |
- Specifies the initial and primal heuristics level                         | HEURISTICS=                   |
- Specifies the maximum allowed difference between an integer variable’s value and an integer | LOGFREQ=                      |
- Specifies the log level for the traveling salesman algorithm              | LOGLEVEL=                     |
- Specifies the maximum number of branch-and-bound nodes to be processed   | MAXNODES=                     |
- Specifies the maximum number of solutions to be found                    | MAXSOLS=                      |
- Specifies the maximum amount of time to spend in the algorithm           | MAXTIME=                      |
- Specifies whether to use a mixed integer linear programming solver        | MILP=                         |
- Specifies the branch-and-bound node selection strategy                    | NODESEL=                      |
- Specifies the output data set for traveling salesman                     | OUT=                          |
- Specifies the probing level                                               | PROBE=                        |
- Specifies the stopping criterion that is based on relative objective gap | RELOBJGAP=                    |
- Specifies the number of simplex iterations to be performed on each variable in the strong branching strategy | STRONGITER=                  |
- Specifies the number of candidates for the strong branching strategy      | STRONGLEN=                    |
- Specifies the stopping criterion based on the target objective value     | TARGET=                       |
- Specifies the rule for selecting branching variable                       | VARSEL=                       |
Table 2.3 lists the valid input formats, `GRAPH_DIRECTION=` values, and `GRAPH_INTERNAL_FORMAT=` values for each statement in the OPTNET procedure.

**Table 2.3**  Supported Input Formats and Graph Types by Statement

<table>
<thead>
<tr>
<th>Statement</th>
<th>Input Format</th>
<th>DIRECTION</th>
<th>INTERNAL_FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>UNDIRECTED</td>
<td>DIRECTED</td>
</tr>
<tr>
<td>BICONCOMP</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>CLIQUE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>CONCOMP ALGORITHM=</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>DFS</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>UNION_FIND</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>CYCLE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>LINEAR_ASSIGNMENT</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>MINCOSTFLOW</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>MINCUT</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>MINSPANTREE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>SHORTPATH</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>TRANSITIVE_CLOSURE</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>TSP</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 2.4 indicates for each algorithm statement in the OPTNET procedure which output data set options you can specify and whether the algorithm populates the data sets specified in the `OUT_NODES=` and `OUT_LINKS=` options in the PROC OPTNET statement.

**Table 2.4**  Output Options by Statement

<table>
<thead>
<tr>
<th>Statement</th>
<th>OUT_NODES</th>
<th>OUT_LINKS</th>
<th>Algorithm Statement Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>BICONCOMP</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>CLIQUE</td>
<td>X</td>
<td></td>
<td>OUT=</td>
</tr>
<tr>
<td>CONCOMP</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CYCLE</td>
<td>X</td>
<td></td>
<td>OUT=</td>
</tr>
<tr>
<td>LINEAR_ASSIGNMENT</td>
<td></td>
<td>X</td>
<td>OUT=</td>
</tr>
<tr>
<td>MINCOSTFLOW</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINCUT</td>
<td>X</td>
<td></td>
<td>OUT=</td>
</tr>
<tr>
<td>MINSPANTREE</td>
<td>X</td>
<td></td>
<td>OUT=</td>
</tr>
<tr>
<td>SHORTPATH</td>
<td>X</td>
<td></td>
<td>OUT_PATHS=, OUT_WEIGHTS=</td>
</tr>
<tr>
<td>TRANSITIVE_CLOSURE</td>
<td></td>
<td>X</td>
<td>OUT=</td>
</tr>
<tr>
<td>TSP</td>
<td>X</td>
<td></td>
<td>OUT=</td>
</tr>
</tbody>
</table>
PROC OPTNET Statement

PROC OPTNET < options > ;

The PROC OPTNET statement invokes the OPTNET procedure. You can specify the following options to define the input and output data sets, the log levels, and various other processing controls:

- **DATA_LINKS=** SAS-data-set
  - LINKS=SAS-data-set
  - Specifies the input data set that contains the graph link information, where the links are defined as a list. See the section “Link Input Data” on page 35 for more information.

- **DATA_MATRIX=** SAS-data-set
  - MATRIX=SAS-data-set
  - Specifies the input data set that contains the matrix to be processed. This is a generic matrix (as opposed to an adjacency matrix, which defines an underlying graph). See the section “Matrix Input Data” on page 40 for more information.

- **DATA_NODES=** SAS-data-set
  - NODES=SAS-data-set
  - Specifies the input data set that contains the graph node information. See the section “Node Input Data” on page 39 for more information.

- **DATA_NODES_SUB=** SAS-data-set
  - NODES_SUB=SAS-data-set
  - Specifies the input data set that contains the graph node subset information. See the section “Node Subset Input Data” on page 40 for more information.

- **GRAPH_DIRECTION=DIRECTED | UNDIRECTED**
  - DIRECTION=DIRECTED | UNDIRECTED
  - Specifies whether the input graph should be considered directed or undirected.

<table>
<thead>
<tr>
<th>Option Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIRECTED</td>
<td>Specifies the graph as directed. In a directed graph, each link ((i, j)) has a direction that defines how something (for example, information) might flow over that link. In link ((i, j)), information flows from node (i) to node (j) ((i \rightarrow j)). The node (i) is called the source (tail) node, and (j) is called the sink (head) node.</td>
</tr>
<tr>
<td>UNDIRECTED</td>
<td>Specifies the graph as undirected. In an undirected graph, each link ({i, j}) has no direction and information can flow in either direction. That is, ({i, j} = {j, i}). This is the default.</td>
</tr>
</tbody>
</table>

By default, GRAPH_DIRECTION=UNDIRECTED. See the section “Graph Input Data” on page 35 for more information.
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GRAPH_INTERNAL_FORMAT=THIN | FULL

requests the internal graph format for the algorithms to use.

Table 2.6  Values for the GRAPH_INTERNAL_FORMAT= Option

<table>
<thead>
<tr>
<th>Option Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FULL</td>
<td>Stores the graph in standard (full) format. This is the default.</td>
</tr>
<tr>
<td>THIN (experimental)</td>
<td>Stores the graph in thin format. This option can improve performance in some cases both by reducing memory and by simplifying the construction of the internal data structures. The thin format causes PROC OPTNET to skip the removal of duplicate links when it reads in the graph. So this option should be used with caution. For some algorithms, the thin format is not allowed and this option is ignored. The THIN option can often be helpful when you do calculations that are decomposed by subgraph.</td>
</tr>
</tbody>
</table>

See the section “Graph Input Data” on page 35 for more information.

INCLUDE_SELFLINK

includes self links—for example, $(i, i)$—when an input graph is read. By default, when PROC OPTNET reads the DATA_LINKS= data set, it removes all self links.

LOGLEVEL=number | string

controls the amount of information that is displayed in the SAS log. Each algorithm has its own specific log level. This setting sets the log level for all algorithms except those for which you specify the LOGLEVEL= option in the algorithm statement. Table 2.7 describes the valid values for this option.

Table 2.7  Values for LOGLEVEL= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all procedure-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the input, output, and algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the input, output, and algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the input, output, and algorithmic processing</td>
</tr>
</tbody>
</table>

By default, LOGLEVEL=BASIC.

OUT_LINKS=SAS-data-set

specifies the output data set to contain the graph link information along with any results from the various algorithms that calculate metrics on links.

See the various algorithm sections for examples of the content of this output data set.

OUT_NODES=SAS-data-set

specifies the output data set to contain the graph node information along with any results from the various algorithms that calculate metrics on nodes.

See the various algorithm sections for examples of the content of this output data set.
TIMETYPE=number | string
specifies whether CPU time or real time is used for the MAXTIME= option for each applicable algorithm. Table 2.8 describes the valid values of the TIMETYPE= option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>CPU</td>
<td>Specifies units of CPU time</td>
</tr>
<tr>
<td>1</td>
<td>REAL</td>
<td>Specifies units of real time</td>
</tr>
</tbody>
</table>

By default, TIMETYPE=CPU.

BICONCOMP Statement

BICONCOMP < option >;

The BICONCOMP statement requests that PROC OPTNET find biconnected components and articulation points of an undirected input graph.

See the section “Biconnected Components and Articulation Points” on page 41 for more information.

You can specify the following option in the BICONCOMP statement.

LOGLEVEL=number | string
controls the amount of information that is displayed in the SAS log. Table 2.9 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).
CLIQUE Statement

CLIQUE < options > ;

The CLIQUE statement invokes an algorithm that finds maximal cliques on the input graph. Maximal cliques are described in the section “Clique” on page 45.

You can specify the following options in the CLIQUE statement:

LOGLEVEL=number | string
controls the amount of information that is displayed in the SAS log. Table 2.10 describes the valid values for this option.

Table 2.10 Values for LOGLEVEL= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

MAXCLIQUES=number
specifies the maximum number of cliques to return during clique calculations. The default is the positive number that has the largest absolute value that can be represented in your operating environment.

MAXTIME=number
specifies the maximum amount of time to spend calculating cliques. The type of time (either CPU time or real time) is determined by the value of the TIMETYPE= option. The value of number can be any positive number; the default value is the positive number that has the largest absolute value that can be represented in your operating environment.

OUT=SAS-data-set
specifies the output data set to contain the maximal cliques.
CONCOMP Statement

CONCOMP < options > ;

The CONCOMP statement invokes an algorithm that finds the connected components of the input graph. Connected components are described in the section “Connected Components” on page 48.

You can specify the following options in the CONCOMP statement:

**ALGORITHM=DFS | UNION_FIND**

specifies the algorithm to use for calculating connected components.

**Table 2.11  Values for the ALGORITHM= Option**

<table>
<thead>
<tr>
<th>Option Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFS</td>
<td>Uses the depth-first search algorithm for connected components. You cannot specify this value when you specify GRAPH_INTERNAL_FORMAT=THIN in the PROC OPTNET statement.</td>
</tr>
<tr>
<td>UNION_FIND</td>
<td>Uses the union-find algorithm for connected components. You can specify this value with either the THIN or FULL value for the GRAPH_INTERNAL_FORMAT option in the PROC OPTNET statement. This value can be faster than DFS when used with GRAPH_INTERNAL_FORMAT=THIN. However, you can use it only with undirected graphs.</td>
</tr>
</tbody>
</table>

By default, ALGORITHM=DFS.

**LOGLEVEL=number | string**

controls the amount of information that is displayed in the SAS log. Table 2.12 describes the valid values for this option.

**Table 2.12  Values for LOGLEVEL= Option**

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).
Chapter 2: The OPTNET Procedure

CYCLE Statement

```
CYCLE < options > ;
```

The CYCLE statement invokes an algorithm that finds the cycles (or the existence of a cycle) in the input graph. Cycles are described in the section “Cycle” on page 52.

You can specify the following `options` in the CYCLE statement:

- **LOGLEVEL=** `number | string`
  
  controls the amount of information that is displayed in the SAS log. Table 2.13 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the `LOGLEVEL=` option in the PROC OPTNET statement (or BASIC if that option is not specified).

- **MAXCYCLES=** `number`

  specifies the maximum number of cycles to return. The default is the positive number that has the largest absolute value representable in your operating environment. This option works only when you also specify `MODE=ALL_CYCLES`.

- **MAXLENGTH=** `number`

  specifies the maximum number of links to allow in a cycle. Any cycle whose length is greater than `number` is removed from the results. The default is the positive number that has the largest absolute value that can be represented in your operating environment. By default, nothing is removed from the results. This option works only when you also specify `MODE=ALL_CYCLES`.

- **MAXLINKWEIGHT=** `number`

  specifies the maximum sum of link weights to allow in a cycle. Any cycle whose sum of link weights is greater than `number` is removed from the results. The default is the positive number that has the largest absolute value that can be represented in your operating environment. By default, nothing is filtered. This option works only when you also specify `MODE=ALL_CYCLES`.

- **MAXNODEWEIGHT=** `number`

  specifies the maximum sum of node weights to allow in a cycle. Any cycle whose sum of node weights is greater than `number` is removed from the results. The default is the positive number that has the largest absolute value that can be represented in your operating environment. By default, nothing is filtered. This option works only when you also specify `MODE=ALL_CYCLES`. 
**MAXTIME**=number

specifies the maximum amount of time to spend finding cycles. The type of time (either CPU time or real time) is determined by the value of the **TIMETYPE**= option. The value of number can be any positive number; the default value is the positive number that has the largest absolute value that can be represented in your operating environment. This option works only when you also specify **MODE**=ALL_CYCLES.

**MINLENGTH**=number

specifies the minimum number of links to allow in a cycle. Any cycle that has fewer links than number is removed from the results. By default MINLENGTH=1 and nothing is filtered. This option works only when you also specify **MODE**=ALL_CYCLES.

**MINLINKWEIGHT**=number

specifies the minimum sum of link weights to allow in a cycle. Any cycle whose sum of link weights is less than number is removed from the results. The default is the negative number that has the largest absolute value that can be represented in your operating environment. By default, nothing is filtered. This option works only when you also specify **MODE**=ALL_CYCLES.

**MINNODEWEIGHT**=number

specifies the minimum sum of node weights to allow in a cycle. Any cycle whose sum of node weights is less than number is removed from the results. The default is the negative number that has the largest absolute value that can be represented in your operating environment. By default, nothing is filtered. This option works only when you also specify **MODE**=ALL_CYCLES.

**MODE**=option

specifies the mode for processing cycles.

<table>
<thead>
<tr>
<th>Option Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL_CYCLES</td>
<td>Returns all (unique, elementary) cycles found.</td>
</tr>
<tr>
<td>FIRST_CYCLES</td>
<td>Returns the first cycle found.</td>
</tr>
</tbody>
</table>

By default, **MODE**=FIRST_CYCLES.

**OUT**=SAS-data-set

specifies the output data set to contain the cycles found.
DATA_LINKS_VAR Statement

DATA_LINKS_VAR < options > ;
LINKS_VAR < options > ;

The DATA_LINKS_VAR statement enables you to explicitly define the data set variable names for PROC OPTNET to use when it reads the data set that is specified in the DATA_LINKS= option in the PROC OPTNET statement. The format of the links input data set is defined in the section “Link Input Data” on page 35.

You can specify the following options in the DATA_LINKS_VAR statement:

FROM=column
  specifies the data set variable name for the from nodes. The value of column can be numeric or character.

LOWER=column
  specifies the data set variable name for the link flow lower bounds. The value of column must be numeric.

TO=column
  specifies the data set variable name for the to nodes. The value of column can be numeric or character.

UPPER=column
  specifies the data set variable name for the link flow upper bounds. The value of column must be numeric.

WEIGHT=column
  specifies the data set variable name for the link weights. The value of column must be numeric.
DATA_MATRIX_VAR Statement

DATA_MATRIX_VAR column1 <,column2,...> ;
MATRIX_VAR column1 <,column2,...> ;

The DATA_MATRIX_VAR statement enables you to explicitly define the data set variable names for PROC OPTNET to use when it reads the data set that is specified in the DATA_MATRIX= option in the PROC OPTNET statement. The format of the matrix input data set is defined in the section “Matrix Input Data” on page 40. The value of each column variable must be numeric.

DATA_NODES_VAR Statement

DATA_NODES_VAR < options > ;
NODES_VAR < options > ;

The DATA_NODES_VAR statement enables you to explicitly define the data set variable names for PROC OPTNET to use when it reads the data set that is specified in the DATA_NODES= option in the PROC OPTNET statement. The format of the node input data set is defined in the section “Node Input Data” on page 39.

You can specify the following options in the DATA_NODES_VAR statement:

NODE=column
    specifies the data set variable name for the nodes. The value of column can be numeric or character.

WEIGHT=column
    specifies the data set variable name for node weights. The value of column must be numeric.

WEIGHT2=column
    specifies the data set variable name for auxiliary node weights. The value of column must be numeric.
The LINEAR_ASSIGNMENT statement invokes an algorithm that solves the minimal-cost linear assignment problem. In graph terms, this problem is also known as the minimum link-weighted matching problem on a bipartite graph. The input data (the cost matrix) is typically defined in the input data set that is specified in the DATA_MATRIX= option in the PROC OPTNET statement. The data can also be defined as a directed graph by specifying the DATA_LINKS= option in the PROC OPTNET statement, where the costs are defined as link weights. Internally, the graph is treated as a bipartite graph in which the from nodes define one part and the to nodes define the other part.

The linear assignment problem is described in the section “Linear Assignment (Matching)” on page 59.

You can specify the following options in the LINEAR_ASSIGNMENT statement:

**ID=(column1 <,column2,...>)**
specifies the data set variable names that identify the matrix rows (from nodes). The information in these columns is carried to the output data set that is specified in the OUT= option. The value of each column variable can be numeric or character.

**LOGLEVEL=number | string**
controls the amount of information that is displayed in the SAS log. Table 2.15 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

**OUT=SAS-data-set**
specifies the output data set to contain the solution to the linear assignment problem.

**WEIGHT=(column1 <,column2,...>)**
specifies the data set variable names for the cost matrix. The value of each column variable must be numeric. If this option is not specified, the matrix is assumed to be defined by all of the numeric variables in the data set (excluding those specified in the ID= option).
MINCOSTFLOW Statement

MINCOSTFLOW < options > ;
MCF < options > ;

The MINCOSTFLOW statement invokes an algorithm that solves the minimum-cost network flow problem on an input graph.

The minimum-cost network flow problem is described in the section “Minimum-Cost Network Flow” on page 60.

You can specify the following options in the MINCOSTFLOW statement:

LOGFREQ=number
controls the frequency for displaying iteration logs for minimum-cost network flow calculations that use the network simplex algorithm. For graphs that contain one component, this option displays progress every number simplex iterations, and the default is 10,000. For graphs that contain multiple components, when you also specify LOGLEVEL=Moderate, this option displays progress after processing every number components, and the default is based on the number of components. When you also specify LOGLEVEL=Aggressive, the simplex iteration log for each component is displayed with frequency number.

The value of number can be any integer greater than or equal to 1. Setting this value too low can hurt performance on large-scale graphs.

LOGLEVEL=number | string
controls the amount of information that is displayed in the SAS log. Table 2.16 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing including a progress log using the interval that is specified in the LOGFREQ option</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing including a progress log using the interval that is specified in the LOGFREQ option</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

MAXTIME=option
specifies the maximum amount of time to spend calculating minimum-cost network flows. The type of time (either CPU time or real time) is determined by the value of the TIMETYPE= option. The value of number can be any positive number; the default value is the positive number that has the largest absolute value that can be represented in your operating environment.
MINCUT Statement

MINCUT < options > ;

The MINCUT statement invokes an algorithm that finds the minimum link-weighted cut of an input graph. The minimum cut problem is described in the section “Minimum Cut” on page 65.

You can specify the following options in the MINCUT statement:

LOGLEVEL=number | string
controls the amount of information that is displayed in the SAS log. Table 2.17 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

MAXNUMCUTS=number
specifies the maximum number of cuts to return from the algorithm. The minimal cut and any others found during the search, up to number, are returned. By default, MAXNUMCUTS=1.

MAXWEIGHT=number
specifies the maximum weight of the cuts to return from the algorithm. Only cuts that have weight less than or equal to number are returned. The default is the positive number that has the largest absolute value that can be represented in your operating environment.

OUT=SAS-data-set
specifies the output data set to contain the solution to the minimum cut problem.
MINSPANTREE Statement

MINSPANTREE < options > ;

MST < options > ;

The MINSPANTREE statement invokes an algorithm that solves the minimum link-weighted spanning tree problem on an input graph.

The minimum spanning tree problem is described in the section “Minimum Spanning Tree” on page 69.

You can specify the following options in the MINSPANTREE statement:

LOGLEVEL=number | string
controls the amount of information that is displayed in the SAS log. Table 2.18 describes the valid values for this option.

Table 2.18  Values for LOGLEVEL= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

OUT=SAS-data-set
specifies the output data set to contain the solution to the minimum link-weighted spanning tree problem.

SHORTPATH Statement

SHORTPATH < options > ;

The SHORTPATH statement invokes an algorithm that calculates shortest paths between sets of nodes on the input graph.

The shortest path algorithm is described in the section “Shortest Path” on page 72.

You can specify the following options in the SHORTPATH statement:

LOGFREQ=number

- displays iteration logs for shortest path calculations every number nodes. The value of number can be any integer greater than or equal to 1. The default is determined automatically based on the size of the graph. Setting this value too low can hurt performance on large-scale graphs.
**LOGLEVEL=** \textit{number}  
controls the amount of information that is displayed in the SAS log. Table 2.19 describes the valid values for this option.

<table>
<thead>
<tr>
<th>\textit{number}</th>
<th>\textit{string}</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

**OUT\_PATHS=** \textit{SAS-data-set}  
**OUT=** \textit{SAS-data-set}  
specifies the output data set to contain the shortest paths.

**OUT\_WEIGHTS=** \textit{SAS-data-set}  
specifies the output data set to contain the shortest path summaries.

**PATHS=** \textit{ALL | SHORTEST | LONGEST}  
specifies the type of output to produce in the output data set that is specified in the OUT\_PATHS= option.

<table>
<thead>
<tr>
<th>Option Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL</td>
<td>Outputs shortest paths for all pairs of source-sinks.</td>
</tr>
<tr>
<td>LONGEST</td>
<td>Outputs shortest paths for the source-sink pair with the longest (finite) length. If other source-sink pairs (up to 100) have equally long length, they are also output.</td>
</tr>
<tr>
<td>SHORTEST</td>
<td>Outputs shortest paths for the source-sink pair with the shortest length. If other source-sink pairs (up to 100) have equally short length, they are also output.</td>
</tr>
</tbody>
</table>

By default, PATHS=ALL.

**SINK=** \textit{sink-node}  
specifies the sink node for shortest paths calculations. This setting overrides the use of the variable sink in the data set that is specified in the DATA\_NODES\_SUB= option in the PROC OPTNET statement.

**SOURCE=** \textit{source-node}  
specifies the source node for shortest paths calculations. This setting overrides the use of the variable source in the data set that is specified in the DATA\_NODES\_SUB= option in the PROC OPTNET statement.
USEWEIGHT=YES | NO
specifies whether to use link weights (if they exist) in calculating shortest paths.

<table>
<thead>
<tr>
<th>Option Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>YES</td>
<td>Uses weights (if they exist) in shortest path calculations. This is the default.</td>
</tr>
<tr>
<td>NO</td>
<td>Does not use weights in shortest path calculations.</td>
</tr>
</tbody>
</table>

WEIGHT2=column
specifies the data set variable name for the auxiliary link weights. The value of column must be numeric.

TRANSITIVE_CLOSURE Statement

TRANSITIVE_CLOSURE < options >;
TRANSCL < options >;

The TRANSITIVE_CLOSURE statement invokes an algorithm that calculates the transitive closure of an input graph.

Transitive closure is described in the section “Transitive Closure” on page 84.

You can specify the following options in the TRANSITIVE_CLOSURE statement:

LOGLEVEL=number
controls the amount of information that is displayed in the SAS log. Table 2.22 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all algorithm-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a basic summary of the algorithmic processing</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Displays a summary of the algorithmic processing</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Displays a detailed summary of the algorithmic processing</td>
</tr>
</tbody>
</table>

The default is the value that is specified in the LOGLEVEL= option in the PROC OPTNET statement (or BASIC if that option is not specified).

OUT=SAS-data-set
specifies the output data set to contain the transitive closure results.
TSP Statement

TSP < options > ;

The TSP statement invokes an algorithm that solves the traveling salesman problem.

The traveling salesman problem is described in the section “Traveling Salesman Problem” on page 86. The algorithm that is used to solve this problem is built around the same method as is used in PROC OPTMILP: a branch-and-cut algorithm. Many of the following options are the same as those described for the OPTMILP procedure in the SAS/OR User’s Guide: Mathematical Programming.

You can specify the following options:

**ABSOBJGAP=** number

specifies a stopping criterion. When the absolute difference between the best integer objective and the objective of the best remaining branch-and-bound node becomes less than the value of number, the solver stops. The value of number can be any nonnegative number; the default value is 1E–6.

**CONFLICTSEARCH=** number | string

specifies the level of conflict search that PROC OPTNET performs. The solver performs a conflict search to find clauses that result from infeasible subproblems that arise in the search tree. Table 2.23 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>–1</td>
<td>AUTOMATIC</td>
<td>Performs a conflict search based on a strategy that is determined by PROC OPTNET</td>
</tr>
<tr>
<td>0</td>
<td>NONE</td>
<td>Disables conflict search</td>
</tr>
<tr>
<td>1</td>
<td>MODERATE</td>
<td>Performs a moderate conflict search</td>
</tr>
<tr>
<td>2</td>
<td>AGGRESSIVE</td>
<td>Performs an aggressive conflict search</td>
</tr>
</tbody>
</table>

By default, CONFLICTSEARCH=AUTOMATIC.

**CUTOFF=** number

cuts off any branch-and-bound nodes in a minimization problem that has an objective value that is greater than number. The value of number can be any number; the default value is the positive number that has the largest absolute value that can be represented in your operating environment.

**CUTSTRATEGY=** number | string

specifies the level of mixed integer linear programming cutting planes to be generated by PROC OPTNET. TSP-specific cutting planes are always generated. Table 2.24 describes the valid values for this option.

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>–1</td>
<td>AUTOMATIC</td>
<td>Generates cutting planes based on a strategy determined by the mixed integer linear programming solver</td>
</tr>
<tr>
<td>0</td>
<td>NONE</td>
<td>Disables generation of mixed integer linear programming cutting planes (some TSP-specific cutting planes are still active for validity)</td>
</tr>
</tbody>
</table>
Table 2.24 (continued)

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MODERATE</td>
<td>Uses a moderate cut strategy</td>
</tr>
<tr>
<td>2</td>
<td>AGGRESSIVE</td>
<td>Uses an aggressive cut strategy</td>
</tr>
</tbody>
</table>

By default, CUTSTRATEGY=NONE.

EMPHASIS=number | string

specifies a search emphasis option. Table 2.25 describes the valid values for this option.

Table 2.25 Values for EMPHASIS= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>BALANCE</td>
<td>Performs a balanced search</td>
</tr>
<tr>
<td>1</td>
<td>OPTIMAL</td>
<td>Emphasizes optimality over feasibility</td>
</tr>
<tr>
<td>2</td>
<td>FEASIBLE</td>
<td>Emphasizes feasibility over optimality</td>
</tr>
</tbody>
</table>

By default, EMPHASIS=BALANCE.

HEURISTICS=number | string

controls the level of initial and primal heuristics that PROC OPTNET applies. This level determines how frequently PROC OPTNET applies primal heuristics during the branch-and-bound tree search. It also affects the maximum number of iterations that are allowed in iterative heuristics. Some computationally expensive heuristics might be disabled by the solver at less aggressive levels. Table 2.26 lists the valid values for this option.

Table 2.26 Values for HEURISTICS= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>AUTOMATIC</td>
<td>Applies the default level of heuristics</td>
</tr>
<tr>
<td>0</td>
<td>NONE</td>
<td>Disables all initial and primal heuristics</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Applies basic initial and primal heuristics at low frequency</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Applies most initial and primal heuristics at moderate frequency</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Applies all initial primal heuristics at high frequency</td>
</tr>
</tbody>
</table>

By default, HEURISTICS=AUTOMATIC.

LOGFREQ=number

specifies how often to print information in the branch-and-bound node log. The value of number can be any nonnegative integer up to the largest four-byte signed integer, which is $2^{31} - 1$. The default value is 100. If number is set to 0, then the node log is disabled. If number is positive, then an entry is made in the node log at the first node, at the last node, and at intervals that are controlled by the value of number. An entry is also made each time a better integer solution is found.

LOGLEVEL=number | string

controls the amount of information displayed in the SAS log by the solver, from a short description of presolve information and summary to details at each branch-and-bound node. Table 2.27 describes the valid values for this option.
Chapter 2: The OPTNET Procedure

Table 2.27 Values for LOGLEVEL= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NONE</td>
<td>Turns off all solver-related messages in the SAS log</td>
</tr>
<tr>
<td>1</td>
<td>BASIC</td>
<td>Displays a solver summary after stopping</td>
</tr>
<tr>
<td>2</td>
<td>MODERATE</td>
<td>Prints a solver summary and a node log by using the interval that is specified in the LOGFREQ= option</td>
</tr>
<tr>
<td>3</td>
<td>AGGRESSIVE</td>
<td>Prints a detailed solver summary and a node log by using the interval that is specified in the LOGFREQ= option</td>
</tr>
</tbody>
</table>

The default value is MODERATE.

**MAXNODES=** *number*

specifies the maximum number of branch-and-bound nodes to be processed. The value of *number* can be any nonnegative integer up to the largest four-byte signed integer, which is $2^{31} - 1$. The default value is $2^{31} - 1$.

**MAXSOLS=** *number*

specifies a stopping criterion. If *number* solutions have been found, then the procedure stops. The value of *number* can be any positive integer up to the largest four-byte signed integer, which is $2^{31} - 1$. The default value is $2^{31} - 1$.

**MAXTIME=** *number*

specifies the maximum amount of time to spend solving the traveling salesman problem. The type of time (either CPU time or real time) is determined by the value of the TIMETYPE= option. The value of *number* can be any positive number; the default value is the positive number that has the largest absolute value that can be represented in your operating environment.

**MILP=** *number* | *string*

specifies whether to use a mixed integer linear programming (MILP) solver for solving the traveling salesman problem. The MILP solver attempts to find the overall best TSP tour by using a branch-and-bound based algorithm. This algorithm can be expensive for large-scale problems. If MILP=OFF, then PROC OPTNET uses its initial heuristics to find a feasible, but not necessarily optimal, tour as quickly as possible. Table 2.28 describes the valid values for this option.

Table 2.28 Values for MILP= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ON</td>
<td>Uses a mixed integer linear programming solver</td>
</tr>
<tr>
<td>0</td>
<td>OFF</td>
<td>Does not use a mixed integer linear programming solver</td>
</tr>
</tbody>
</table>

By default, MILP=ON.

**NODESEL=** *number* | *string*

specifies the branch-and-bound node selection strategy option. Table 2.29 describes the valid values for this option.
Table 2.29  Values for NODESEL= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>–1</td>
<td>AUTOMATIC</td>
<td>Uses automatic node selection</td>
</tr>
<tr>
<td>0</td>
<td>BESTBOUND</td>
<td>Chooses the node that has the best relaxed objective (best-bound-first strategy)</td>
</tr>
<tr>
<td>1</td>
<td>BESTESTIMATE</td>
<td>Chooses the node that has the best estimate of the integer objective value (best-estimate-first strategy)</td>
</tr>
<tr>
<td>2</td>
<td>DEPTH</td>
<td>Chooses the most recently created node (depth-first strategy)</td>
</tr>
</tbody>
</table>

By default, NODESEL=AUTOMATIC. For more information about node selection, see Chapter 12, “The OPTMILP Procedure” (SAS/OR User’s Guide: Mathematical Programming).

OUT=SAS-data-set specifies the output data set to contain the solution to the traveling salesman problem.

PROBE=number | string specifies a probing option. Table 2.30 describes the valid values for this option.

Table 2.30  Values for PROBE= Option

<table>
<thead>
<tr>
<th>number</th>
<th>string</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>–1</td>
<td>AUTOMATIC</td>
<td>Uses an automatic probing strategy</td>
</tr>
<tr>
<td>0</td>
<td>NONE</td>
<td>Disables probing</td>
</tr>
<tr>
<td>1</td>
<td>MODERATE</td>
<td>Uses the probing moderately</td>
</tr>
<tr>
<td>2</td>
<td>AGGRESSIVE</td>
<td>Uses the probing aggressively</td>
</tr>
</tbody>
</table>

By default, PROBE=NONE.

RELOBJGAP=number specifies a stopping criterion that is based on the best integer objective (BestInteger) and the objective of the best remaining node (BestBound). The relative objective gap is equal to

\[
\frac{| \text{BestInteger} - \text{BestBound} |}{(10^{-10} + | \text{BestBound} |)}
\]

When this value becomes less than the specified gap size number, the solver stops. The value of number can be any nonnegative number. By default, RELOBJGAP=1E–4.

STRONGITER=number | AUTOMATIC specifies the number of simplex iterations that PROC OPTNET performs for each variable in the candidate list when it uses the strong branching variable selection strategy. The value of number can be any positive integer up to the largest four-byte signed integer, which is \(2^{31} - 1\). If you specify the keyword AUTOMATIC or the value –1, PROC OPTNET uses the default value, which it calculates automatically.

STRONGLNEN=number | AUTOMATIC specifies the number of candidates that PROC OPTNET considers when it uses the strong branching variable selection strategy. The value of number can be any positive integer up to the largest four-byte signed integer, which is \(2^{31} - 1\). If you specify the keyword AUTOMATIC or the value –1, PROC OPTNET uses the default value, which it calculates automatically.
**TARGET=** `number`

specifies a stopping criterion for minimization problems. If the best integer objective is better than or equal to `number`, the solver stops. The value of `number` can be any number; the default is the negative number that has the largest absolute value that can be represented in your operating environment.

**VARSEL=** `number` `|` `string`

specifies the rule for selecting the branching variable. Table 2.31 describes the valid values for this option.

<table>
<thead>
<tr>
<th><code>number</code></th>
<th><code>string</code></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>AUTOMATIC</td>
<td>Uses automatic branching variable selection</td>
</tr>
<tr>
<td>0</td>
<td>MAXINFEAS</td>
<td>Chooses the variable that has maximum infeasibility</td>
</tr>
<tr>
<td>1</td>
<td>MININFEAS</td>
<td>Chooses the variable that has minimum infeasibility</td>
</tr>
<tr>
<td>2</td>
<td>PSEUDO</td>
<td>Chooses a branching variable based on pseudocost</td>
</tr>
<tr>
<td>3</td>
<td>STRONG</td>
<td>Uses the strong branching variable selection strategy</td>
</tr>
</tbody>
</table>

By default, `VARSEL=AUTOMATIC`. For more information about variable selection, see Chapter 12, “The OPTMILP Procedure” (*SAS/OR User’s Guide: Mathematical Programming*).
Details: OPTNET Procedure

Graph Input Data

This section describes how to input a graph for analysis by PROC OPTNET. Let $G = (N, A)$ define a graph with a set $N$ of nodes and a set $A$ of links.

Consider the directed graph shown in Figure 2.5.

**Figure 2.5 A Simple Directed Graph**

Notice that each node and link has associated attributes: a node label and a link weight.

Link Input Data

The DATA_LINKS= option in the PROC OPTNET statement defines the data set that contains the list of links in the graph. A link is represented as a pair of nodes, which are defined by using either numeric or character labels. The links data set is expected to contain some combination of the following possible variables:

- from: the *from* node (this variable can be numeric or character)
- to: the *to* node (this variable can be numeric or character)
- weight: the link weight (this variable must be numeric)
- lower: the link flow lower bound (this variable must be numeric)
- upper: the link flow upper bound (this variable must be numeric)

As described in the GRAPH_DIRECTION= option, if the graph is undirected, the *from* and *to* labels are interchangeable. If the weights are not given for algorithms that call for link weights, they are all assumed to be 1.
The data set variable names can have any values that you want. If you use nonstandard names, you must identify the variables by using the DATA_LINKS_VAR statement, as described in the section “DATA_LINKS_VAR Statement” on page 22.

For example, the following two data sets identify the same graph:

```plaintext
data LinkSetInA;
  input from $ to $ weight;
  datalines;
  A B 1
  A C 2
  A D 4
;

data LinkSetInB;
  input source_node $ sink_node $ value;
  datalines;
  A B 1
  A C 2
  A D 4
;
```

These data sets can be presented to PROC OPTNET by using the following equivalent statements:

```plaintext
proc optnet
data_links = LinkSetInA;
run;

proc optnet
data_links = LinkSetInB;
  data_links_var
    from   = source_node
    to     = sink_node
    weight = value;
run;
```

The directed graph $G$ shown in Figure 2.5 can be represented by the following links data set LinkSetIn:

```plaintext
data LinkSetIn;
  input from $ to $ weight @@;
  datalines;
  A B 1 A C 2 A D 4 B C 1 B E 2
  B F 5 C E 1 D E 1 E D 1 E F 2
  F G 6 G H 1 G I 1 H G 2 H I 3
;```
The following statements read in this graph, declare it as a directed graph, and output the resulting links and nodes data sets. These statements do not run any algorithms, so the resulting output contains only the input graph.

```
proc optnet
    graph_direction = directed
data_links = LinkSetIn
out_nodes = NodeSetOut
out_links = LinkSetOut;
run;
```

The data set `NodeSetOut`, shown in Figure 2.6, now contains the nodes that are read from the input link data set. The variable `node` shows the label associated with each node.

**Figure 2.6** Node Data Set of a Simple Directed Graph

<table>
<thead>
<tr>
<th>node</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>G</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>I</td>
</tr>
</tbody>
</table>

The data set `LinkSetOut`, shown in Figure 2.7, contains the links that were read from the input link data set. The variables `from` and `to` show the associated node labels.

**Figure 2.7** Link Data Set of a Simple Directed Graph

<table>
<thead>
<tr>
<th>Obs</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>D</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>E</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>F</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>D</td>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>E</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>E</td>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>F</td>
<td>G</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>G</td>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>G</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>H</td>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>H</td>
<td>I</td>
<td>3</td>
</tr>
</tbody>
</table>
If you define this graph as undirected, then reciprocal links (for example, \(D \to E\) and \(D \leftarrow E\)) are treated as the same link, and duplicates are removed. PROC OPTNET takes the first occurrence of the link and ignores the others. By default, GRAPH_DIRECTION=UNDIRECTED, so you can just remove this option to declare the graph as undirected.

```plaintext
proc optnet
  data_links = LinkSetIn
  out_nodes = NodeSetOut
  out_links = LinkSetOut;
run;
```

The progress of the procedure is shown in Figure 2.8. The log now shows the links (and their observation identifiers) that were declared as duplicates and removed.

**Figure 2.8** PROC OPTNET Log: Link Data Set of a Simple Undirected Graph

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE: Data input used 0.01 (cpu: 0.02) seconds.</td>
</tr>
<tr>
<td>WARNING: Link (E,D) in observation 9 of the DATA_LINKS= data set is a duplicate and is ignored.</td>
</tr>
<tr>
<td>WARNING: Link (H,G) in observation 14 of the DATA_LINKS= data set is a duplicate and is ignored.</td>
</tr>
<tr>
<td>NOTE: The number of nodes in the input graph is 9.</td>
</tr>
<tr>
<td>NOTE: The number of links in the input graph is 13.</td>
</tr>
<tr>
<td>NOTE: Data output used 0.00 (cpu: 0.00) seconds.</td>
</tr>
<tr>
<td>NOTE: The data set WORK.NODESETOUT has 9 observations and 1 variables.</td>
</tr>
<tr>
<td>NOTE: The data set WORK.LINKSETOUT has 13 observations and 3 variables.</td>
</tr>
</tbody>
</table>

The data set NodeSetOut is equivalent to the one shown in Figure 2.6. However, the new links data set LinkSetOut shown in Figure 2.9 contains two fewer links than before, because duplicates are removed.

**Figure 2.9** Link Data Set of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>Obs</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>D</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>E</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>F</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>D</td>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>E</td>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>F</td>
<td>G</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>G</td>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>G</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>H</td>
<td>I</td>
<td>3</td>
</tr>
</tbody>
</table>
Certain algorithms can perform more efficiently when you specify `GRAPH_INTERNAL_FORMAT=THIN` in the PROC OPTNET statement. However, when you specify this option, PROC OPTNET does not remove duplicate links. Instead, you should use appropriate DATA steps to clean your data before calling PROC OPTNET.

**Node Input Data**

The `DATA_NODES=` option in the PROC OPTNET statement defines the data set that contains the list of nodes in the graph. This data set is used to assign node weights.

The nodes data set is expected to contain some combination of the following possible variables:

- `node`: the node label (this variable can be numeric or character)
- `weight`: the node weight (this variable must be numeric)
- `weight2`: the auxiliary node weight (this variable must be numeric)

You can specify any values that you want for the data set variable names. If you use nonstandard names, you must identify the variables by using the `DATA_NODES_VAR` statement, as described in the section “DATA_NODES_VAR Statement” on page 23.

The data set that is specified in the `DATA_LINKS=` option defines the set of nodes that are incident to some link. If the graph contains a node that has no links (called a singleton node), then this node must be defined in the `DATA_NODES` data set. The following is an example of a graph with three links but four nodes, including a singleton node D:

```plaintext
data NodeSetIn;
  input label $ @@;
  datalines;
  A B C D
;

data LinkSetInS;
  input from $ to $ weight;
  datalines;
  A B 1
  A C 2
  B C 1
;
```

If you specify duplicate entries in the node data set, PROC OPTNET takes the first occurrence of the node and ignores the others. A warning is printed to the log.
Node Subset Input Data

For some algorithms you might want to process only a subset of the nodes in the input graph. You can accomplish this by using the DATA NODES SUB= option in the PROC OPTNET statement. You can use the node subset data set in conjunction with the SHORTPATH statement (see the section “Shortest Path” on page 72. The node subset data set is expected to contain some combination of the following variables:

- **node**: the node label (this variable can be numeric or character)
- **source**: whether to process this node as a source node in shortest path algorithms (this variable must be numeric)
- **sink**: whether to process this node as a sink node in shortest path algorithms (this variable must be numeric)

The values in the node subset data set determine how to process nodes when the SHORTPATH statement is processed. A value of 0 for the source variable designates that the node is not to be processed as a source; a value of 1 designates that the node is to be processed as a source. The same values can be used for the sink variable to designate whether the node is to be processed as a sink. The missing indicator (.) can also be used in place of 0 to designate that a node is not to be processed.

A representative example of a node subset data set that might be used with the graph in Figure 2.5 is as follows:

```sql
data NodeSubSetIn;
  input node $ source sink;
  datalines;
  A 1 .
  F . 1
  E 1 .
;
```

The data set NodeSubSetIn indicates that you want to process the shortest paths from nodes A and E and the shortest paths to node F.

Matrix Input Data

This section describes the matrix input format that you can use with some of the algorithms in PROC OPTNET. The DATA MATRIX= option in the PROC OPTNET statement defines the data set that contains the matrix values. You can specify any values that you want for the data set variable names (the columns) by using the DATA MATRIX VAR statement, as described in the section “DATA MATRIX VAR Statement” on page 23. If you do not specify any names, then PROC OPTNET assumes that all numeric variables in the data set are to be used in defining the matrix.
The following statements solve the linear assignment problem for the cost matrix that is defined in the data set CostMatrix:

```plaintext
data CostMatrix;
   input back breast fly free;
datalines;
  35.1 36.7 28.3 36.1
  34.6 32.6 26.9 26.2
  31.3 33.9 27.1 31.2
  28.6 34.1 29.1 30.3
  32.9 32.2 26.6 24.0
  27.8 32.5 27.8 27.0
  26.3 27.6 23.5 22.4
  29.0 24.0 27.9 25.4
  27.2 33.8 25.2 24.1
  27.0 29.2 23.0 21.9
;
proc optnet
data_matrix = CostMatrix;
   data_matrix_var
      back--free;
   linear_assignment
      out = LinearAssign;
run;
```

**Biconnected Components and Articulation Points**

A *biconnected component* of a graph $G = (N, A)$ is a connected subgraph that cannot be broken into disconnected pieces by deleting any single node (and its incident links). An *articulation point* is a node of a graph whose removal would cause an increase in the number of connected components. Articulation points can be important when you analyze any graph that represents a communications network. Consider an articulation point $i \in N$ which, if removed, disconnects the graph into two components $C^1$ and $C^2$. All paths in $G$ between some nodes in $C^1$ and some nodes in $C^2$ must pass through node $i$. In this sense, articulation points are critical to communication. Examples of where articulation points are important are airline hubs, electric circuits, network wires, protein bonds, traffic routers, and numerous other industrial applications.

In PROC OPTNET, you can find biconnected components and articulation points of an input graph by invoking the BICONCOMP statement. This algorithm works only with undirected graphs.

The results for the biconnected components algorithm are written to the output links data set that is specified in the OUT_LINKS= option in the PROC OPTNET statement. For each link in the links data set, the variable biconcomp identifies its component. The component identifiers are numbered sequentially starting from 1. The results for the articulation points are written to the output nodes data set that is specified in the OUT_NODES= option in the PROC OPTNET statement. For each node in the nodes data set, the variable artpoint is either 1 (if the node is an articulation point) or 0 (otherwise).
The biconnected components algorithm reports status information in a macro variable called _OROPTNET_BICONCOMP_. See the section “Macro Variable _OROPTNET_BICONCOMP_” on page 95 for more information about this macro variable.

The algorithm that PROC OPTNET uses to compute biconnected components is a variant of depth-first search (Tarjan 1972). This algorithm runs in time $O(|V| + |A|)$ and therefore should scale to very large graphs.

### Biconnected Components of a Simple Undirected Graph

This section illustrates the use of the biconnected components algorithm on the simple undirected graph $G$ that is shown in Figure 2.10.

**Figure 2.10** A Simple Undirected Graph $G$

![Figure 2.10 A Simple Undirected Graph $G$](image)

The undirected graph $G$ can be represented by the following links data set LinkSetInBiCC:

```plaintext
data LinkSetInBiCC;
   input from $ to $ @@;
datalines;
   A B A F A G B C B D
   B E C D E F G I G H
   H I
;
```
The following statements calculate the biconnected components and articulation points and output the results in the data sets LinkSetOut and NodeSetOut:

```plaintext
proc optnet
  data_links = LinkSetInBiCC
  out_links  = LinkSetOut
  out_nodes  = NodeSetOut;
  biconcomp;
run;
```

The data set LinkSetOut now contains the biconnected components of the input graph, as shown in Figure 2.11.

**Figure 2.11** Biconnected Components of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>from</th>
<th>to</th>
<th>biconcomp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>G</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>E</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>I</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>H</td>
<td>3</td>
</tr>
<tr>
<td>H</td>
<td>I</td>
<td>3</td>
</tr>
</tbody>
</table>

In addition, the data set NodeSetOut contains the articulation points of the input graph, as shown in Figure 2.12.

**Figure 2.12** Articulation Points of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>node</th>
<th>artpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>0</td>
</tr>
</tbody>
</table>
The biconnected components are shown graphically in Figure 2.13 and Figure 2.14.

**Figure 2.13** Biconnected Components $C^1$ and $C^2$

$C^1 = \{B, C, D\}$  \hspace{2cm} $C^2 = \{A, B, E, F\}$

For a more detailed example, see “Example 2.1: Articulation Points in a Terrorist Network” on page 101.
Clique

A **clique** of a graph \( G = (N, A) \) is an induced subgraph that is a complete graph. Every node in a clique is connected to every other node in that clique. A **maximal clique** is a clique that is not a subset of the nodes of any larger clique. That is, it is a set \( C \) of nodes such that every pair of nodes in \( C \) is connected by a link and every node not in \( C \) is missing a link to at least one node in \( C \). The number of maximal cliques in a particular graph can be very large and can grow exponentially with every node added. Finding cliques in graphs has applications in numerous industries including bioinformatics, social networks, electrical engineering, and chemistry.

You can find the maximal cliques of an input graph by invoking the **CLIQUE statement**. The options for this statement are described in the section “**CLIQUE Statement**” on page 18. The clique algorithm works only with undirected graphs.

The results for the clique algorithm are written to the output data set that is specified in the **OUT=** option in the CLIQUE statement. Each node of each clique is listed in the output data set along with the variable **clique** to identify the clique to which it belongs. A node can appear multiple times in this data set if it belongs to multiple cliques.

The clique algorithm reports status information in a macro variable called **_OROPTNET_CLIQUE_**. See the section “**Macro Variable _OROPTNET_CLIQUE_**” on page 95 for more information about this macro variable.

The algorithm that PROC OPTNET uses to compute maximal cliques is a variant of the Bron-Kerbosch algorithm (Bron and Kerbosch 1973; Harley 2003). Enumerating all maximal cliques is NP-hard, so this algorithm typically does not scale to very large graphs.

**Maximal Cliques of a Simple Undirected Graph**

This section illustrates the use of the clique algorithm on the simple undirected graph \( G \) that is shown in **Figure 2.15**.

**Figure 2.15** A Simple Undirected Graph \( G \)
The undirected graph $G$ can be represented by the following links data set LinkSetIn:

```plaintext
data LinkSetIn;
  input from to @@;
  datalines;
  0 1 0 2 0 3 0 4 0 5
  0 6 1 2 1 3 1 4 2 3
  2 4 2 5 2 6 2 7 2 8
  3 4 5 6 7 8 8 9
;
```

The following statements calculate the maximal cliques, output the results in the data set Cliques, and use the SQL procedure as a convenient way to create a table CliqueSizes of clique sizes:

```plaintext
proc optnet
  data_links = LinkSetIn;
  clique
    out = Cliques;
run;

proc sql;
  create table CliqueSizes as
  select clique, count(*) as size
  from Cliques
  group by clique
  order by size desc;
quit;
```

The data set Cliques now contains the maximal cliques of the input graph; it is shown in Figure 2.16.

---

**Figure 2.16** Maximal Cliques of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>clique</th>
<th>node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>
In addition, the data set CliqueSizes contains the number of nodes in each clique; it is shown in Figure 2.17.

**Figure 2.17** Sizes of Maximal Cliques of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>clique</th>
<th>size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

The maximal cliques are shown graphically in Figure 2.18 and Figure 2.19.

**Figure 2.18** Maximal Cliques $C^1$ and $C^2$

$C^1 = \{0, 1, 2, 3, 4\}$

$C^2 = \{0, 2, 5, 6\}$

**Figure 2.19** Maximal Cliques $C^3$ and $C^4$

$C^3 = \{2, 7, 8\}$

$C^4 = \{8, 9\}$
Connected Components

A connected component of a graph is a set of nodes that are all reachable from each other. That is, if two nodes are in the same component, then there exists a path between them. For a directed graph, there are two types of components: a strongly connected component has a directed path between any two nodes, and a weakly connected component ignores direction and requires only that a path exists between any two nodes.

In PROC OPTNET, you can invoke connected components by using the CONCOMP statement. The options for this statement are described in the section “CONCOMP Statement” on page 19.

There are two main algorithms for finding connected components in an undirected graph: a depth-first search algorithm (ALGORITHM=DFS) and a union-find algorithm (ALGORITHM=UNION_FIND). Given a graph \( G = (N, A) \), both algorithms run in time \( O(|N| + |A|) \) and can usually scale to very large graphs. The default, depth-first search, works only with a full graph structure (GRAPH_INTERNAL_FORMAT=FULL) and for this reason can sometimes be slower than the union-find algorithm. For directed graphs, the only algorithm available is depth-first search.

The results for the connected components algorithm are written to the output node data set that is specified in the OUT_NODES= option in the PROC OPTNET statement. For each node in the node data set, the variable concomp identifies its component. The component identifiers are numbered sequentially starting from 1.

The connected components algorithm reports status information in a macro variable called _OROPTNET_CONCOMP_. See the section “Macro Variable _OROPTNET_CONCOMP_” on page 96 for more information about this macro variable.

Connected Components of a Simple Undirected Graph

This section illustrates the use of the connected components algorithm on the simple undirected graph \( G \) that is shown in Figure 2.20.

Figure 2.20 A Simple Undirected Graph \( G \)

The undirected graph \( G \) can be represented by the following links data set LinkSetIn:

```plaintext
data LinkSetIn;
  input from $ to $ @@;
datalines;
  A B  A C  B C  C H  D E  D F  D G  F E  G I  K L
;
```
The following statements calculate the connected components and output the results in the data set NodeSetOut:

```sas
proc optnet
  data_links = LinkSetIn
  out_nodes = NodeSetOut;
  concomp;
run;
```

The data set NodeSetOut contains the connected components of the input graph and is shown in Figure 2.21.

**Figure 2.21** Connected Components of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>node</th>
<th>concomp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
</tr>
<tr>
<td>K</td>
<td>3</td>
</tr>
<tr>
<td>L</td>
<td>3</td>
</tr>
</tbody>
</table>

Notice that the graph was defined by using only the links data set. As seen in Figure 2.20, this graph also contains a singleton node labeled J, which has no associated links. By definition, this node defines its own component. But because the input graph was defined by using only the links data set, it did not show up in the results data set. To define a graph by using nodes that have no associated links, you should also define the input nodes data set. In this case, define the nodes data set NodeSetIn as follows:

```sas
data NodeSetIn;
  input node $ @@;
datalines;
  A B C D E F G H I J K L
;
```

Now, when you calculate the connected components, you define the input graph by using both the nodes input data set and the links input data set:

```sas
proc optnet
  data_nodes = NodeSetIn
  data_links = LinkSetIn
  out_nodes = NodeSetOut;
  concomp;
run;
```

The resulting data set, NodeSetOut, includes the singleton node J as its own component, as shown in Figure 2.22.
Figure 2.22  Connected Components of a Simple Undirected Graph

<table>
<thead>
<tr>
<th>node</th>
<th>concomp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
</tr>
<tr>
<td>J</td>
<td>3</td>
</tr>
<tr>
<td>K</td>
<td>4</td>
</tr>
<tr>
<td>L</td>
<td>4</td>
</tr>
</tbody>
</table>

Connected Components of a Simple Directed Graph

This section illustrates the use of the connected components algorithm on the simple directed graph $G$ that is shown in Figure 2.23.

Figure 2.23  A Simple Directed Graph $G$
The directed graph $G$ can be represented by the following links data set LinkSetIn:

```plaintext
data LinkSetIn;
  input from $ to $ @@;
datalines;
  A B B C B E B F C G
  C D D C D H E A E F
  F G G F H G H D
;```

The following statements calculate the connected components and output the results in the data set NodeSetOut:

```plaintext
proc optnet
  graph_direction = directed
  data_links = LinkSetIn
  out_nodes = NodeSetOut;
  concomp;
run;
```

The data set NodeSetOut, shown in Figure 2.24, now contains the connected components of the input graph.

**Figure 2.24** Connected Components of a Simple Directed Graph

<table>
<thead>
<tr>
<th>node</th>
<th>concomp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
</tr>
</tbody>
</table>
The connected components are represented graphically in Figure 2.25.

**Figure 2.25** Strongly Connected Components of $G$

---

**Cycle**

A path in a graph is a sequence of nodes, each of which has a link to the next node in the sequence. A cycle is a path in which the start node and end node are the same.

In PROC OPTNET, you can find the cycles (or just count the cycles) of an input graph by invoking the CYCLE statement. The options for this statement are described in the section “CYCLE Statement” on page 20. To find the cycles and report them in an output data set, use the OUT= option. To simply count the cycles, do not use the OUT= option.

For undirected graphs, each link represents two directed links. For this reason, the following cycles are filtered out: trivial cycles ($A \rightarrow B \rightarrow A$) and duplicate cycles that are found by traversing a cycle in both directions ($A \rightarrow B \rightarrow C \rightarrow A$ and $A \rightarrow C \rightarrow B \rightarrow A$).

The results for the cycle detection algorithm are written to the output data set that is specified in the OUT= option in the CYCLE statement. Each node of each cycle is listed in the OUT= data set along with the variable cycle to identify the cycle to which it belongs. The variable order defines the order (sequence) of the node in the cycle.

The cycle detection algorithm reports status information in a macro variable called _OROPTNET_CYCLE_. See the section “Macro Variable _OROPTNET_CYCLE_” on page 96 for more information about this macro variable.

The algorithm that PROC OPTNET uses to compute all cycles is a variant of the algorithm found in Johnson 1975. This algorithm runs in time $O((|N| + |A|)(c + 1))$, where $c$ is the number of elementary cycles in the
Cycle Detection of a Simple Directed Graph

This section provides a simple example for using the cycle detection algorithm on the simple directed graph $G$ that is shown in Figure 2.26. Two other examples are “Example 2.2: Cycle Detection for Kidney Donor Exchange” on page 103, which shows the use of cycle detection for optimizing a kidney donor exchange, and “Example 2.6: Transitive Closure for Identification of Circular Dependencies in a Bug Tracking System” on page 115, which shows another application of cycle detection.

**Figure 2.26** A Simple Directed Graph $G$

The directed graph $G$ can be represented by the following links data set LinkSetIn:

```latex
\begin{verbatim}
data LinkSetIn;
  input from $ to $ @@;
datalines;
A B A E B C C A C D
D E D F E B E C F E
;
\end{verbatim}
```
The following statements check whether the graph has a cycle:

```
proc optnet
  graph_direction = directed
data_links = LinkSetIn;
cycle
  mode = first_cycle;
run;
%put &_OROPTNET_;  %put &_OROPTNET_CYCLE_;;
```

The result is written to the log of the procedure, as shown Figure 2.27.

**Figure 2.27** PROC OPTNET Log: Check the Existence of a Cycle in a Simple Directed Graph

```
NOTE: Data input used 0.01 (cpu: 0.02) seconds.
NOTE: The number of nodes in the input graph is 6.
NOTE: The number of links in the input graph is 10.
NOTE: Processing cycle detection.
NOTE: The graph does have a cycle.
NOTE: Processing cycle detection used 0.00 (cpu: 0.00) seconds.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.
NOTE: STATUS=OK CYCLE=OK
STATUS=OK NUM_CYCLES=1 CPU_TIME=0.00 REAL_TIME=0.00
```

The following statements count the number of cycles in the graph:

```
proc optnet
  graph_direction = directed
data_links = LinkSetIn;
cycle
  mode = all_cycles;
run;
%put &_OROPTNET_;  %put &_OROPTNET_CYCLE_;;
```
The result is written to the log of the procedure, as shown in Figure 2.28.

**Figure 2.28** PROC OPTNET Log: Count the Number of Cycles in a Simple Directed Graph

The following statements return the first cycle found in the graph:

```plaintext
proc optnet
  graph_direction = directed
  data_links = LinkSetIn;
  cycle
    out = Cycles
    mode = first_cycle;
run;
```

The data set *Cycles* now contains the first cycle found in the input graph; it is shown in Figure 2.29.

**Figure 2.29** First Cycle Found in a Simple Directed Graph

<table>
<thead>
<tr>
<th>cycle</th>
<th>order</th>
<th>node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>A</td>
</tr>
</tbody>
</table>
The first cycle found in the input graph is shown graphically in Figure 2.30.

**Figure 2.30**  \( A \rightarrow B \rightarrow C \rightarrow A \)

The following statements return all of the cycles in the graph:

```plaintext
proc optnet
  graph_direction = directed
  data_links = LinkSetIn;
  cycle
    out = Cycles
    mode = all_cycles;
run;
```
The data set Cycles now contains all of the cycles in the input graph; it is shown in Figure 2.31.

**Figure 2.31** All Cycles in a Simple Directed Graph

<table>
<thead>
<tr>
<th>cycle</th>
<th>order</th>
<th>node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>E</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>E</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>E</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>F</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>B</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>E</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>E</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>E</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>F</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>E</td>
</tr>
</tbody>
</table>
The six additional cycles are shown graphically in Figure 2.32 through Figure 2.34.

**Figure 2.32** Cycles

\[ A \rightarrow E \rightarrow B \rightarrow C \rightarrow A \]

\[ A \rightarrow E \rightarrow C \rightarrow A \]

**Figure 2.33** Cycles

\[ B \rightarrow C \rightarrow D \rightarrow E \rightarrow B \]

\[ B \rightarrow C \rightarrow D \rightarrow F \rightarrow E \rightarrow B \]
Linear Assignment (Matching)

The linear assignment problem (LAP) is a fundamental problem in combinatorial optimization that involves assigning workers to tasks at minimal costs. In graph theoretic terms, LAP is equivalent to finding a minimum-weight matching in a weighted bipartite directed graph. In a bipartite graph, the nodes can be divided into two disjoint sets $S$ (workers) and $T$ (tasks) such that every link connects a node in $S$ to a node in $T$. That is, the node sets $S$ and $T$ are independent. The concept of assigning workers to tasks can be generalized to the assignment of any abstract object from one group to some abstract object from a second group.

The linear assignment problem can be formulated as an integer programming optimization problem. The form of the problem depends on the sizes of the two input sets, $S$ and $T$. Let $A$ represent the set of possible assignments between sets $S$ and $T$. In the bipartite graph, these assignments are the links. If $|S| \geq |T|$, then the following optimization problem is solved:

**minimize** $\sum_{(i,j) \in A} c_{ij} x_{ij}$

**subject to** $\sum_{(i,j) \in A} x_{ij} \leq 1 \quad i \in S$

$\sum_{(i,j) \in A} x_{ij} = 1 \quad j \in T$

$x_{ij} \in \{0, 1\} \quad (i, j) \in A$
This model allows for some elements of set $S$ (workers) to go unassigned (if $|S| > |T|$). However, if $|S| < |T|$, then the following optimization problem is solved:

\[
\begin{align*}
\text{minimize} & \quad \sum_{(i,j) \in A} c_{ij} x_{ij} \\
\text{subject to} & \quad \sum_{(i,j) \in A} x_{ij} = 1 \quad i \in S \\
& \quad \sum_{(i,j) \in A} x_{ij} \leq 1 \quad j \in T \\
& \quad x_{ij} \in \{0, 1\} \quad (i, j) \in A
\end{align*}
\]

This model allows for some elements of set $T$ (tasks) to go unassigned.

In PROC OPTNET, you can invoke the linear assignment problem solver by using the LINEAR_ASSIGNMENT statement. The options for this statement are described in the section “LINEAR_ASSIGNMENT Statement” on page 24. The algorithm that the PROC OPTNET uses for solving a LAP is based on augmentation of shortest paths (Jonker and Volgenant 1987). This algorithm can be applied to either matrix data input (see the section “Matrix Input Data” on page 40) or graph data input (see the section “Graph Input Data” on page 35) as long as the graph is bipartite.

The resulting assignment (or matching) is contained in the output data set that is specified in the OUT= option in the LINEAR_ASSIGNMENT statement.

The linear assignment problem solver reports status information in a macro variable called _OROPTNET_LAP_. See the section “Macro Variable _OROPTNET_LAP_” on page 96 for more information about this macro variable.

For a detailed example, see “Example 2.3: Linear Assignment Problem for Minimizing Swim Times” on page 108.

---

**Minimum-Cost Network Flow**

The minimum-cost network flow problem (MCF) is a fundamental problem in network analysis that involves sending flow over a network at minimal cost. Let $G = (N, A)$ be a directed graph. For each link $(i, j) \in A$, associate a cost per unit of flow, designated by $c_{ij}$. The demand (or supply) at each node $i \in N$ is designated as $b_i$, where $b_i \geq 0$ denotes a supply node and $b_i < 0$ denotes a demand node. These values must be within $[b_i^l, b_i^u]$. Define decision variables $x_{ij}$ that denote the amount of flow sent between node $i$ and node $j$. The amount of flow that can be sent across each link is bounded to be within $[l_{ij}, u_{ij}]$. The problem can be modeled as a linear programming problem as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{(i,j) \in A} c_{ij} x_{ij} \\
\text{subject to} & \quad b_i^l \leq \sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} \leq b_i^u \quad i \in N \\
& \quad l_{ij} \leq x_{ij} \leq u_{ij} \quad (i, j) \in A
\end{align*}
\]

When $b_i = b_i^l = b_i^u$ for all nodes $i \in N$, the problem is called a pure network flow problem. For these problems, the sum of the supplies and demands must be equal to 0 to ensure that a feasible solution exists.
In PROC OPTNET, you can invoke the minimum-cost network flow solver by using the MINCOSTFLOW statement. The options for this statement are described in the section “MINCOSTFLOW Statement” on page 25.

The minimum-cost network flow solver reports status information in a macro variable called _OROPTNET_MCF_. See the section “Macro Variable _OROPTNET_MCF_” on page 97 for more information about this macro variable.

The algorithm that the PROC OPTNET uses for solving MCF is a variant of the primal network simplex algorithm (Ahuja, Magnanti, and Orlin 1993). Sometimes the directed graph $G$ is disconnected. In this case, the problem is first decomposed into its weakly connected components, and then each minimum-cost flow problem is solved separately.

The input for the network is the standard graph input, which is described in the section “Graph Input Data” on page 35. The links data set, which is specified in the DATA_LINKS= option in the PROC OPTNET statement, contains the following columns:

- weight defines the link cost $c_{ij}$.
- lower defines the link lower bound $l_{ij}$. The default is 0.
- upper defines the link upper bound $u_{ij}$. The default is $\infty$.

The nodes data set, which is specified in the DATA_NODES= option in the PROC OPTNET statement, can contain the following columns:

- weight defines the node supply lower bound $b_{li}^l$. The default is 0.
- weight2 defines the node supply upper bound $b_{li}^u$. The default is $\infty$.

To define a pure network where the node supply must be met exactly, use the weight variable only. You do not need to specify all the node supply bounds. For any missing node, the solver uses a lower and upper bound of 0.

The resulting optimal flow through the network is written to the links output data set, which is specified in the OUT_LINKS= option in the PROC OPTNET statement.
Minimum Cost Network Flow for a Simple Directed Graph

The following example demonstrates how to use the network simplex solver to find a minimum-cost flow in a directed graph. Consider the directed graph in Figure 2.35, which appears in Ahuja, Magnanti, and Orlin (1993).

Figure 2.35 Minimum-Cost Network Flow Problem: Data

The directed graph $G$ can be represented by the following links data set LinkSetIn and nodes data set NodeSetIn.

```plaintext
data LinkSetIn;
  input from to weight upper;
  datalines;
  1 4 2 15
  2 1 1 10
  2 3 0 10
  2 6 6 10
  3 4 1 5
  3 5 4 10
  4 7 5 10
  5 6 2 20
  5 7 7 15
  6 8 8 10
  7 8 9 15
;
```
data NodeSetIn;
  input node weight;
datalines;
1  10
2  20
4  -5
7  -15
8  -10
;

You can use the following call to PROC OPTNET to find a minimum-cost flow:

proc optnet
  loglevel = moderate
  graph_direction = directed
  data_links = LinkSetIn
  data_nodes = NodeSetIn
  out_links = LinkSetOut;
  mincostflow
    logfreq = 1;
run;
%put &_OROPTNET_;
%put &_OROPTNET_MCF_;
Chapter 2: The OPTNET Procedure

The progress of the procedure is shown in Figure 2.36.

Figure 2.36  PROC OPTNET Log for Minimum-Cost Network Flow

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Objective</th>
<th>Infeasibility</th>
<th>Infeasibility</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000E+00</td>
<td>2.000000E+01</td>
<td>8.900000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.000000E+00</td>
<td>2.000000E+01</td>
<td>8.900000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>5.000000E+00</td>
<td>1.500000E+01</td>
<td>8.400000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>5.000000E+00</td>
<td>1.500000E+01</td>
<td>8.300000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>7.500000E+00</td>
<td>1.500000E+01</td>
<td>8.300000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>7.500000E+00</td>
<td>1.500000E+01</td>
<td>7.900000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>1.300000E+02</td>
<td>1.000000E+01</td>
<td>7.600000E+01</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>2.700000E+02</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

NOTE: The Network Simplex solve time is 0.00 seconds.

The network has 1 connected component.

Status=OK  MCF=OPTIMAL
STATUS=OPTIMAL  OBJECTIVE=270  CPU_TIME=0.00  REAL_TIME=0.00
The optimal solution is displayed in Figure 2.37.

**Figure 2.37** Minimum-Cost Network Flow Problem: Optimal Solution

<table>
<thead>
<tr>
<th>Obs</th>
<th>from</th>
<th>to</th>
<th>upper</th>
<th>weight</th>
<th>mcf_flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>15</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>6</td>
<td>20</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>7</td>
<td>15</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td>8</td>
<td>15</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

The optimal solution is represented graphically in Figure 2.38.

**Figure 2.38** Minimum-Cost Network Flow Problem: Optimal Solution

---

**Minimum Cut**

A cut is a partition of the nodes of a graph into two disjoint subsets. The cut-set is the set of links whose from and to nodes are in different subsets of the partition. A minimum cut of an undirected graph is a cut whose cut-set has the smallest link metric, which is measured as follows: For an unweighted graph, the link metric is the number of links in the cut-set. For a weighted graph, the link metric is the sum of the link weights in the cut-set.

In PROC OPTNET, you can invoke the minimum cut algorithm by using the MINCUT statement. The options for this statement are described in the section “MINCUT Statement” on page 26. This algorithm can be used only on undirected graphs.

If the value of the MAXNUMCUTS= option is greater than 1, then the algorithm can return more than one set of cuts. The resulting cuts can be described in terms of partitions of the nodes of the graph or the links in the cut-sets. The node partition is specified by the mincut_i variable, for each cut i, in the data set that is specified in the OUT_NODES= option in the PROC OPTNET statement. Each node is assigned the value 0
or 1, which defines the side of the partition to which it belongs. The cut-set is defined in the output data set that is specified in the OUT= option in the MINCUT statement. This data set lists the links and their weights for each cut.

The minimum cut algorithm reports status information in a macro variable called _OROPTNET_MINCUT_. See the section “Macro Variable _OROPTNET_MINCUT_” on page 97 for more information about this macro variable.

PROC OPTNET uses the Stoer-Wagner algorithm (Stoer and Wagner 1997) to compute the minimum cuts. This algorithm runs in time $O(|N||A| + |N|^2 \log |N|)$.

**Minimum Cut for a Simple Undirected Graph**

As a simple example, consider the weighted undirected graph in Figure 2.39.

*Figure 2.39* A Simple Undirected Graph
The links data set can be represented as follows:

```latex
\begin{verbatim}
data LinkSetIn;
    input from to weight @0;
datalines;
    1  2  2  1  5  3  2  3  3  2  5  2  2  6  2
    3  4  4  3  7  2  4  7  2  4  8  2  5  6  3
    6  7  1  7  8  3
;
\end{verbatim}
```

The following statements calculate minimum cuts in the graph and output the results in the data set MinCut:

```latex
\begin{verbatim}
proc optnet
    loglevel   = moderate
    out_nodes  = NodeSetOut
    data_links = LinkSetIn;
    mincut
        out    = MinCut
        maxnumcuts = 3;
run;
%put &_OROPTNET_;
%put &_OROPTNET_MINCUT_;
\end{verbatim}
```
The progress of the procedure is shown in Figure 2.40.

**Figure 2.40** PROC OPTNET Log for Minimum Cut

```
NOTE: Reading the links data set.
NOTE: There were 12 observations read from the data set WORK.LINKSETIN.
NOTE: Building the input graph storage used 0.00 (cpu: 0.00) seconds.
NOTE: The input graph storage is using 0.0 MBs of memory.
NOTE: The number of nodes in the input graph is 8.
NOTE: The number of links in the input graph is 12.
NOTE: Processing the minimum cut problem.
NOTE: The minimum cut algorithm found 3 cuts.
NOTE: The cut 1 has weight 4.
NOTE: The cut 2 has weight 5.
NOTE: The cut 3 has weight 5.
NOTE: Processing the minimum cut problem used 0.00 (cpu: 0.02) seconds.
NOTE: Creating nodes data set output.
NOTE: Creating minimum cut data set output.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.
NOTE: The data set WORK.NODESETOUT has 8 observations and 4 variables.
NOTE: The data set WORK.MINCUT has 6 observations and 4 variables.
STATUS=OK  MINCUT=OPTIMAL
STATUS=OPTIMAL  OBJECTIVE=4  CPU_TIME=0.02  REAL_TIME=0.00
```

The data set NodeSetOut now contains the partition of the nodes for each cut, shown in Figure 2.41.

**Figure 2.41** Minimum Cut Node Partition

<table>
<thead>
<tr>
<th>node</th>
<th>mincut_1</th>
<th>mincut_2</th>
<th>mincut_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
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<td>5</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
<td>3</td>
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<td>1</td>
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<tr>
<td>6</td>
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<tr>
<td>4</td>
<td>0</td>
<td>1</td>
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</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The data set MinCut contains the links in the cut-sets for each cut. This data set is shown in Figure 2.42, which also shows each cut separately.

**Figure 2.42** Minimum Cut Sets

<table>
<thead>
<tr>
<th>mincut</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
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<td>7</td>
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<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

---

**mincut=1**

<table>
<thead>
<tr>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
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<tr>
<td>6</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

**mincut=2**

<table>
<thead>
<tr>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

**mincut=3**

<table>
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<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

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**Minimum Spanning Tree**

A *spanning tree* of a connected undirected graph is a subgraph that is a tree that connects all the nodes together. When weights have been assigned to the links, a *minimum spanning tree* (MST) is a spanning tree whose sum of link weights is less than or equal to the sum of link weights of every other spanning tree. More generally, any undirected graph (not necessarily connected) has a *minimum spanning forest*, which is a union
of minimum spanning trees of its connected components.

In PROC OPTNET, you can invoke the minimum spanning tree algorithm by using the MINSPANTREE statement. The options for this statement are described in the section “MINSPANTREE Statement” on page 27. This algorithm can be used only on undirected graphs.

The resulting minimum spanning tree is contained in the output data set that is specified in the OUT= option in the MINSPANTREE statement.

The minimum spanning tree algorithm reports status information in a macro variable called _OROPTNET_MST_. See the section “Macro Variable _OROPTNET_MST_” on page 98 for more information about this macro variable.

PROC OPTNET uses Kruskal’s algorithm (Kruskal 1956) to compute the minimum spanning tree. This algorithm runs in time $O(|A| \log |N|)$ and therefore should scale to very large graphs.

Minimum Spanning Tree for a Simple Undirected Graph

As a simple example, consider the weighted undirected graph in Figure 2.43.

![Figure 2.43 A Simple Undirected Graph](image)

The links data set can be represented as follows:

```plaintext
data LinkSetIn;
  input from $ to $ weight @@;
datalines;
  A B 7 A D 5 B C 8 B D 9 B E 7
  C E 5 D E 15 D F 6 E F 8 E G 9
  F G 11 H I 1 I J 3 H J 2
;
```

The following statements calculate a minimum spanning forest and output the results in the data set \textit{MinSpanForest}:

```sql
proc optnet
data_links = LinkSetIn;
  minspantree
    out = MinSpanForest;
run;
```

The data set \textit{MinSpanForest} now contains the links that belong to a minimum spanning forest, which is shown in \textit{Figure 2.44}.

\textbf{Figure 2.44} Minimum Spanning Forest

<table>
<thead>
<tr>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
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<td>I</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>J</td>
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<tr>
<td>C</td>
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<tr>
<td>A</td>
<td>D</td>
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<tr>
<td>D</td>
<td>F</td>
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<tr>
<td>B</td>
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<tr>
<td>A</td>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>9</td>
</tr>
</tbody>
</table>

\quad \text{========}
\quad 42

The minimal cost links are shown in green in \textit{Figure 2.45}.

\textbf{Figure 2.45} Minimum Spanning Forest

For a more detailed example, see “Example 2.5: Minimum Spanning Tree for Computer Network Topology” on page 114.
Chapter 2: The OPTNET Procedure

Shortest Path

A shortest path between two nodes $u$ and $v$ in a graph is a path that starts at $u$ and ends at $v$ and has the lowest total link weight. The starting node is called the source node, and the ending node is the sink node.

In PROC OPTNET, shortest paths can be calculated by invoking the SHORTPATH statement. The options for this statement are described in the section “SHORTPATH Statement” on page 27.

The shortest path algorithm reports status information in a macro variable called _OROPTNET_SHORTPATH_. See the section “Macro Variable _OROPTNET_SHORTPATH_” on page 98 for more information about this macro variable.

By default, PROC OPTNET finds shortest paths for all pairs. That is, it finds a shortest path for each possible combination of source and sink nodes. Alternatively, you can use the SOURCE= option to fix a particular source node and find shortest paths from the fixed source node to all possible sink nodes. Conversely, by using the SINK= option, you can fix a sink node and find shortest paths from all possible source nodes to the fixed sink node. Using both options together, you can request one particular shortest path for a specific source-sink pair. In addition, you can use the DATA_NODES_SUB= option to define a list of source-sink pairs to process, as described in the section “Node Subset Input Data” on page 40. The following sections show examples of these options.

The algorithm that the PROC OPTNET uses for finding shortest paths is a variant of Dijkstra’s algorithm (Ahuja, Magnanti, and Orlin 1993). For unweighted graphs, PROC OPTNET uses a variant of breadth-first search. Dijkstra’s algorithm on weighted graphs runs in time $O(|N| \log |N| + |A|)$ for each source node. Breadth-first search runs in time $O(|N| + |A|)$ for each source node.

For weighted graphs, the algorithm uses the weight variable that is defined in the links data set to evaluate a path’s total weight (cost). You can also use the WEIGHT2= option in the SHORTPATH statement to define an auxiliary weight. The auxiliary weight is not used in the algorithm to evaluate a path’s total weight. It is simply calculated for the sake of reporting the total auxiliary weight for each shortest path.
Output Data Sets

The shortest path algorithm produces up to two output data sets. The output data set that is specified in the OUT_PATHS= option contains the links of a shortest path for each source-sink pair combination. The output data set that is specified in the OUT_WEIGHTS= option contains the total weight for the shortest path for each source-sink pair combination.

**OUT_PATHS= Data Set**

This data set contains the links present in the shortest path for each of the source-sink pairs. For large graphs and a large requested number of source-sink pairs, this output data set can be extremely large. For extremely large sets, generating the output can sometimes take longer than computing the shortest paths. For example, using the US road network data for the state of New York, the data contain a directed graph that has 264,346 nodes. Finding the shortest path for all pairs from only one source node results in 140,969,120 observations, which is a data set of size 11 GB. Finding shortest paths for all pairs from all nodes would produce an enormous output data set.

The OUT_PATHS= data set contains the following columns:

- **source**: the source node label of this shortest path
- **sink**: the sink node label of this shortest path
- **order**: for this source-sink pair, the order of this link in a shortest path
- **from**: the *from* node label of this link in a shortest path
- **to**: the *to* node label of this link in a shortest path
- **weight**: the weight of this link in a shortest path
- **weight2**: the auxiliary weight of this link

**OUT_WEIGHTS= Data Set**

This data set contains the total weight (and total auxiliary weight) for the shortest path for each of the source-sink pair.

The data set contains the following columns:

- **source**: the source node label of this shortest path
- **sink**: the sink node label of this shortest path
- **path_weight**: the total weight of the shortest path for this source-sink pair
- **path_weight2**: the total auxiliary weight of the shortest path for this source-sink pair
Shortest Paths for All Pairs

This example illustrates the use of the shortest path algorithm for all source-sink pairs on the simple undirected graph $G$ shown in Figure 2.46.

Figure 2.46 A Simple Undirected Graph $G$
The undirected graph $G$ can be represented by the following links data set LinkSetIn:

```plaintext
data LinkSetIn;
  input from $ to $ weight @@;
datalines;
  A B 3   A C 2   A D 6   A E 4   B D 5
  B F 5   C E 1   D E 2   D F 1   E F 4
;
```

The following statements calculate shortest paths for all source-sink pairs:

```plaintext
proc optnet
  data_links   = LinkSetIn;
  shortpath
    out_weights = ShortPathW
    out_paths  = ShortPathP;
run;
```

The data set ShortPathP contains the shortest paths and is shown in Figure 2.47.
Figure 2.47  All-Pairs Shortest Paths

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>order</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>1</td>
<td>A</td>
<td>B</td>
<td>3</td>
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<tr>
<td>A</td>
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<td>C</td>
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<td>F</td>
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<td>1</td>
<td>F</td>
<td>D</td>
<td>1</td>
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<td>1</td>
<td>F</td>
<td>D</td>
<td>1</td>
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<td>F</td>
<td>E</td>
<td>2</td>
<td>D</td>
<td>E</td>
<td>2</td>
</tr>
</tbody>
</table>
The data set ShortPathW contains the path weight for the shortest paths of each source-sink pair and is shown in Figure 2.48.

**Figure 2.48** All-Pairs Shortest Paths Summary

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>path weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>D</td>
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<td>F</td>
<td>6</td>
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<tr>
<td>B</td>
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<tr>
<td>C</td>
<td>F</td>
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<tr>
<td>D</td>
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<tr>
<td>D</td>
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<td>D</td>
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<td>D</td>
<td>1</td>
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<tr>
<td>F</td>
<td>E</td>
<td>3</td>
</tr>
</tbody>
</table>

When you are interested only in the source-sink pair that has the longest shortest path, you can use the PATHS= option. This option affects only the output processing; it does not affect the computation. All of the designated source-sink shortest paths are calculated, but only the longest ones are written to the output data set.

The following statements display only the longest shortest paths:

```plaintext
proc optnet
  data_links = LinkSetIn;
  shortpath
    paths = longest
    out_paths = ShortPathLong;
run;
```
The data set `ShortPathLong` now contains the longest shortest paths and is shown in Figure 2.49.

**Figure 2.49** Longest Shortest Paths

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>order</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>F</td>
<td>1</td>
<td>A</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>F</td>
<td>2</td>
<td>C</td>
<td>E</td>
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<tr>
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<td>F</td>
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<tr>
<td>B</td>
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<tr>
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<td>C</td>
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<td>2</td>
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<tr>
<td>E</td>
<td>B</td>
<td>3</td>
<td>A</td>
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<tr>
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<tr>
<td>F</td>
<td>A</td>
<td>4</td>
<td>C</td>
<td>A</td>
<td>2</td>
</tr>
</tbody>
</table>

**Shortest Paths for a Subset of Source-Sink Pairs**

This section illustrates the use of a node subset data set, the `DATAstrcasecmp_SUB=` option, and the shortest path algorithm for calculating shortest paths between a subset of source-sink pairs. The data set variables `source` and `sink` are used as indicators to specify which pairs to process. The marked source nodes define a set `S`, and the marked sink nodes define a set `T`. PROC OPTNET then calculates all the source-sink pairs in the cross product of these two sets.

For example, the following DATA step tells PROC OPTNET to calculate the pairs in `S \times T = \{A, C\} \times \{B, F\}`:

```plaintext
data NodeSetInSub;
  input node $ source sink;
datalines;
  A 1 0
  C 1 0
  B 0 1
  F 0 1
;
```

The following statements calculate a shortest path for the four combinations of source-sink pairs:

```plaintext
proc optnet
data_nodes_sub = NodeSetInSub
data_links = LinkSetIn;
shortpath
  out_paths = ShortPath;
run;
```
The data set ShortPath contains the shortest paths and is shown in Figure 2.50.

**Figure 2.50** Shortest Paths for a Subset of Source-Sink Pairs

<table>
<thead>
<tr>
<th>ShortPath</th>
</tr>
</thead>
<tbody>
<tr>
<td>source</td>
</tr>
<tr>
<td>A</td>
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<tr>
<td>A</td>
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<tr>
<td>A</td>
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<tr>
<td>A</td>
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<tr>
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<tr>
<td>C</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

**Shortest Paths for a Subset of Source or Sink Pairs**

This section illustrates the use of the shortest path algorithm for calculating shortest paths between a subset of source (or sink) nodes and all other sink (or source) nodes.

In this case, you designate the subset of source (or sink) nodes in the node subset data set by specifying the source (or sink). By specifying only one of the variables, you indicate that you want PROC OPTNET to calculate all pairs from a subset of source nodes (or to calculate all pairs to a subset of sink nodes).

For example, the following DATA step designates nodes B and E as source nodes:

```plaintext
data NodeSetInSub;
  input node $ source;
datalines;
B 1
E 1
;
```
You can use the same PROC OPTNET call as is used in the section “Shortest Paths for a Subset of Source-Sink Pairs” on page 78 to calculate all the shortest paths from nodes B and E. The data set ShortPath contains the shortest paths and is shown in Figure 2.51.

**Figure 2.51** Shortest Paths for a Subset of Source Pairs

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>order</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>A</td>
<td>1</td>
<td>B</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>1</td>
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<td>A</td>
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<td>B</td>
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<td>B</td>
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<td>C</td>
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<td>B</td>
<td>F</td>
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<td>1</td>
<td>E</td>
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<tr>
<td>E</td>
<td>F</td>
<td>2</td>
<td>D</td>
<td>F</td>
<td>1</td>
</tr>
</tbody>
</table>

Conversely, the following DATA step designates nodes B and E as sink nodes:

```plaintext
data NodeSetInSub;
  input node $ sink;
  datalines;
  B 1
  E 1
;
```
You can use the same PROC OPTNET call again to calculate all the shortest paths to nodes \( B \) and \( E \). The data set ShortPath contains the shortest paths and is shown in Figure 2.52.

**Figure 2.52** Shortest Paths for a Subset of Sink Pairs

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>order</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>1</td>
<td>A</td>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>A</td>
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<td>B</td>
<td>E</td>
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<td>B</td>
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<tr>
<td>C</td>
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<td>1</td>
<td>C</td>
<td>E</td>
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<tr>
<td>D</td>
<td>B</td>
<td>1</td>
<td>D</td>
<td>B</td>
<td>5</td>
</tr>
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<tr>
<td>E</td>
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<td>C</td>
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<td>2</td>
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<tr>
<td>F</td>
<td>E</td>
<td>2</td>
<td>D</td>
<td>E</td>
<td>2</td>
</tr>
</tbody>
</table>

**Shortest Paths for One Source-Sink Pair**

This section illustrates the use of the shortest path algorithm for calculating shortest paths between one source-sink pair by using the SOURCE= and SINK= options.

The following statements calculate a shortest path between node \( C \) and node \( F \):

```plaintext
proc optnet
    data_links  = LinkSetIn;
    shortpath
        source  = C
        sink    = F
        out_paths = ShortPath;
run;
```

The data set ShortPath contains this shortest path and is shown in Figure 2.53.

**Figure 2.53** Shortest Paths for One Source-Sink Pair

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>order</th>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>F</td>
<td>1</td>
<td>C</td>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>F</td>
<td>2</td>
<td>E</td>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>F</td>
<td>3</td>
<td>D</td>
<td>F</td>
<td>1</td>
</tr>
</tbody>
</table>
The shortest path is shown graphically in Figure 2.54.

**Figure 2.54** Shortest Path between Nodes \( C \) and \( F \)

---

**Shortest Paths with Auxiliary Weight Calculation**

This section illustrates the use of the shortest path algorithm with auxiliary weights for calculating the shortest paths between all source-sink pairs.

Consider a links data set in which the auxiliary weight is a counter for each link:

```plaintext
data LinkSetIn;
  input from $ to $ weight count @@;
datalines;
A B 3 1 A C 2 1 A D 6 1 A E 4 1 B D 5 1
B F 5 1 C E 1 1 D E 2 1 D F 1 1 E F 4 1
;
```
The following statements calculate shortest paths for all source-sink pairs:

```plaintext
proc optnet
    data_links = LinkSetIn;
    shortpath
        weight2 = count
        out_weights = ShortPathW;
run;
```

The data set `ShortPathW` contains the total path weight for shortest paths in each source-sink pair and is shown in Figure 2.55. Because the variable `count` in `LinkSetIn` is 1 for all links, the value in the output data set variable `path_weights2` contains the number of links in each shortest path.

**Figure 2.55** Shortest Paths Including Auxiliary Weights in Calculation

<table>
<thead>
<tr>
<th>source</th>
<th>sink</th>
<th>path_weight</th>
<th>path_weight2</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>A</td>
<td>C</td>
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<tr>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>E</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The section “Road Network Shortest Path” on page 7 shows an example of using the shortest path algorithm for minimizing travel to and from work based on traffic conditions.
Transitive Closure

The transitive closure of a graph $G$ is a graph $G^T = (N, A^T)$ such that for all $i, j \in N$ there is a link $(i, j) \in A^T$ if and only if there exists a path from $i$ to $j$ in $G$.

The transitive closure of a graph can help to efficiently answer questions about reachability. Suppose you want to answer the question of whether you can get from node $i$ to node $j$ in the original graph $G$. Given the transitive closure $G^T$ of $G$, you can simply check for the existence of link $(i, j)$ to answer the question. Transitive closure has many applications, including speeding up the processing of structured query languages, which are often used in databases.

In PROC OPTNET, you can invoke the transitive closure algorithm by using the TRANSITIVE_CLOSURE statement. The options for this statement are described in the section “TRANSITIVE_CLOSURE Statement” on page 29.

The results for the transitive closure algorithm are written to the output data set that is specified in the OUT= option in the TRANSITIVE_CLOSURE statement. The links that define the transitive closure are listed in the output data set with variable names from and to.

The transitive closure algorithm reports status information in a macro variable called _OROPTNET_TRANSCL_. See the section “Macro Variable _OROPTNET_TRANSCL_” on page 99 for more information about this macro variable.

The algorithm that the PROC OPTNET uses to compute transitive closure is a sparse version of the Floyd-Warshall algorithm (Cormen, Leiserson, and Rivest 1990). This algorithm runs in time $O(|N|^3)$ and therefore might not scale to very large graphs.

Transitive Closure of a Simple Directed Graph

This example illustrates the use of the transitive closure algorithm on the simple directed graph $G$, which is shown in Figure 2.56.

![Figure 2.56 A Simple Directed Graph G](image)
The directed graph $G$ can be represented by the following links data set LinkSetIn:

```r
data LinkSetIn;
    input from $ to $ @@;
datalines;
    B C B D C B D A D C
;
```

The following statements calculate the transitive closure and output the results in the data set TransClosure:

```r
proc optnet
    graph_direction = directed
    data_links = LinkSetIn;
    transitive_closure
        out = TransClosure;
run;
```

The data set TransClosure contains the transitive closure of $G$ and is shown in Figure 2.57.

**Figure 2.57** Transitive Closure of a Simple Directed Graph

<table>
<thead>
<tr>
<th>Transitive Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>from</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>
The transitive closure of $G$ is shown graphically in Figure 2.58.

**Figure 2.58** Transitive Closure of $G$

For a more detailed example, see “Example 2.6: Transitive Closure for Identification of Circular Dependencies in a Bug Tracking System” on page 115.

**Traveling Salesman Problem**

The *traveling salesman problem* (TSP) finds a minimum-cost tour in an undirected graph $G$ that has a node set, $N$, and link set, $A$. A *tour* is a connected subgraph for which each node has degree two. The goal is then to find a tour of minimum total cost, where the total cost is the sum of the costs of the links in the tour. Associated with each link $(i, j) \in A$ are a binary variable $x_{ij}$, which indicates whether link $x_{ij}$ is part of the tour, and a cost $c_{ij}$. Let $\delta(S) = \{(i, j) \in A \mid i \in S, \ j \notin S\}$. Then an integer linear programming formulation of the TSP is as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{(i,j) \in A} c_{ij} x_{ij} \\
\text{subject to} & \quad \sum_{(i,j) \in \delta(i)} x_{ij} = 2 \quad i \in N \quad \text{(two_match)} \\
& \quad \sum_{(i,j) \in \delta(S)} x_{ij} \geq 2 \quad S \subseteq N, \ 2 \leq |S| \leq |N| - 1 \quad \text{(subtour_elim)} \\
& \quad x_{ij} \in \{0, 1\} \quad (i,j) \in A
\end{align*}
\]
The equations (two_match) are the matching constraints, which ensure that each node has degree two in the subgraph. The inequalities (subtour_elim) are the subtour elimination constraints (SECs), which enforce connectivity.

In practical terms, you can think of the TSP in the context of a routing problem in which each node is a city and the links are roads that connect cities. Given the pairwise distances between each city, the goal is to find the shortest possible route that visits each city exactly once. The TSP has applications in planning, logistics, manufacturing, genomics, and many other areas.

In PROC OPTNET, you can invoke the traveling salesman problem solver by using the TSP statement. The options for this statement are described in the section “TSP Statement” on page 30.

The traveling salesman problem solver reports status information in a macro variable called _OROPTNET_TSP_. See the section “Macro Variable _OROPTNET_TSP_” on page 99 for more information about this macro variable.

The algorithm that PROC OPTNET uses for solving a TSP is based on a variant of the branch-and-cut process described in Applegate et al. (2006).

The resulting tour is represented in two ways: In the data set that is specified in the OUT_NODES= option in the PROC OPTNET statement, the tour is specified as a sequence of nodes. In the data set that is specified in the OUT= option of the TSP statement, the tour is specified as a list of links in the optimal tour.

**Traveling Salesman Problem of a Simple Undirected Graph**

As a simple example, consider the weighted undirected graph in Figure 2.59.

**Figure 2.59  A Simple Undirected Graph**

![A Simple Undirected Graph](image)
Chapter 2: The OPTNET Procedure

The links data set can be represented as follows:

```sql
data LinkSetIn;
  input from $ to $ weight @0;
  datalines;
  A B 1.0 A C 1.0 A D 1.5 B C 2.0 B D 4.0
  B E 3.0 C D 3.0 C F 3.0 C H 4.0 D E 1.5
  D F 3.0 D G 4.0 E F 1.0 E G 1.0 F G 2.0
  F H 4.0 H I 3.0 I J 1.0 C J 5.0 F J 3.0
  F I 1.0 H J 1.0
;
```

The following statements calculate an optimal traveling salesman tour and output the results in the data sets `TSPTour` and `NodeSetOut`:

```sql
proc optnet
  loglevel  = moderate
  data_links = LinkSetIn
  out_nodes  = NodeSetOut;
  tsp
    out  = TSPTour;
run;
%put &_OROPTNET_;
%put &_OROPTNET_TSP_;
```

%put & _OROPTNET _ TSP ;
The progress of the procedure is shown in Figure 2.60.

Figure 2.60  PROC OPTNET Log: Optimal Traveling Salesman Tour of a Simple Undirected Graph

```
NOTE: --------------------------------------------------------------------------
NOTE: --------------------------------------------------------------------------
NOTE: Reading the links data set.
NOTE: There were 22 observations read from the data set WORK.LINKSETIN.
NOTE: Data input used 0.01 (cpu: 0.00) seconds.
NOTE: Building the input graph storage used 0.00 (cpu: 0.00) seconds.
NOTE: The input graph storage is using 0.0 MBs of memory.
NOTE: The number of nodes in the input graph is 10.
NOTE: The number of links in the input graph is 22.
NOTE: --------------------------------------------------------------------------
NOTE: Processing the traveling salesman problem.
NOTE: The initial TSP heuristics found a tour with cost 16 using 0.00 (cpu: 0.00) seconds.
NOTE: The MILP presolver value NONE is applied.
NOTE: The MILP solver is called.

<table>
<thead>
<tr>
<th>Node</th>
<th>Active</th>
<th>Sols</th>
<th>BestInteger</th>
<th>BestBound</th>
<th>Gap</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>16.0000000</td>
<td>16.0000000</td>
<td>0.00%</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>16.0000000</td>
<td>16.0000000</td>
<td>0.00%</td>
<td>0</td>
</tr>
</tbody>
</table>

NOTE: Optimal.
NOTE: Objective = 16.
NOTE: Processing the traveling salesman problem used 0.01 (cpu: 0.00) seconds.
NOTE: --------------------------------------------------------------------------
NOTE: Creating nodes data set output.
NOTE: Creating traveling salesman data set output.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.
NOTE: --------------------------------------------------------------------------
NOTE: The data set WORK.NODESETOUT has 10 observations and 2 variables.
NOTE: The data set WORK.TSPTOUR has 10 observations and 3 variables.
STATUS=OK  TSP=OPTIMAL
STATUS=OPTIMAL  OBJECTIVE=16  RELATIVE_GAP=0  ABSOLUTE_GAP=0
PRIMAL_INFEASIBILITY=0  BOUND_INFEASIBILITY=0  INTEGER_INFEASIBILITY=0
BEST_BOUND=16  NODES=1  ITERATIONS=14  CPU_TIME=0.00  REAL_TIME=0.01
```
The data set **NodeSetOut** now contains a sequence of nodes in the optimal tour and is shown in **Figure 2.61**.

**Figure 2.61** Nodes in the Optimal Traveling Salesman Tour

```
<table>
<thead>
<tr>
<th>Traveling Salesman Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>tsp_</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>J</td>
</tr>
<tr>
<td>I</td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>G</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>
```

The data set **TSPTour** now contains the links in the optimal tour and is shown in **Figure 2.62**.

**Figure 2.62** Links in the Optimal Traveling Salesman Tour

```
<table>
<thead>
<tr>
<th>Traveling Salesman Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>from</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>I</td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
```
The minimum-cost links are shown in green in Figure 2.63.

**Figure 2.63** Optimal Traveling Salesman Tour

---

**Macro Variables**

**Macro Variable _OROPTNET_**

The OPTNET procedure defines a macro variable named _OROPTNET_. This variable contains a character string that indicates the status of the OPTNET procedure upon termination. The various terms of the variable are interpreted as follows:

**STATUS**

indicates the status of the procedure at termination. The STATUS term can take one of the following values:

- **OK** The procedure terminated normally.
- **OUT_OF_MEMORY** Insufficient memory was allocated to the procedure.
- **ERROR** The procedure encountered an error.

**BICONCOMP**

indicates the status of the biconnected components algorithm at termination. This algorithm is described in the section “Biconnected Components and Articulation Points” on page 41. The BICONCOMP term can take one of the following values:

- **OK** The algorithm terminated normally.
- **ERROR** The algorithm encountered an error.
**CLIQUE**
indicates the status of the clique-finding algorithms at termination. These algorithms are described in the section “Clique” on page 45. The CLIQUE term can take one of the following values:

- **OK** The algorithm terminated normally.
- **TIMELIMIT** The algorithm reached its execution time limit, which is specified in the MAXTIME= option in the CLIQUE statement.
- **SOLUTION_LIM** The algorithm reached its limit on the number of cliques found, which is specified in the MAXCLIQUES= option in the CLIQUE statement.
- **ERROR** The algorithm encountered an error.

**CONCOMP**
indicates the status of the connected components algorithm at termination. This algorithm is described in the section “Connected Components” on page 48. The CONCOMP term can take one of the following values:

- **OK** The algorithm terminated normally.
- **ERROR** The algorithm encountered an error.

**CYCLE**
indicates the status of the cycle detection algorithm at termination. This algorithm is described in the section “Cycle” on page 52. The CYCLE term can take one of the following values:

- **OK** The algorithm terminated normally.
- **TIMELIMIT** The algorithm reached its execution time limit, which is specified in the MAXTIME= option in the CYCLE statement.
- **SOLUTION_LIM** The algorithm reached its limit on the number of cycles found, which is specified in the MAXCYCLES= option in the CYCLE statement.
- **ERROR** The algorithm encountered an error.

**LAP**
indicates the status of the linear assignment solver at termination. This solver is described in the section “Linear Assignment (Matching)” on page 59. The LAP term can take one of the following values:

- **OPTIMAL** The solution is optimal.
- **INFEASIBLE** The problem is infeasible.
- **ERROR** The solver encountered an error.
MCF
indicates the status of the minimum-cost network flow solver at termination. This solver is described in
the section “Minimum-Cost Network Flow” on page 60. The MCF term can take one of the following
values:

OPTIMAL The solution is optimal.
OPTIMAL_COND The solution is optimal, but some infeasibilities (primal or bound) exceed
tolerances because of scaling.
INFEASIBLE The problem is infeasible.
UNBOUNDED The problem is unbounded.
TIMELIMIT The solver reached its execution time limit, which is specified in the MAX-
TIME= option in the MINCOSTFLOW statement.
ERROR The solver encountered an error.

MINCUT
indicates the status of the minimum cut solver at termination. This solver is described in the section
“Minimum Cut” on page 65. The MINCUT term can take one of the following values:

OPTIMAL The solution is optimal.
INFEASIBLE The problem is infeasible.
INTERRUPTED The solver was interrupted by the user.
ERROR The solver encountered an error.

MST
indicates the status of the minimum spanning tree solver at termination. This solver is described in the
section “Minimum Spanning Tree” on page 69. The MST term can take one of the following values:

OPTIMAL The solution is optimal.
ERROR The solver encountered an error.

SHORTPATH
indicates the status of the shortest path algorithms at termination. These algorithms are described in
the section “Shortest Path” on page 72. The SHORTPATH term can take one of the following values:

OK The algorithm terminated normally.
INTERRUPTED The algorithm was interrupted by the user.
ERROR The algorithm encountered an error.
TRANSCL
indicates the status of the transitive closure algorithm at termination. This algorithm is described in the section “Transitive Closure” on page 84. The TRANSCL term can take one of the following values:

OK The algorithm terminated normally.
ERROR The algorithm encountered an error.

TSP
indicates the status of the traveling salesman problem solver at termination. This algorithm is described in the section “Traveling Salesman Problem” on page 86. The TSP term can take one of the following values:

OPTIMAL The solution is optimal.
OPTIMAL_AGAP The solution is optimal within the absolute gap that is specified in the ABSOBJGAP= option.
OPTIMAL_RGAP The solution is optimal within the relative gap that is specified in the RELOBJGAP= option.
OPTIMAL_COND The solution is optimal, but some infeasibilities (primal, bound, or integer) exceed tolerances because of scaling.
TARGET The solution is not worse than the target that is specified in the TARGET= option.
INFEASIBLE The problem is infeasible.
UNBOUNDED The problem is unbounded.
INFEASIBLE_OR_UNBOUNDED The problem is infeasible or unbounded.
SOLUTION_LIM The solver reached the maximum number of solutions specified in the MAXSOLS= option.
NODE_LIM_SOL The solver reached the maximum number of nodes specified in the MAXNODES= option and found a solution.
NODE_LIM_NOSOL The solver reached the maximum number of nodes specified in the MAXNODES= option and did not find a solution.
TIME_LIM_SOL The solver reached the execution time limit specified in the MAXTIME= option and found a solution.
TIME_LIM_NOSOL The solver reached the execution time limit specified in the MAXTIME= option and did not find a solution.
HEURISTIC_SOL The solver used only heuristics and found a solution.
HEURISTIC_NOSOL The solver used only heuristics and did not find a solution.
ABORT_SOL The solver was stopped by the user but still found a solution.
ABORT_NOSOL The solver was stopped by the user and did not find a solution.
OUTMEM_SOL The solver ran out of memory but still found a solution.
OUTMEM_NOSOL The solver ran out of memory and either did not find a solution or failed to output the solution due to insufficient memory.
FAIL_SOL  The solver stopped due to errors but still found a solution.
FAIL_NOSOL  The solver stopped due to errors and did not find a solution.

Each algorithm reports its own status information in an additional macro variable. The following sections provide more information about these macro variables.

Macro Variable _OROPTNET_BICONCOMP_

The OPTNET procedure defines a macro variable named _OROPTNET_BICONCOMP_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to calculate biconnected components. The various terms of the variable are interpreted as follows:

STATUS  indicates the status of the algorithm at termination. The STATUS term takes the same value as the term BICONCOMP in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

NUM_COMPONENTS  indicates the number of biconnected components found by the algorithm.

NUM_ARTICULATION_POINTS  indicates the number of articulation points found by the algorithm.

CPU_TIME  indicates the CPU time (in seconds) taken by the algorithm.

REAL_TIME  indicates the real time (in seconds) taken by the algorithm.

Macro Variable _OROPTNET_CLIQUE_

The OPTNET procedure defines a macro variable named _OROPTNET_CLIQUE_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to calculate cliques. The various terms of the variable are interpreted as follows:

STATUS  indicates the status of the algorithm at termination. The STATUS term takes the same value as the term CLIQUE in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

NUM_CLIQUES  indicates the number of cliques found by the algorithm.

CPU_TIME  indicates the CPU time (in seconds) taken by the algorithm.

REAL_TIME  indicates the real time (in seconds) taken by the algorithm.
Macro Variable _OROPTNET_CONCOMP_

The OPTNET procedure defines a macro variable named _OROPTNET_CONCOMP_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to calculate connected components. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the algorithm at termination. The STATUS term takes the same value as the term CONCOMP in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

**NUM_COMPONENTS**
indicates the number of connected components found by the algorithm.

**CPU_TIME**
indicates the CPU time (in seconds) taken by the algorithm.

**REAL_TIME**
indicates the real time (in seconds) taken by the algorithm.

Macro Variable _OROPTNET_CYCLE_

The OPTNET procedure defines a macro variable named _OROPTNET_CYCLE_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to calculate cycles. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the algorithm at termination. The STATUS term takes the same value as the term CYCLE in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

**NUM_CYCLES**
indicates the number of cycles found by the algorithm.

**CPU_TIME**
indicates the CPU time (in seconds) taken by the algorithm.

**REAL_TIME**
indicates the real time (in seconds) taken by the algorithm.

Macro Variable _OROPTNET_LAP_

The OPTNET procedure defines a macro variable named _OROPTNET_LAP_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to solve the linear assignment problem. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the solver at termination. The STATUS term takes the same value as the term LAP in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.
OBJECTIVE
indicates the total weight of the minimum linear assignment.

CPU_TIME
indicates the CPU time (in seconds) taken by the solver.

REAL_TIME
indicates the real time (in seconds) taken by the solver.

Macro Variable _OROPTNET_MCF_

The OPTNET procedure defines a macro variable named _OROPTNET_MCF_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to solve the minimum-cost network flow problem. The various terms of the variable are interpreted as follows:

STATUS
indicates the status of the solver at termination. The STATUS term takes the same value as the term MCF in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

OBJECTIVE
indicates the total link weight of the minimum-cost network flow.

CPU_TIME
indicates the CPU time (in seconds) taken by the solver.

REAL_TIME
indicates the real time (in seconds) taken by the solver.

Macro Variable _OROPTNET_MINCUT_

The OPTNET procedure defines a macro variable named _OROPTNET_MINCUT_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to find the minimum cut. The various terms of the variable are interpreted as follows:

STATUS
indicates the status of the algorithm at termination. The STATUS term takes the same value as the term MINCUT in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

OBJECTIVE
indicates the total link weight of the minimum cut.

CPU_TIME
indicates the CPU time (in seconds) taken by the algorithm.

REAL_TIME
indicates the real time (in seconds) taken by the algorithm.
Macro Variable _OROPTNET_MST_

The OPTNET procedure defines a macro variable named _OROPTNET_MST_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to solve the minimum spanning tree problem. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the solver at termination. The STATUS term takes the same value as the term MST in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

**OBJECTIVE**
indicates the total link weight of the minimum spanning tree.

**CPU_TIME**
indicates the CPU time (in seconds) taken by the solver.

**REAL_TIME**
indicates the real time (in seconds) taken by the solver.

Macro Variable _OROPTNET_SHORTPATH_

The OPTNET procedure defines a macro variable named _OROPTNET_SHORTPATH_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to calculate shortest paths. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the algorithm at termination. The STATUS term takes the same value as the term SHORTPATH in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

**NUM_PATHS**
indicates the number of shortest paths that the algorithm finds.

**CPU_TIME**
indicates the CPU time (in seconds) taken by the algorithm.

**REAL_TIME**
indicates the real time (in seconds) taken by the algorithm.
**Macro Variable _OROPTNET_TRANSCL_**

The OPTNET procedure defines a macro variable named _OROPTNET_TRANSCL_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to calculate transitive closure. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the algorithm at termination. The STATUS term takes the same value as the term TRANSCL in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

**CPU_TIME**
indicates the CPU time (in seconds) taken by the algorithm.

**REAL_TIME**
indicates the real time (in seconds) taken by the algorithm.

**Macro Variable _OROPTNET_TSP_**

The OPTNET procedure defines a macro variable named _OROPTNET_TSP_. This variable contains a character string that indicates the status and some basic statistics about the results of the algorithm that PROC OPTNET uses to solve the traveling salesman problem. The various terms of the variable are interpreted as follows:

**STATUS**
indicates the status of the solver at termination. The STATUS term takes the same value as the term TSP in the _OROPTNET_ macro as defined in the section “Macro Variable _OROPTNET_” on page 91.

**OBJECTIVE**
indicates the objective value that the solver obtains at termination.

**RELATIVE_GAP**
specifies the relative gap between the best integer objective (BestInteger) and the objective of the best remaining node (BestBound) upon termination of the solver. The relative gap is equal to

\[
\frac{|\text{BestInteger} - \text{BestBound}|}{(1E-10 + |\text{BestBound}|)}
\]

**ABSOLUTE_GAP**
specifies the absolute gap between the best integer objective (BestInteger) and the objective of the best remaining node (BestBound) upon termination of the solver. The absolute gap is equal to

| BestInteger – BestBound |

**PRIMAL_INFEASIBILITY**
indicates the maximum (absolute) violation of the primal constraints by the solution.
BOUND_INFEASIBILITY
indicates the maximum (absolute) violation by the solution of the lower or upper bounds (or both).

INTEGER_INFEASIBILITY
indicates the maximum (absolute) violation of the integrality of integer variables that are returned by
the solver.

BEST_BOUND
specifies the best linear programming objective value of all unprocessed nodes in the branch-and-bound
tree at the end of execution. A missing value indicates that the solver has processed either all or none
of the nodes in the branch-and-bound tree.

NODES
specifies the number of nodes enumerated by the solver by using the branch-and-bound algorithm.

ITERATIONS
indicates the number of simplex iterations taken to solve the problem.

CPU_TIME
indicates the CPU time (in seconds) taken by the algorithm.

REAL_TIME
indicates the real time (in seconds) taken by the algorithm.

NOTE: The time reported in PRESOLVE_TIME and SOLUTION_TIME is either CPU time (default) or real
time. The type is determined by the TIMETYPE= option.
Example 2.1: Articulation Points in a Terrorist Network

This example considers the terrorist communications network from the attacks on the United States on September 11, 2001, described in Krebs 2002. Figure 2.64 shows this network, which was constructed after the attacks, based on collected intelligence information. The image was created using SAS/GRAPH Network Visualization Workshop 2.1. See the SAS/GRAPH: Network Visualization Workshop User’s Guide.

Figure 2.64  Terrorist Communications Network from 9/11
The full network data include 153 links. The following statements show a small subset to illustrate the use of the BICONCOMP statement in this context:

```sas
data LinkSetInTerror911;
  length from $25 to $32;
  input from to;
  datalines;
  Abu_Zubeida  Djamal_Beghal
  Jean-Marc_Grandvisir Djamal_Beghal
  Nizar_Trabelsi Djamal_Beghal
  Abu_Walid Djamal_Beghal
  Abu_Qatada Djamal_Beghal
  Zacarias_Moussaoui Djamal_Beghal
  Jerome_Courtaillier Djamal_Beghal
  Kamel_Daoudi Djamal_Beghal
  Abu_Walid Kamel_Daoudi
  Abu_Walid Abu_Qatada
  Kamel_Daoudi Zacarias_Moussaoui
  Kamel_Daoudi Jerome_Courtaillier
  Jerome_Courtaillier Zacarias_Moussaoui
  Jerome_Courtaillier David_Courtaillier
  Zacarias_Moussaoui David_Courtaillier
  Zacarias_Moussaoui Ahmed_Ressam
  Zacarias_Moussaoui Abu_Qatada
  Zacarias_Moussaoui Ramzi_Bin_al-Shibh
  Zacarias_Moussaoui Mahamed_Atta
  Ahmed_Ressam Haydar_Abu_Doha
  Mehdi_Khammoun Haydar_Abu_Doha
  Essid_Sami_Ben_Khemais Haydar_Abu_Doha
  Mehdi_Khammoun Essid_Sami_Ben_Khemais
  Mehdi_Khammoun Mohamed_Bensakhria
...;
```

Suppose that this communications network had been discovered before the attack on 9/11. If the investigators’ goal was to disrupt the flow of communication between different groups within the organization, then they would want to focus on the people who are articulation points in the network.

To find the articulation points, use the following statements:

```sas
proc optnet
  data_links = LinkSetInTerror911
  out_nodes = NodeSetOut;
  biconcomp;
run;

data ArtPoints;
  set NodeSetOut;
  where artpoint=1;
run;
```
The data set ArtPoints contains members of the network who are articulation points. Focusing investigations on cutting off these particular members could have caused a great deal of disruption in the terrorists’ ability to communicate when formulating the attack.

Output 2.1.1 Articulation Points of Terrorist Communications Network from 9/11

<table>
<thead>
<tr>
<th>node</th>
<th>artpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Djamal_Beghal</td>
<td>1</td>
</tr>
<tr>
<td>Zacarias_Moussaoui</td>
<td>1</td>
</tr>
<tr>
<td>Essid_Sami_Ben_Khemaiss</td>
<td>1</td>
</tr>
<tr>
<td>Mohamed_Atta</td>
<td>1</td>
</tr>
<tr>
<td>Mamoun_Darkazanli</td>
<td>1</td>
</tr>
<tr>
<td>Nawaf_Alhazmi</td>
<td>1</td>
</tr>
</tbody>
</table>

Example 2.2: Cycle Detection for Kidney Donor Exchange

This example looks at an application of cycle detection to help create a kidney donor exchange. Suppose someone needs a kidney transplant and a family member is willing to donate one. If the donor and recipient are incompatible (because of blood types, tissue mismatch, and so on), the transplant cannot happen. Now suppose two donor-recipient pairs A and B are in this situation, but donor A is compatible with recipient B and donor B is compatible with recipient A. Then two transplants can take place in a two-way swap, shown graphically in Figure 2.65. More generally, an $n$-way swap can be performed involving $n$ donors and $n$ recipients (Willingham 2009).

To model this problem, define a directed graph as follows. Each node is an incompatible donor-recipient pair. Link $(i, j)$ exists if the donor from node $i$ is compatible with the recipient from node $j$. The link weight is a measure of the quality of the match. By introducing dummy links whose weight is 0, you can also include altruistic donors who have no recipients or recipients who have no donors. The idea is to find a maximum-weight node-disjoint union of directed cycles. You want the union to be node-disjoint so that no kidney is donated more than once, and you want cycles so that the donor from node $i$ gives up a kidney if and only if the recipient from node $i$ receives a kidney.

Without any other constraints, the problem could be solved as a linear assignment problem, as described in the section “Linear Assignment (Matching)” on page 59. But doing so would allow arbitrarily long cycles in
the solution. Because of practical considerations (such as travel) and to mitigate risk, each cycle must have no more than \( L \) links. The kidney exchange problem is to find a maximum-weight node-disjoint union of short directed cycles.

One way to solve this problem is to explicitly generate all cycles whose length is at most \( L \) and then solve a set packing problem. You can use PROC OPTNET to generate the cycles and then PROC OPTMODEL (see \textit{SAS/OR User's Guide: Mathematical Programming}) to read the PROC OPTNET output, formulate the set packing problem, call the mixed integer linear programming solver, and output the optimal solution.

The following DATA step sets up the problem, first creating a random graph on \( n \) nodes with link probability \( p \) and Uniform(0,1) weight:

```sas
/* create random graph on \( n \) nodes with arc probability \( p \) and uniform(0,1) weight */
%let n = 100;
%let p = 0.02;
data LinkSetIn;
  do from = 0 to &n - 1;
    do to = 0 to &n - 1;
      if from eq to then continue;
      else if ranuni(1) < &p then do;
        weight = ranuni(2);
        output;
      end;
    end;
  end;
run;
```

The following statements use PROC OPTNET to generate all cycles whose length is greater than or equal to 2 and less than or equal to 10:

```sas
/* generate all cycles with 2 <= length <= \( \text{max\_length} \) */
%let max\_length = 10;
proc optnet
  loglevel = moderate
  graph_direction = directed
  data_links = LinkSetIn;
  cycle
    minLength = 2
    maxLength = &max\_length
  out = Cycles
  mode = all\_cycles;
run;
%put &_OROPTNET_;
%put &_OROPTNET\_CYCLE_;```

PROC OPTNET finds 224 cycles of the appropriate length, as shown in Output 2.2.1.

**Output 2.2.1** Cycles for Kidney Donor Exchange PROC OPTNET Log

```plaintext
NOTE: Reading the links data set.
NOTE: There were 194 observations read from the data set WORK.LINKSETIN.
NOTE: Data input used 0.01 (cpu: 0.00) seconds.
NOTE: Building the input graph storage used 0.00 (cpu: 0.00) seconds.
NOTE: The input graph storage is using 0.0 MBs of memory.
NOTE: The number of nodes in the input graph is 97.
NOTE: The number of links in the input graph is 194.
NOTE: Processing cycle detection.
NOTE: The graph has 224 cycles.
NOTE: Processing cycle detection used 2.13 (cpu: 2.14) seconds.
NOTE: Creating cycle data set output.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.
NOTE: The data set WORK.CYCLES has 2124 observations and 3 variables.
STATUS=OK CYCLE=OK
STATUS=OK NUM_CYCLES=224 CPU_TIME=2.14 REAL_TIME=2.13
```

From the resulting data set Cycles, use the following DATA step to convert the cycles into one observation per arc:

```plaintext
/* convert Cycles into one observation per arc */
data Cycles0(keep=c i j);
  set Cycles;
  retain last;
  c   = cycle;
  i   = last;
  j   = node;
  last = j;
  if order ne 1 then output;
run;
```
Given the set of cycles, you can now formulate a mixed integer linear program (MILP) to maximize the total cycle weight. Let $C$ define the set of cycles of appropriate length, $N_c$ define the set of nodes in cycle $c$, $A_c$ define the set of links in cycle $c$, and $w_{ij}$ denote the link weight for link $(i, j)$. Define a binary decision variable $x_c$. Set $x_c$ to 1 if cycle $c$ is used in the solution; otherwise, set it to 0. Then, the following MILP defines the problem that you want to solve to maximize the quality of the kidney exchange:

$$\minimize \sum_{c \in C} \left( \sum_{(i,j) \in A_c} w_{ij} \right) x_c$$

subject to

$$\sum_{c \in C : i \in N_c} x_c \leq 1 \quad i \in N$$

$$(\text{incomp\_pair})$$

$$x_c \in \{0, 1\} \quad c \in C$$

The constraint (incomp_pair) ensures that each node (incompatible pair) in the graph is intersected at most once. That is, a donor can donate a kidney only once. You can use PROC OPTMODEL to solve this mixed integer linear programming problem as follows:

```optmodel
/* solve set packing problem to find maximum weight node-disjoint union of short directed cycles */
proc optmodel;
/* declare index sets and parameters, and read data */
set <num,num> ARCS;
num weight {ARCS};
read data LinkSetIn into ARCS= [from to] weight;
set <num,num,num> TRIPLES;
read data Cycles0 into TRIPLES= [c i j];
set CYCLES = setof {<c,i,j> in TRIPLES} c;
set ARCS_c {c in CYCLES} = setof {<(c),i,j> in TRIPLES} <i,j>;
set NODES_c {c in CYCLES} = union {<i,j> in ARCS_c[c]} {i,j};
set NODES = union {c in CYCLES} NODES_c[c];
um cycle_weight {c in CYCLES} = sum {<i,j> in ARCS_c[c]} weight[i,j];

/* UseCycle[c] = 1 if cycle c is used, 0 otherwise */
var UseCycle {CYCLES} binary;

/* declare objective */
max TotalWeight = sum {c in CYCLES} cycle_weight[c] * UseCycle[c];

/* each node appears in at most one cycle */
con node_packing {i in NODES}:
  sum {c in CYCLES: i in NODES_c[c]} UseCycle[c] <= 1;

/* call solver */
solve with milp;

/* output optimal solution */
create data Solution from
  [c]={c in CYCLES: UseCycle[c].sol > 0.5} cycle_weight;
quit;
%put &_OROPTMODEL_;```
PROC OPTMODEL solves the problem by using the mixed integer linear programming solver. As shown in Output 2.2.2, it was able to find a total weight (quality level) of 26.02.

### Output 2.2.2  Cycles for Kidney Donor Exchange

PROC OPTMODEL Log

```
NOTE: There were 194 observations read from the data set WORK.LINKSETIN.
NOTE: There were 1900 observations read from the data set WORK.CYCLES0.
NOTE: Problem generation will use 4 threads.
NOTE: The problem has 224 variables (0 free, 0 fixed).
NOTE: The problem has 224 binary and 0 integer variables.
NOTE: The problem has 63 linear constraints (63 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 1900 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 35 constraints.
NOTE: The MILP presolver modified 116 constraint coefficients.
NOTE: The presolved problem has 224 variables, 28 constraints, and 1382 constraint coefficients.
NOTE: The MILP solver is called.
```

<table>
<thead>
<tr>
<th>Node</th>
<th>Active</th>
<th>Sols</th>
<th>BestInteger</th>
<th>BestBound</th>
<th>Gap</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>22.7780692</td>
<td>1080.2049611</td>
<td>97.89%</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>22.7780692</td>
<td>26.5638757</td>
<td>14.25%</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>4</td>
<td>23.2158345</td>
<td>26.0202979</td>
<td>10.78%</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>4</td>
<td>23.2158345</td>
<td>26.0202956</td>
<td>10.78%</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
<td>26.0202871</td>
<td>26.0202871</td>
<td>0.00%</td>
<td>0</td>
</tr>
</tbody>
</table>

NOTE: The MILP solver added 5 cuts with 591 cut coefficients at the root.

NOTE: Objective = 26.020287142.

NOTE: The data set WORK.SOLUTION has 6 observations and 2 variables.

```
STATUS=OK ALGORITHM=BAC SOLUTION_STATUS=OPTIMAL OBJECTIVE=26.020287142
RELATIVE_GAP=0 ABSOLUTE_GAP=0 PRIMAL_INFEASIBILITY=4.440892E-16
BOUND_INFEASIBILITY=4.440892E-16 INTEGER_INFEASIBILITY=4.440892E-16
BEST_BOUND=26.020287142 NODES=1 ITERATIONS=89 PRESOLVE_TIME=0.05
SOLUTION_TIME=0.06
```

The data set Solution, shown in Output 2.2.3, now contains the cycles that define the best exchange and their associated weight (quality).

### Output 2.2.3  Maximum Quality Solution for Kidney Donor Exchange

```
cycle_ c weight
12    5.84985
43    3.90015
71    5.44467
124   7.42574
222   2.28231
224   1.11757
```
Example 2.3: Linear Assignment Problem for Minimizing Swim Times

A swimming coach needs to assign male and female swimmers to each stroke of a medley relay team. The swimmers’ best times for each stroke are stored in a SAS data set. The LINEAR_ASSIGNMENT statement evaluates the times and matches strokes and swimmers to minimize the total relay swim time.

The data are stored in matrix format, where the row identifier is the swimmer’s name (variable name) and each swimming event is a column (variables: back, breast, fly, and free). In the following DATA step, the relay times are split into two categories, male and female:

```sas
data RelayTimes;
  input name $ sex $ back breast fly free;
datalines;
Sue      F 35.1 36.7 28.3 36.1
Karen    F 34.6 32.6 26.9 26.2
Jan      F 31.3 33.9 27.1 31.2
Andrea   F 28.6 34.1 29.1 30.3
Carol    F 32.9 32.2 26.6 24.0
Ellen    F 27.8 32.5 27.8 27.0
Jim      M 26.3 27.6 23.5 22.4
Mike     M 29.0 24.0 27.9 25.4
Sam      M 27.2 33.8 25.2 24.1
Clayton  M 27.0 29.2 23.0 21.9;

data RelayTimesF RelayTimesM;
  set RelayTimes;
  if sex='F' then output RelayTimesF;
  else if sex='M' then output RelayTimesM;
run;
```

The following statements solve the linear assignment problem for both male and female relay teams:

```sas
proc optnet
  data_matrix = RelayTimesF;
  linear_assignment
    out     = LinearAssignF
    id      = (name sex);
run;
%put &_OROPTNET_;
%put &_OROPTNET_LAP_;
proc optnet
  data_matrix = RelayTimesM;
  linear_assignment
    out     = LinearAssignM
    id      = (name sex);
run;
%put &_OROPTNET_;
%put &_OROPTNET_LAP_;
```

The progress of the two PROC OPTNET calls is shown in Output 2.3.1 and Output 2.3.2.
Output 2.3.1 PROC OPTNET Log: Linear Assignment for Female Swim Times

NOTE: The number of columns in the input matrix is 4.
NOTE: The number of rows in the input matrix is 6.
NOTE: Data input used 0.00 (cpu: 0.00) seconds.
NOTE: Processing the linear assignment problem.
NOTE: The minimum cost linear assignment is 111.5.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.
NOTE: The data set WORK.LINEARASSIGNF has 4 observations and 4 variables.
STATUS=OK LAP=OPTIMAL
STATUS=OPTIMAL OBJECTIVE=111.5 CPU_TIME=0.00 REAL_TIME=0.00

Output 2.3.2 PROC OPTNET Log: Linear Assignment for Male Swim Times

NOTE: The number of columns in the input matrix is 4.
NOTE: The number of rows in the input matrix is 4.
NOTE: Data input used 0.00 (cpu: 0.00) seconds.
NOTE: Processing the linear assignment problem.
NOTE: The minimum cost linear assignment is 96.6.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.
NOTE: The data set WORK.LINEARASSIGNM has 4 observations and 4 variables.
STATUS=OK LAP=OPTIMAL
STATUS=OPTIMAL OBJECTIVE=96.6 CPU_TIME=0.00 REAL_TIME=0.00
The data sets `LinearAssignF` and `LinearAssignM` contain the optimal assignments. Note that in the case of the female data, there are more people (set $S$) than there are strokes (set $T$). Therefore, the solver allows for some members of $S$ to remain unassigned.

**Output 2.3.3** Optimal Assignments for Best Female Swim Times

<table>
<thead>
<tr>
<th>name</th>
<th>sex</th>
<th>assign</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karen</td>
<td>F</td>
<td>breast</td>
<td>32.6</td>
</tr>
<tr>
<td>Jan</td>
<td>F</td>
<td>fly</td>
<td>27.1</td>
</tr>
<tr>
<td>Carol</td>
<td>F</td>
<td>free</td>
<td>24.0</td>
</tr>
<tr>
<td>Ellen</td>
<td>F</td>
<td>back</td>
<td>27.8</td>
</tr>
</tbody>
</table>

\[111.5\]

**Output 2.3.4** Optimal Assignments for Best Male Swim Times

<table>
<thead>
<tr>
<th>name</th>
<th>sex</th>
<th>assign</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
<td>M</td>
<td>free</td>
<td>22.4</td>
</tr>
<tr>
<td>Mike</td>
<td>M</td>
<td>breast</td>
<td>24.0</td>
</tr>
<tr>
<td>Sam</td>
<td>M</td>
<td>back</td>
<td>27.2</td>
</tr>
<tr>
<td>Clayton</td>
<td>M</td>
<td>fly</td>
<td>23.0</td>
</tr>
</tbody>
</table>

\[96.6\]

**Example 2.4: Linear Assignment Problem, Sparse Format versus Dense Format**

This example looks at the problem of assigning swimmers to strokes based on their best times. However, in this case certain swimmers are not eligible to perform certain strokes. A missing (.) value in the data matrix identifies an ineligible assignment. For example:

```plaintext
data RelayTimesMatrix;
   input name $ sex $ back breast fly free;
datalines;
Sue   F . 36.7 28.3 36.1
Karen F 34.6 . . 26.2
Jan   F 31.3 . 27.1 .
Andrea F 28.6 . 29.1 .
Carol F 32.9 . 26.6 .
;```

Recall that the linear assignment problem can also be interpreted as the minimum-weight matching in a bipartite graph. The eligible assignments define links between the rows (swimmers) and the columns (strokes), as in Figure 2.66.

Figure 2.66  Bipartite Graph for Linear Assignment Problem

You can represent the same data in RelayTimesMatrix by using a links data set as follows:

```plaintext
data RelayTimesLinks;
  input name $ attr $ cost;
datalines;
Sue   breast 36.7
Sue   fly    28.3
Sue   free   36.1
Karen back  34.6
Karen free  26.2
Jan    back   31.3
Jan    fly    27.1
Andrea back  28.6
Andrea fly  29.1
Carol  back   32.9
Carol  fly    26.6;
```

This graph must be bipartite (such that $S$ and $T$ are disjoint). If it is not, PROC OPTNET returns an error.
Now, you can use either input format to solve the same problem, as follows:

```
proc optnet
  data_matrix = RelayTimesMatrix;
  linear_assignment
    out     = LinearAssignMatrix
    weight  = (back--free)
    id      = (name sex);
run;
```

```
proc optnet
  graph_direction = directed
  data_links     = RelayTimesLinks;
  data_links_var
    from     = name
    to       = attr
    weight   = cost;
  linear_assignment
    out     = LinearAssignLinks;
run;
```

When you use the graph input format, the LINEAR_ASSIGNMENT options WEIGHT= and ID= are not used directly.

The data sets LinearAssignMatrix and LinearAssignLinks now contain the optimal assignments, as shown in Output 2.4.1 and Output 2.4.2.

**Output 2.4.1** Optimal Assignments for Swim Times (Dense Input)

<table>
<thead>
<tr>
<th>name</th>
<th>sex</th>
<th>assign</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>F</td>
<td>breast</td>
<td>36.7</td>
</tr>
<tr>
<td>Karen</td>
<td>F</td>
<td>free</td>
<td>26.2</td>
</tr>
<tr>
<td>Andrea</td>
<td>F</td>
<td>back</td>
<td>28.6</td>
</tr>
<tr>
<td>Carol</td>
<td>F</td>
<td>fly</td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>=====</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>118.1</td>
</tr>
</tbody>
</table>

**Output 2.4.2** Optimal Assignments for Swim Times (Sparse Input)

<table>
<thead>
<tr>
<th>name</th>
<th>attr</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>breast</td>
<td>36.7</td>
</tr>
<tr>
<td>Karen</td>
<td>free</td>
<td>26.2</td>
</tr>
<tr>
<td>Andrea</td>
<td>back</td>
<td>28.6</td>
</tr>
<tr>
<td>Carol</td>
<td>fly</td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>=====</td>
</tr>
<tr>
<td></td>
<td></td>
<td>118.1</td>
</tr>
</tbody>
</table>
The optimal assignments are shown graphically in Figure 2.67.

**Figure 2.67** Optimal Assignments for Swim Times

For large problems where a number of links are forbidden, the sparse format can be faster and can save a great deal of memory. Consider an example that uses the format of the DATA_MATRIX= option with 15,000 columns ($|S| = 15,000$) and 4,000 rows ($|T| = 4,000$). To store the dense matrix in memory, PROC OPTNET needs to allocate approximately $|S||T| \cdot 8/1024/1024 = 457$ MB. If the data have mostly ineligible links, then the sparse (graph) format that uses the DATA_LINKS= option is much more efficient with respect to memory. For example, if the data have only 5% of the eligible links ($15,000 \cdot 4,000 \cdot 0.05 = 3,000,000$), then the dense storage would still need 457 MB. The sparse storage for the same example needs approximately $|S| \cdot |T| \cdot 0.05 \cdot 12/1024/1024 = 34$ MB. If the problem is fully dense (all links are eligible), then the dense format that uses the DATA_MATRİX= option is the most efficient.
Example 2.5: Minimum Spanning Tree for Computer Network Topology

Consider the problem of designing a small network of computers in an office. In designing the network, the goal is to make sure that each machine in the office can reach every other machine. To accomplish this goal, Ethernet lines must be constructed and run between the machines. The construction costs for each possible link are based approximately on distance and are shown in Figure 2.68. Besides distance, the costs also reflect some restrictions due to physical boundaries. To connect all the machines in the office at minimal cost, you need to find a minimum spanning tree on the network of possible links.

Figure 2.68 Potential Office Computer Network

Define the link data set as follows:

```plaintext
data LinkSetInCompNet;
  input from $ to $ weight @@;
datalines;
  A B 1.0 A C 1.0 A D 1.5 B C 2.0 B D 4.0
  B E 3.0 C D 3.0 C F 3.0 C H 4.0 D E 1.5
  D F 3.0 D G 4.0 E F 1.0 E G 1.0 F G 2.0
  F H 4.0 H I 1.0 I J 1.0
;
```

The following statements find a minimum spanning tree:

```plaintext
proc optnet
  data_links = LinkSetInCompNet;
  minspantree
    out = MinSpanTree;
run;
```
Output 2.5.1 shows the resulting data set MinSpanTree, which is displayed graphically in Figure 2.69 with the minimal cost links shown in green.

**Figure 2.69** Minimum Spanning Tree for Office Computer Network

![Minimum Spanning Tree for Office Computer Network](image)

Output 2.5.1 Minimum Spanning Tree of a Computer Network Topology

<table>
<thead>
<tr>
<th>from</th>
<th>to</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>I</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>F</td>
<td>1.0</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>1.0</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>1.0</td>
</tr>
<tr>
<td>I</td>
<td>J</td>
<td>1.0</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>1.5</td>
</tr>
<tr>
<td>A</td>
<td>D</td>
<td>1.5</td>
</tr>
<tr>
<td>C</td>
<td>H</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Example 2.6: Transitive Closure for Identification of Circular Dependencies in a Bug Tracking System

Most software bug tracking systems have some notion of *duplicate bugs* in which one bug is declared to be the same as another bug. If bug A is considered a duplicate (DUP) of bug B, then a fix for B would also fix A. You can represent the DUPs in a bug tracking system as a directed graph where you add a link \( A \rightarrow B \) if A is a DUP of B.

The bug tracking system needs to check for two situations when users declare a bug to be a DUP. The first situation is called a *circular dependence*. Consider bugs A, B, C, and D in the tracking system. The first user declares that A is a DUP of B and that C is a DUP of D. Then, a second user declares that B is a DUP of C, and a third user declares that D is a DUP of A. You now have a circular dependence, and no primary bug is
defined on which the development team should focus. You can easily see this circular dependence in the graph representation, because \( A \rightarrow B \rightarrow C \rightarrow D \rightarrow A \). Finding such circular dependencies can be done using cycle detection, which is described in the section “Cycle” on page 52. However, the second situation that needs to be checked is more general. If a user declares that \( A \) is a DUP of \( B \) and another user declares that \( B \) is a DUP of \( C \), this chain of duplicates is already an issue. The bug tracking system needs to provide one primary bug to which the rest of the bugs are duplicated. The existence of these chains can be identified by calculating the transitive closure of the directed graph that is defined by the DUP links.

Given the original directed graph \( G \) (defined by the DUP links) and its transitive closure \( G^T \), any link in \( G^T \) that is not in \( G \) exists because of some chain that is present in \( G \).

Consider the following data that define some duplicated bugs (called defects) in a small sample of the bug tracking system:

```plaintext
data DefectLinks;
  input defectId $ linkedDefect $ linkType $ when datetime16.;
  format when datetime16.;
  datalines;
  D0096978 S0711218 DUPTO 20OCT10:00:00:00
  S0152674 S0153280 DUPTO 30MAY02:00:00:00
  S0153280 S0153307 DUPTO 30MAY02:00:00:00
  S0153307 S0152674 DUPTO 30MAY02:00:00:00
  S0162973 S0162978 DUPTO 29NOV10:16:13:16
  S0162978 S0165405 DUPTO 29NOV10:16:13:16
  S0325026 S0575748 DUPTO 01JUN10:00:00:00
  S0347945 S0346582 DUPTO 03MAR06:00:00:00
  S0350596 S0346582 DUPTO 21MAR06:00:00:00
  S0539744 S0643230 DUPTO 10MAY10:00:00:00
  S0575748 S0643230 DUPTO 15JUN10:00:00:00
  S0629984 S0643230 DUPTO 01JUN10:00:00:00
;
```

The following statements calculate cycles in addition to the transitive closure of the graph \( G \) that is defined by the duplicated defects in DefectLinks. The output data set Cycles contains any circular dependencies, and the data set TransClosure contains the transitive closure \( G^T \). To identify the chains, you can use PROC SQL to identify the links in \( G^T \) that are not in \( G \).

```plaintext
proc optnet
  loglevel = moderate
  graph_direction = directed
  data_links = DefectLinks;
  data_links_var
    from = defectId
    to = linkedDefect;
  cycle
    out = Cycles
    mode = all_cycles;
  transitive_closure
    out = TransClosure;
run;
```
Example 2.6: Transitive Closure for Identification of Circular Dependencies

```sql
%put &_OROPTNET_;
%put &_OROPTNET_CYCLE_;
%put &_OROPTNET_TRANSCL_;

proc sql;
   create table Chains as
      select defectId, linkedDefect from TransClosure
      except
      select defectId, linkedDefect from DefectLinks;
quit;
```

The progress of the procedure is shown in Output 2.6.1.

**Output 2.6.1** PROC OPTNET Log: Transitive Closure for Identification of Circular Dependencies in a Bug Tracking System

```plaintext
NOTE: Reading the links data set.
NOTE: There were 12 observations read from the data set WORK.DEFECTLINKS.
NOTE: Building the input graph storage used 0.00 (cpu: 0.00) seconds.
NOTE: The number of nodes in the input graph is 16.
NOTE: The number of links in the input graph is 12.
NOTE: Processing cycle detection.
NOTE: The graph has 1 cycle.
NOTE: Processing the transitive closure.
NOTE: Creating transitive closure data set output.
NOTE: The data set WORK.CYCLES has 4 observations and 3 variables.
NOTE: The data set WORK.TRANSCLOSURE has 20 observations and 2 variables.
STATUS=OK  CYCLE=OK  TRANSCL=OK
STATUS=OK  NUM_CYCLES=1  CPU_TIME=0.00  REAL_TIME=0.00
STATUS=OK  CPU_TIME=0.00  REAL_TIME=0.00
NOTE: Table WORK.CHAINS created, with 8 rows and 2 columns.
```
The data set Cycles contains one case of a circular dependence in which the DUPs start and end at S0152674.

**Output 2.6.2** Cycle in Bug Tracking System

<table>
<thead>
<tr>
<th>cycle</th>
<th>order</th>
<th>node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>S0152674</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>S0153280</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>S0153307</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>S0152674</td>
</tr>
</tbody>
</table>

The data set Chains contains the chains in the bug tracking system that come from the links in $G^T$ that are not in $G$.

**Output 2.6.3** Chains in Bug Tracking System

<table>
<thead>
<tr>
<th>defectId</th>
<th>linked</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0152674</td>
<td>S0152674</td>
</tr>
<tr>
<td>S0152674</td>
<td>S0153307</td>
</tr>
<tr>
<td>S0153280</td>
<td>S0152674</td>
</tr>
<tr>
<td>S0153280</td>
<td>S0153280</td>
</tr>
<tr>
<td>S0153307</td>
<td>S0153280</td>
</tr>
<tr>
<td>S0153307</td>
<td>S0153307</td>
</tr>
<tr>
<td>S0162973</td>
<td>S0165405</td>
</tr>
<tr>
<td>S0325026</td>
<td>S0643230</td>
</tr>
</tbody>
</table>

**Example 2.7: Traveling Salesman Tour through US Capital Cities**

Consider a cross-country trip where you want to travel the fewest miles to visit all of the capital cities in all US states except Alaska and Hawai. Finding the optimal route is an instance of the traveling salesman problem, which is described in section “Traveling Salesman Problem” on page 86.

The following PROC SQL statements use the built-in data set maps.uscity to generate a list of the capital cities and their latitude and longitude:

```sql
/* Get a list of the state capital cities (with lat and long) */
proc sql;
  create table Cities as
  select unique statecode as state, city, lat, long
  from maps.uscity
  where capital='Y' and statecode not in ('AK' 'PR' 'HI');
quit;
```
From this list, you can generate a links data set CitiesDist that contains the distances, in miles, between each pair of cities. The distances are calculated by using the SAS function GEODIST.

```sas
/* Create a list of all the possible pairs of cities */
proc sql;
create table CitiesDist as
   select
      a.city as city1, a.lat as lat1, a.long as long1,
      b.city as city2, b.lat as lat2, b.long as long2,
      geodist(lat1, long1, lat2, long2, 'DM') as distance
   from Cities as a, Cities as b
   where a.city < b.city;
quit;
```

The following PROC OPTNET statements find the optimal tour through each of the capital cities:

```sas
/* Find optimal tour using OPTNET */
proc optnet
   loglevel = moderate
   data_links = CitiesDist
   out_nodes = TSPTourNodes;
   data_links_var
      from = city1
      to = city2
      weight = distance;
   tsp
      out = TSPTourLinks;
run;
%put &_OROPTNET_;  
%put &_OROPTNET_TSP_;  
```
The progress of the procedure is shown in Output 2.7.1. The total mileage needed to optimally traverse the capital cities is 10,627.75 miles.

Output 2.7.1  PROC OPTNET Log: Traveling Salesman Tour through US Capital Cities

NOTE: Reading the links data set.
NOTE: There were 1176 observations read from the data set WORK.CITIESDIST.
NOTE: Data input used 0.01 (cpu: 0.00) seconds.
NOTE: Building the input graph storage used 0.00 (cpu: 0.00) seconds.
NOTE: The input graph storage is using 0.1 MBs of memory.
NOTE: The number of nodes in the input graph is 49.
NOTE: The number of links in the input graph is 1176.

NOTE: Processing the traveling salesman problem.
NOTE: The initial TSP heuristics found a tour with cost 10645.918753 using 0.08 (cpu: 0.08) seconds.
NOTE: The MILP presolver value NONE is applied.

Node  Active  Sols    BestInteger  BestBound  Gap    Time
0      1       1  10645.918753 10040.5139714  6.03%  0
0      1       1  10645.918753 10241.6970024  3.95%  0
0      1       1  10645.918753 10293.2995080  3.43%  0
0      1       1  10645.918753 10345.3151313  2.91%  0
0      1       1  10645.918753 10549.5506188  0.91%  0
0      1       1  10645.918753 10576.0823291  0.66%  0
0      1       1  10645.918753 10590.3709358  0.52%  0
0      1       1  10645.918753 10590.8162090  0.52%  0
0      1       1  10645.918753 10590.9748294  0.52%  0
0      1       3  10645.918753 10607.8528157  0.36%  0
0      1       3  10645.918753 10607.8528157  0.36%  0

NOTE: The MILP solver added 16 cuts with 4267 cut coefficients at the root.
1      1       4  10627.7543183 10607.8528157  0.19%  0
2      0       4  10627.7543183 10627.7543183  0.00%  0

NOTE: Optimal.
NOTE: Objective = 10627.754318.
NOTE: Processing the traveling salesman problem used 0.12 (cpu: 0.11) seconds.
NOTE: Creating nodes data set output.
NOTE: Creating traveling salesman data set output.
NOTE: Data output used 0.00 (cpu: 0.00) seconds.

NOTE: The data set WORK.TSPTOURNODES has 49 observations and 2 variables.
NOTE: The data set WORK.TSPTOURLINKS has 49 observations and 3 variables.
STATUS=OK  TSP=OPTIMAL
STATUS=OPTIMAL  OBJECTIVE=10627.754318  RELATIVE_GAP=0  ABSOLUTE_GAP=0
PRIMAL_INFEASIBILITY=0  BOUND_INFEASIBILITY=0  INTEGER_INFEASIBILITY=0
BEST_BOUND=10627.754318  NODES=3  ITERATIONS=166  CPU_TIME=0.11  REAL_TIME=0.12
The following PROC GPROJECT and PROC GMAP statements produce a graphical display of the solution:

```sql
/* Merge latitude and longitude */
proc sql;
  /* merge in the lat & long for city1 */
  create table TSPTourLinksAnno1 as
  select unique TSPTourLinks.*, cities.lat as lat1, cities.long as long1
  from TSPTourLinks left join cities
  on TSPTourLinks.city1=cities.city;
  /* merge in the lat & long for city2 */
  create table TSPTourLinksAnno2 as
  select unique TSPTourLinksAnno1.*, cities.lat as lat2, cities.long as long2
  from TSPTourLinksAnno1 left join cities
  on TSPTourLinksAnno1.city2=cities.city;
quit;

/* Create the annotated data set to draw the path on the map
(convert lat & long degrees to radians, since the map is in radians) */
data anno_path;
  set TSPTourLinksAnno2;
  length function color $8;
  xsys='2'; ysys='2'; hsys='3'; when='a'; anno_flag=1;
  function='move';
  x=atan(1)/45 * long1;
  y=atan(1)/45 * lat1;
  output;
  function='draw';
  color="blue"; size=0.8;
  x=atan(1)/45 * long2;
  y=atan(1)/45 * lat2;
  output;
run;

/* Get a map with only the contiguous 48 states */
data states;
  set maps.states (where=(fipstate(state) not in ('HI' 'AK' 'PR')));
run;

data combined;
  set states anno_path;
run;
```
/* Project the map and annotate the data */
proc gproject data=combined out=combined dupok;
   id state;
run;

data states anno_path;
   set combined;
   if anno_flag=1 then output anno_path;
   else output states;
run;

/* Get a list of the endpoints locations */
proc sql;
   create table anno_dots as
      select unique x, y from anno_path;
quit;

/* Create the final annotate data set */
data anno_dots;
   set anno_dots;
   length function color $8;
   xsys='2'; ysys='2'; when='a'; hsys='3';
   function='pie';
   rotate=360; size=0.8; style='psolid'; color="red";
   output;
   style='pempty'; color="black";
   output;
run;

/* Generate the map with GMAP */
pattern1 v=s c=cxcffcc repeat=100;
proc gmap data=states map=states anno=anno_path all;
   id state;
   choro state / levels=1 nolegend coutline=black
      anno=anno_dots des='1' name="tsp";
run;
The minimal cost tour through the capital cities is shown on the US map in Figure 2.7.2.

**Output 2.7.2** Optimal Traveling Salesman Tour through US Capital Cities

The data set TSPTourLinks contains the links in the optimal tour. To display the links in the order they are to be visited, you can use the following DATA step:

```sas
/* Create the directed optimal tour */
data TSPTourLinksDirected(drop=next);
  set TSPTourLinks;
  retain next;
  if _N_ ne 1 and city1 ne next then do;
    city2 = city1;
    city1 = next;
  end;
  next = city2;
run;
```
The data set TSPTourLinksDirected is shown in Figure 2.70.

**Figure 2.70** Links in the Optimal Traveling Salesman Tour

<table>
<thead>
<tr>
<th>City Name</th>
<th>City Name</th>
<th>distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Montgomery</td>
<td>Tallahassee</td>
<td>177.14</td>
</tr>
<tr>
<td>Tallahassee</td>
<td>Columbia</td>
<td>311.23</td>
</tr>
<tr>
<td>Columbia</td>
<td>Raleigh</td>
<td>182.99</td>
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<tr>
<td>Raleigh</td>
<td>Richmond</td>
<td>135.58</td>
</tr>
<tr>
<td>Richmond</td>
<td>Washington</td>
<td>97.96</td>
</tr>
<tr>
<td>Washington</td>
<td>Annapolis</td>
<td>27.89</td>
</tr>
<tr>
<td>Annapolis</td>
<td>Dover</td>
<td>54.01</td>
</tr>
<tr>
<td>Dover</td>
<td>Trenton</td>
<td>83.88</td>
</tr>
<tr>
<td>Trenton</td>
<td>Hartford</td>
<td>151.65</td>
</tr>
<tr>
<td>Hartford</td>
<td>Providence</td>
<td>65.56</td>
</tr>
<tr>
<td>Providence</td>
<td>Boston</td>
<td>38.41</td>
</tr>
<tr>
<td>Boston</td>
<td>Concord</td>
<td>66.30</td>
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<tr>
<td>Concord</td>
<td>Augusta</td>
<td>117.36</td>
</tr>
<tr>
<td>Augusta</td>
<td>Montpelier</td>
<td>139.32</td>
</tr>
<tr>
<td>Montpelier</td>
<td>Albany</td>
<td>126.19</td>
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<td>Harrisburg</td>
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<td>Olympia</td>
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<td>191.67</td>
</tr>
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<td>Topeka</td>
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<td>Des Moines</td>
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</tr>
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<td>Springfield</td>
<td>243.02</td>
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<tr>
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<td>Indianapolis</td>
<td>186.46</td>
</tr>
<tr>
<td>Indianapolis</td>
<td>Frankfort</td>
<td>129.90</td>
</tr>
<tr>
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<td>Nashville-Davidson</td>
<td>175.58</td>
</tr>
<tr>
<td>Nashville-Davidson</td>
<td>Atlanta</td>
<td>212.61</td>
</tr>
<tr>
<td>Atlanta</td>
<td>Montgomery</td>
<td>145.39</td>
</tr>
</tbody>
</table>

==========

10,627.75
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